Computational Photonics

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course information available at ilias.studium.kit.edu

Organisational details

Lecture: Tuesday, 2 pm–3.30 pm, weekly, 30.22 Physik-Hörsaal Nr. 4 (Kl. HS B)

Tutorial: To be discussed in the first lecture

bi-weekly (extended exercises will be 2 x 90 minutes every second week)

- start with a tutorial in second lecture week with an introduction to Python
- scientific tutorials start in the third lecture week
- Nebenfach: send in afterwards your programs
- tutorial material available on ILIAS as well

Examination

- Register for tutorials (also called units) in CAMPUS
- Depending on context in which you take the course
    - KSOP students: participation in tutorial + oral exam
    - Schwerpunktfach: participation in tutorials
    - Ergänzungsfach: participation in tutorials + oral exam
    - Nebenfach: 50% of problems solved

Questions

- Organisation
- Lectures
- Tutorials
- Registration
- Grading

What is computational photonics?

- rather unspecific name
- different levels of abstraction:
  - ✳ ray optics
  - ✳ wave optics
  - ✳ EM optics
  - ✳ quantum optics

electromagnetic field-based approach on optics

focus on numerical methods for PDEs

research-oriented approach

Why computational photonics?

- its a numerical experiment
- provides insides to inaccessible domains
- permits to interpret / understand experiments
- simplifies the design
- explores applications not realisable

with nowadays computational resources computational photonics became an inevitable tool for micro- and nanooptics

Prerequisites

- good basic knowledge in MATLAB or Python (this is not a basic programming course!)
- solid mathematical background in PDE
- knowledge about electromagnetic field theory (Bachelor level Electrodynamics or Master level Fundamentals of modern optics)
- knowledge about numerical basic techniques on a Bachelor level (see 1st seminar & additional material)

Topics to be discussed

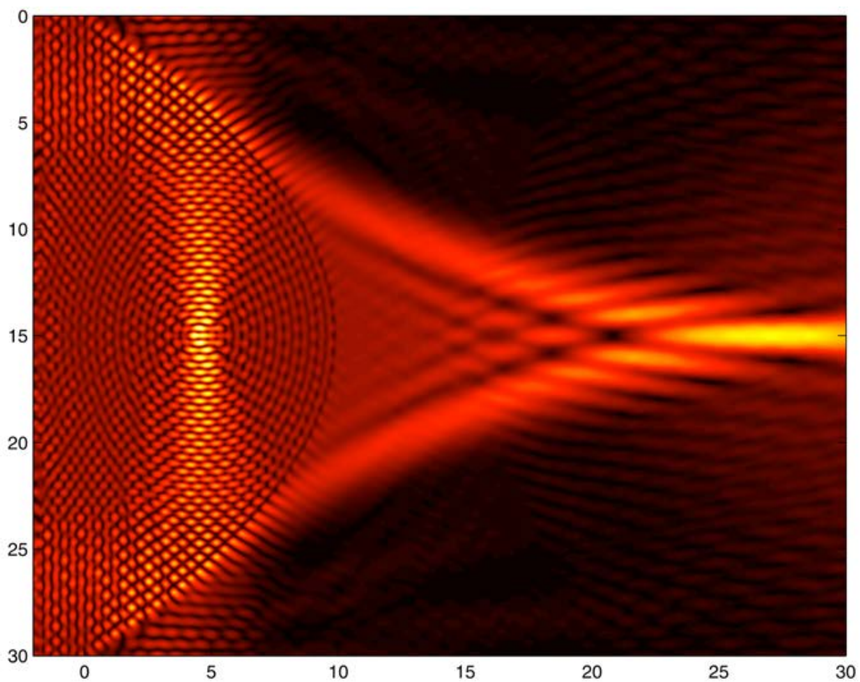
- Transfer Matrix Method
- Finite Differences for Waveguides
- Fiber Waveguides
- Beam propagation method
- Grating methods
- Mie Theory
- Finite-Difference Time-Domain
- Multiple Multipole Method
- Greens Methods
- Boundary Element Method
- Finite Element Method

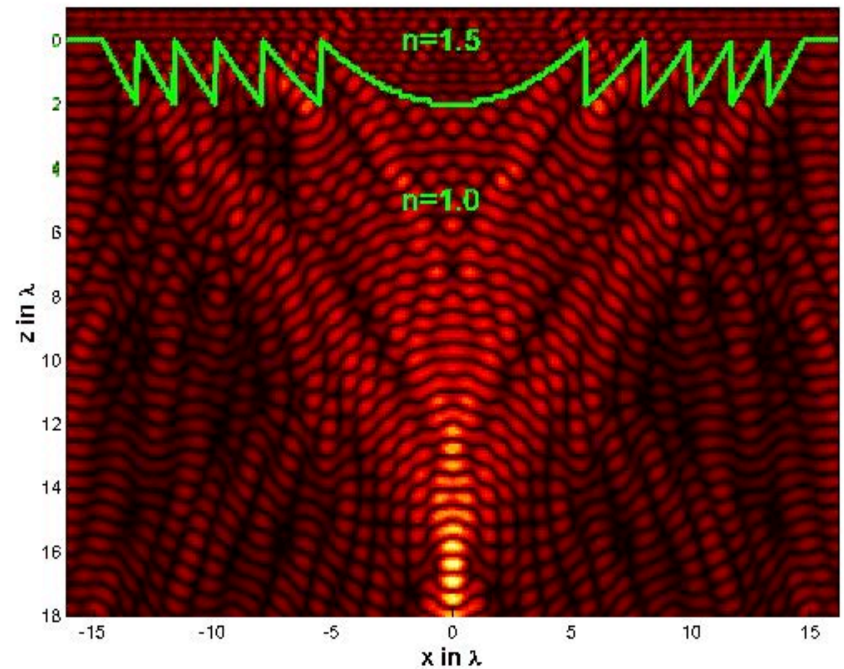
along with selected applications and demonstrations

Computational Photonics

Examples of applications

Transmission functions of a lens

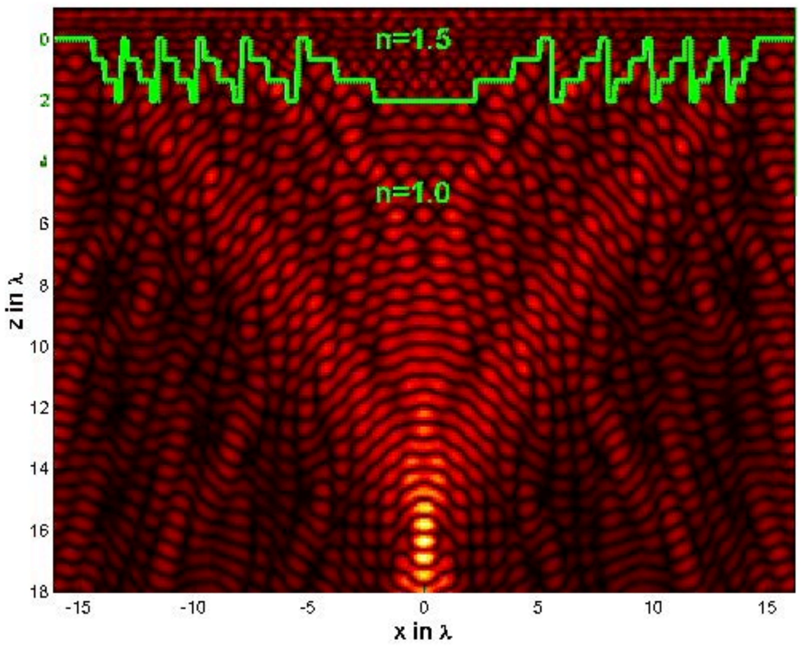


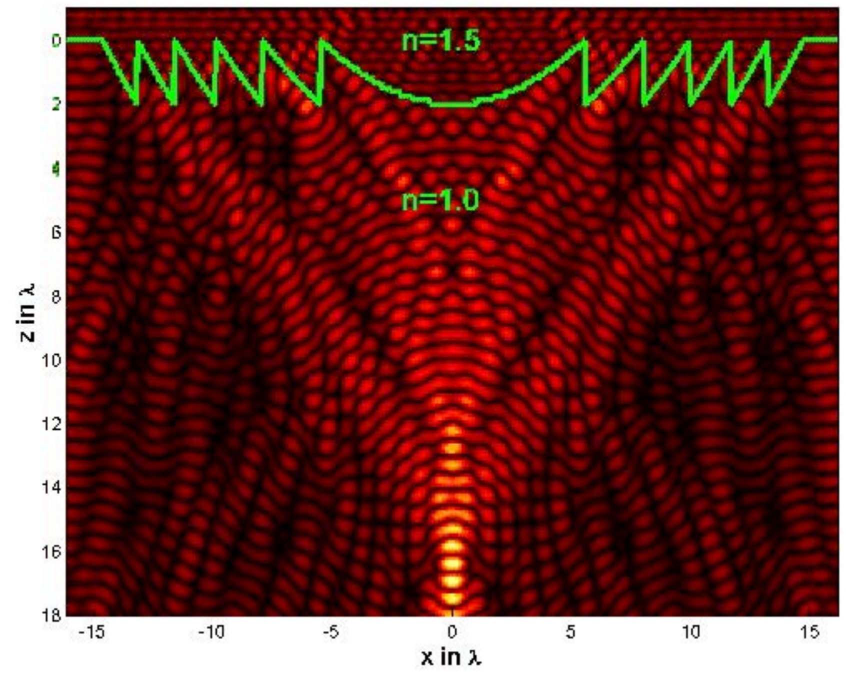


rigorously calculated field focusing properties of a lens

in scalar theory no difference between lens and Fresnel lens!

Transmission functions of a lens

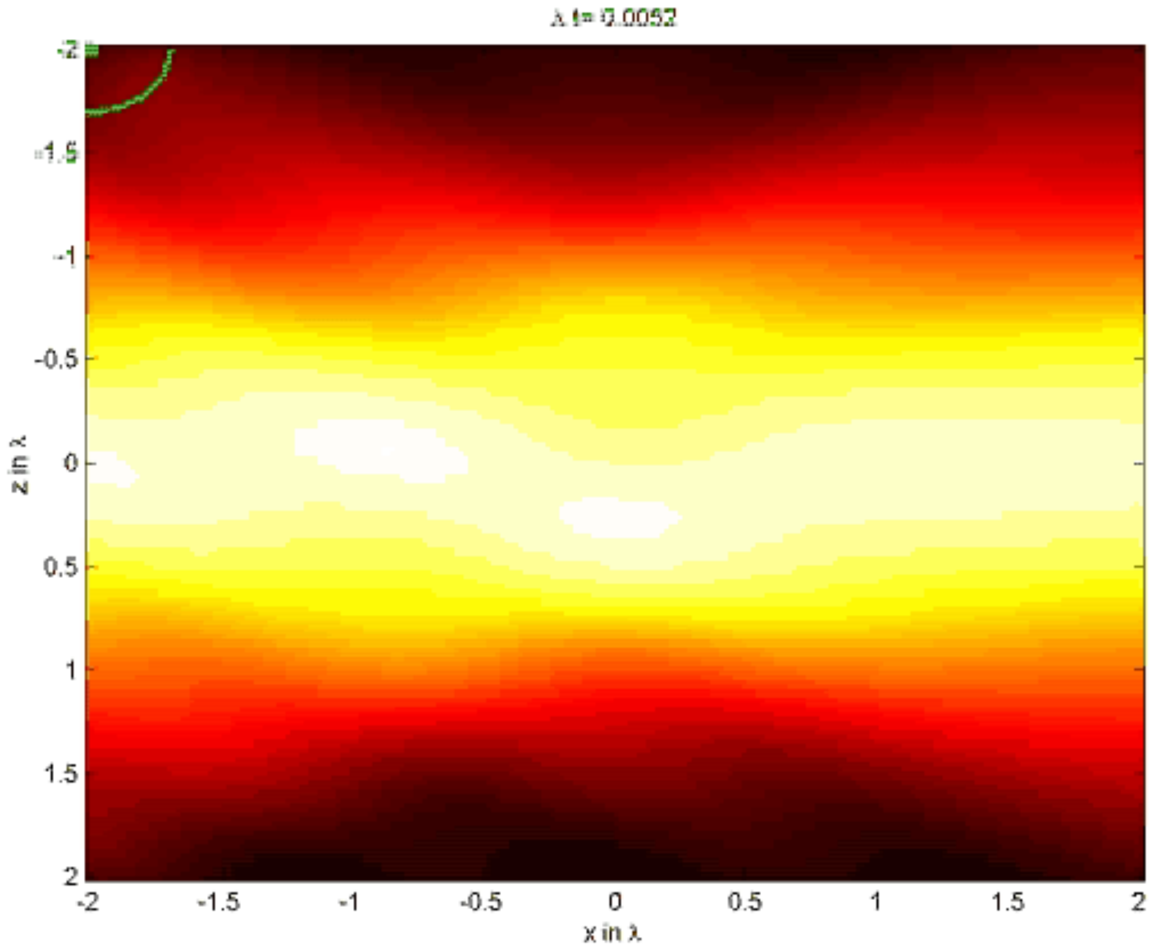




rigorously calculated field focusing properties of a lens

Fresnel lenses can be also stepwise continuously approximated

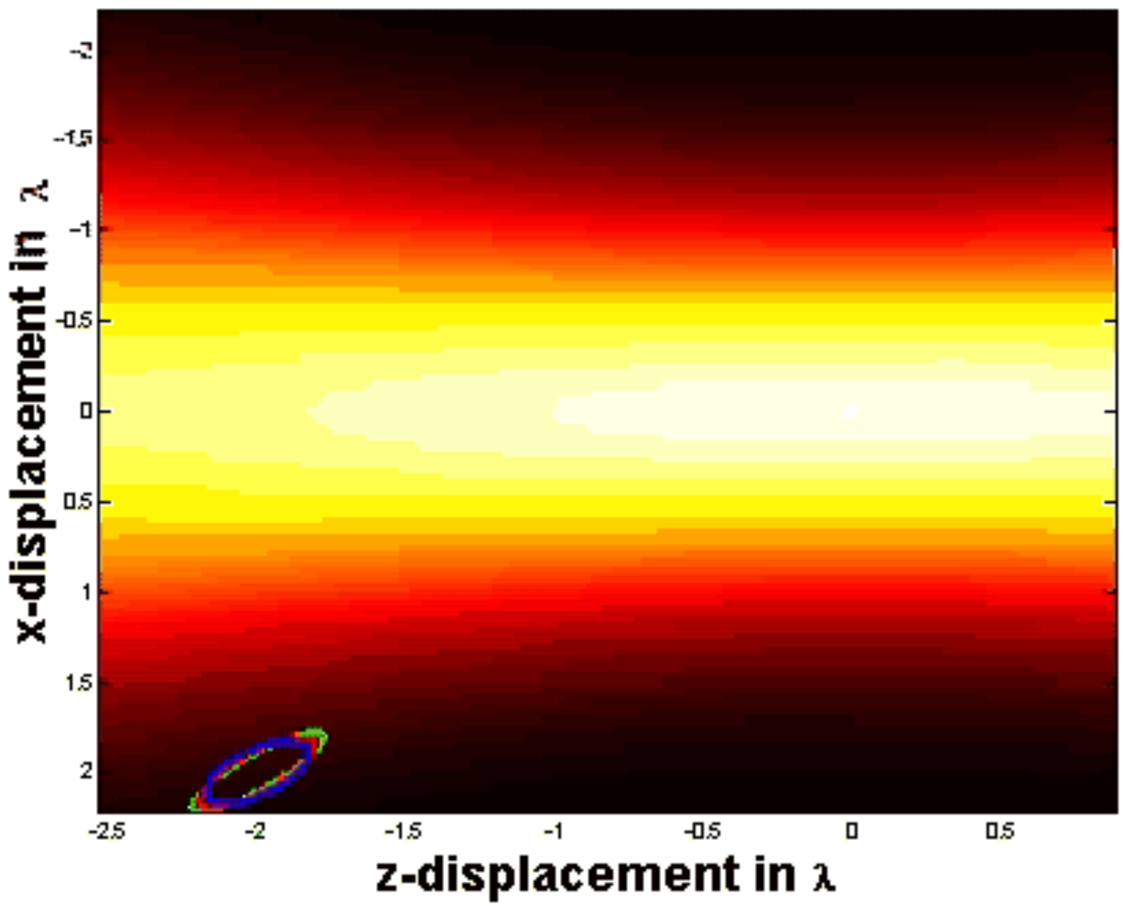
Trapping a particle in a light beam



an optical tweezer can be used to trap particles

scattering strength of the particle is too large ⟶ no trapping

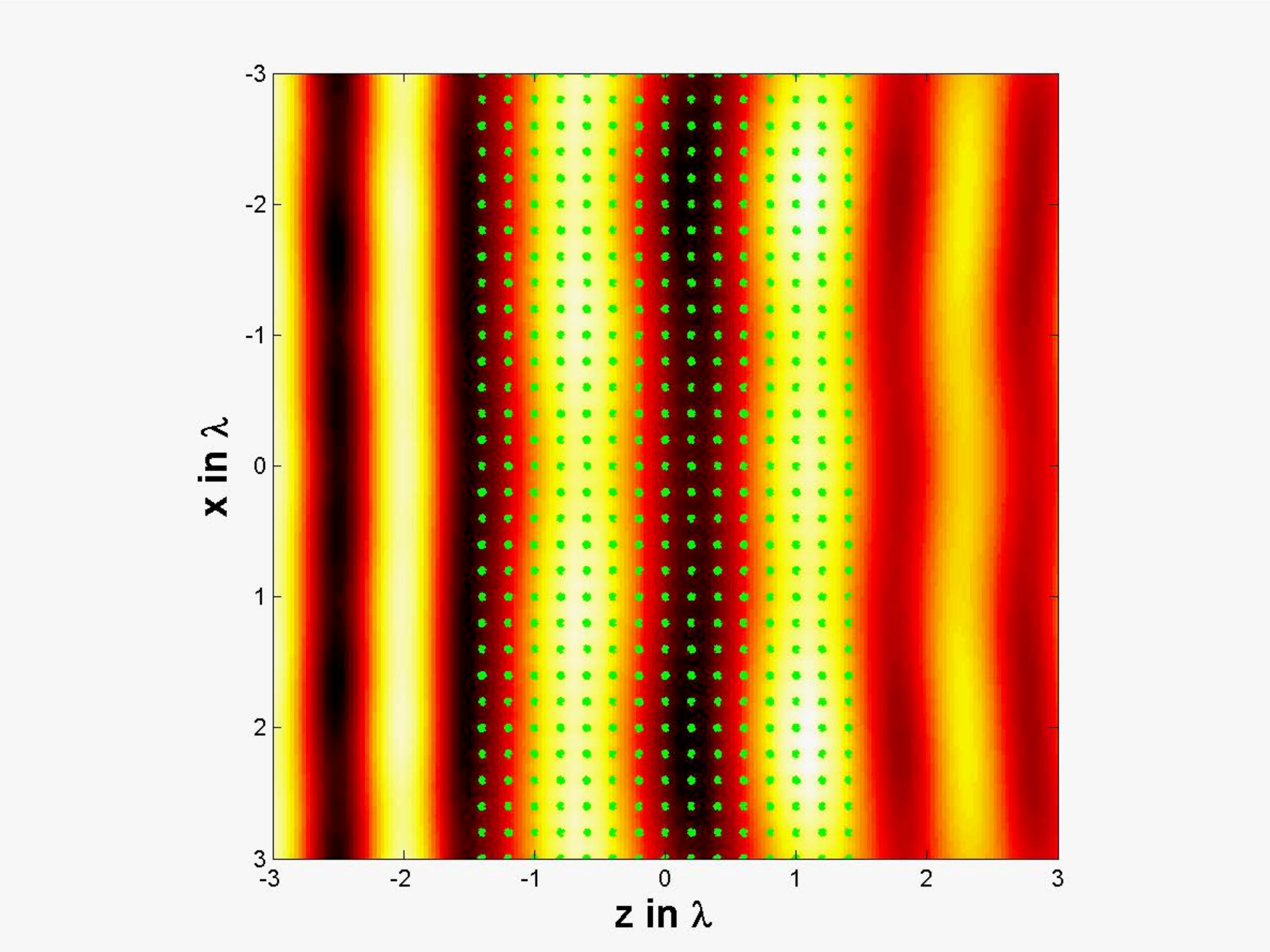
Trapping a particle in a light beam



an optical tweezer can be used to trap particles

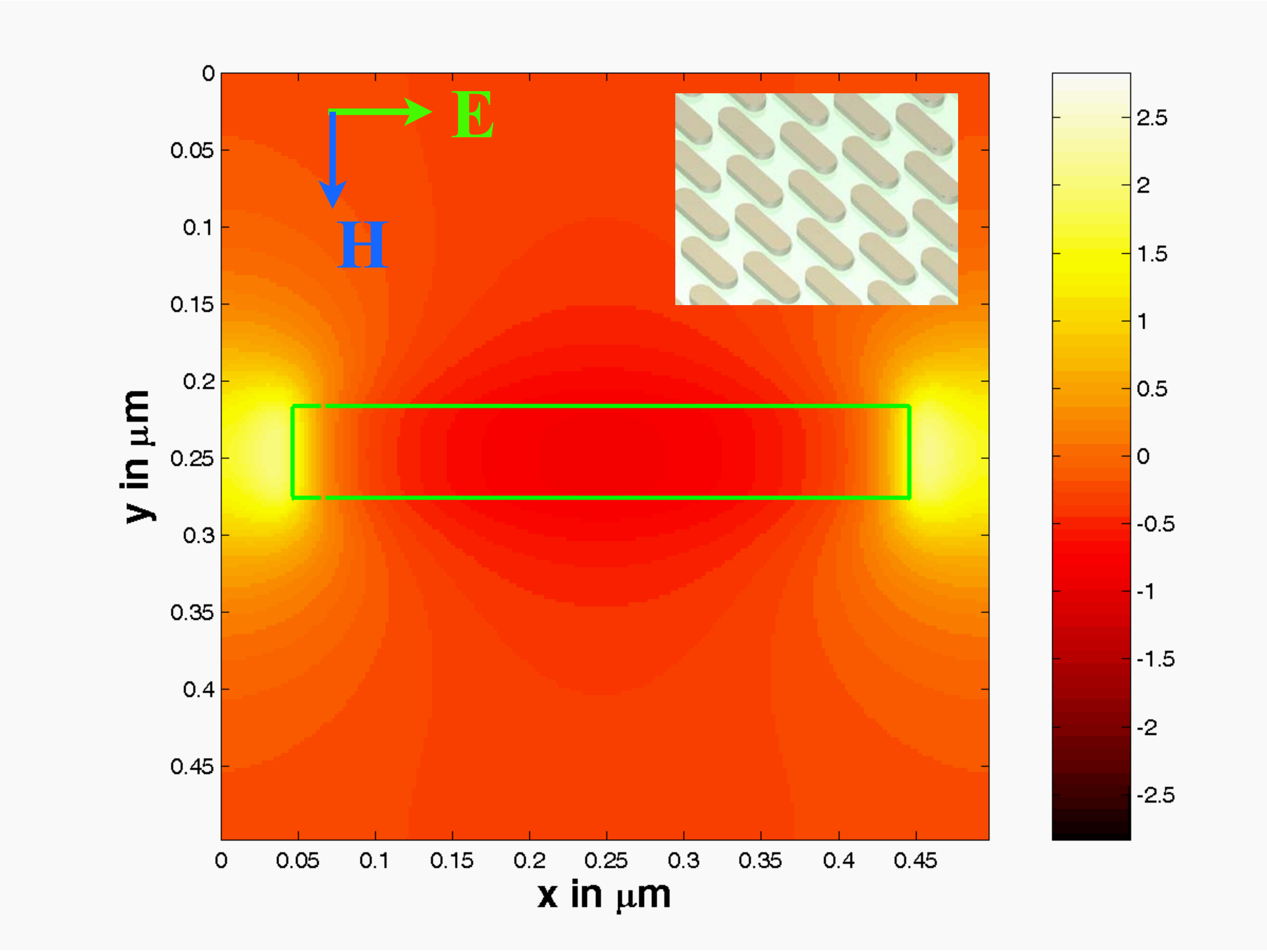
scattering strength of the particle is too large → no trapping

Metamaterials



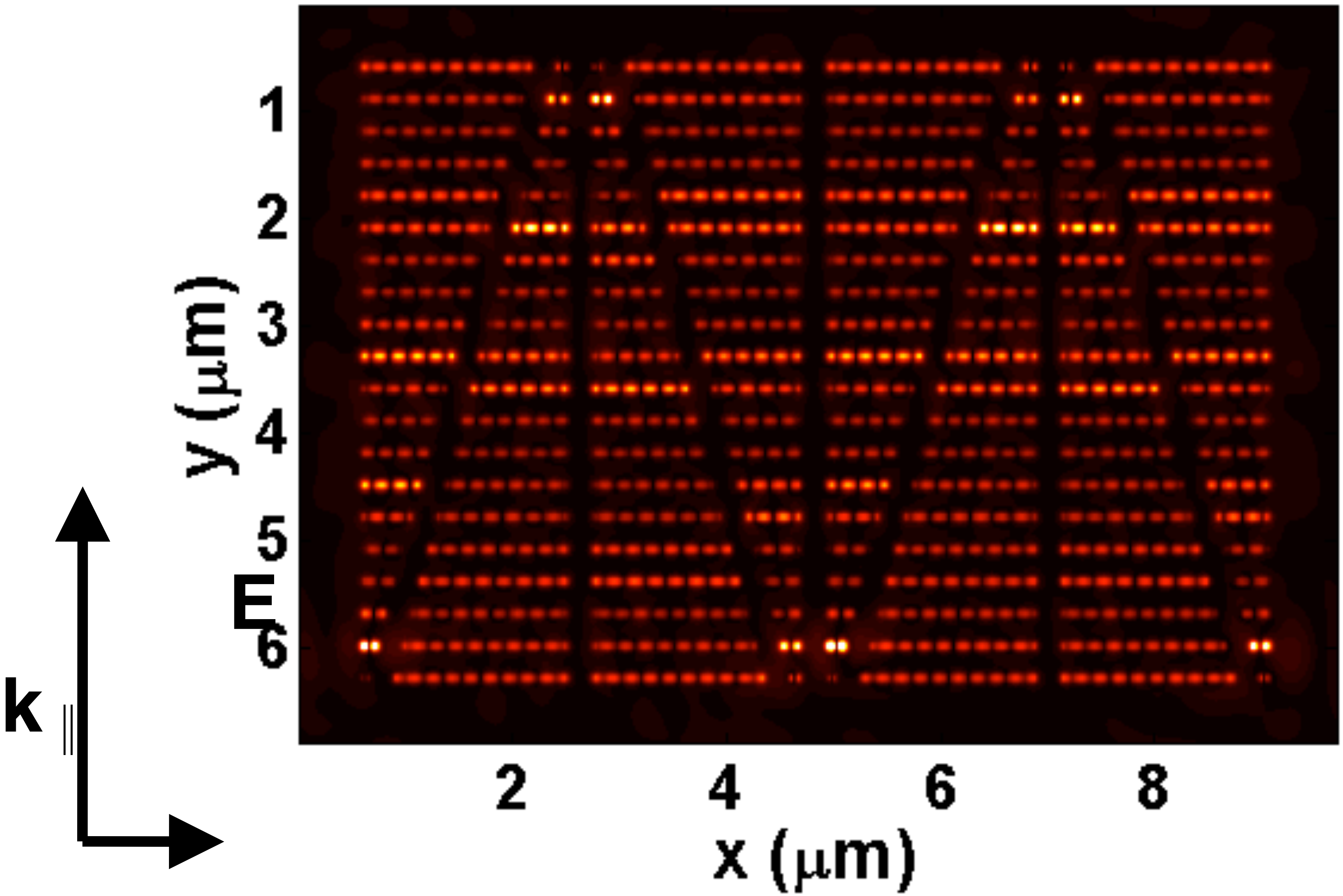
having a media with a refractive index smaller than unity

Metamaterials

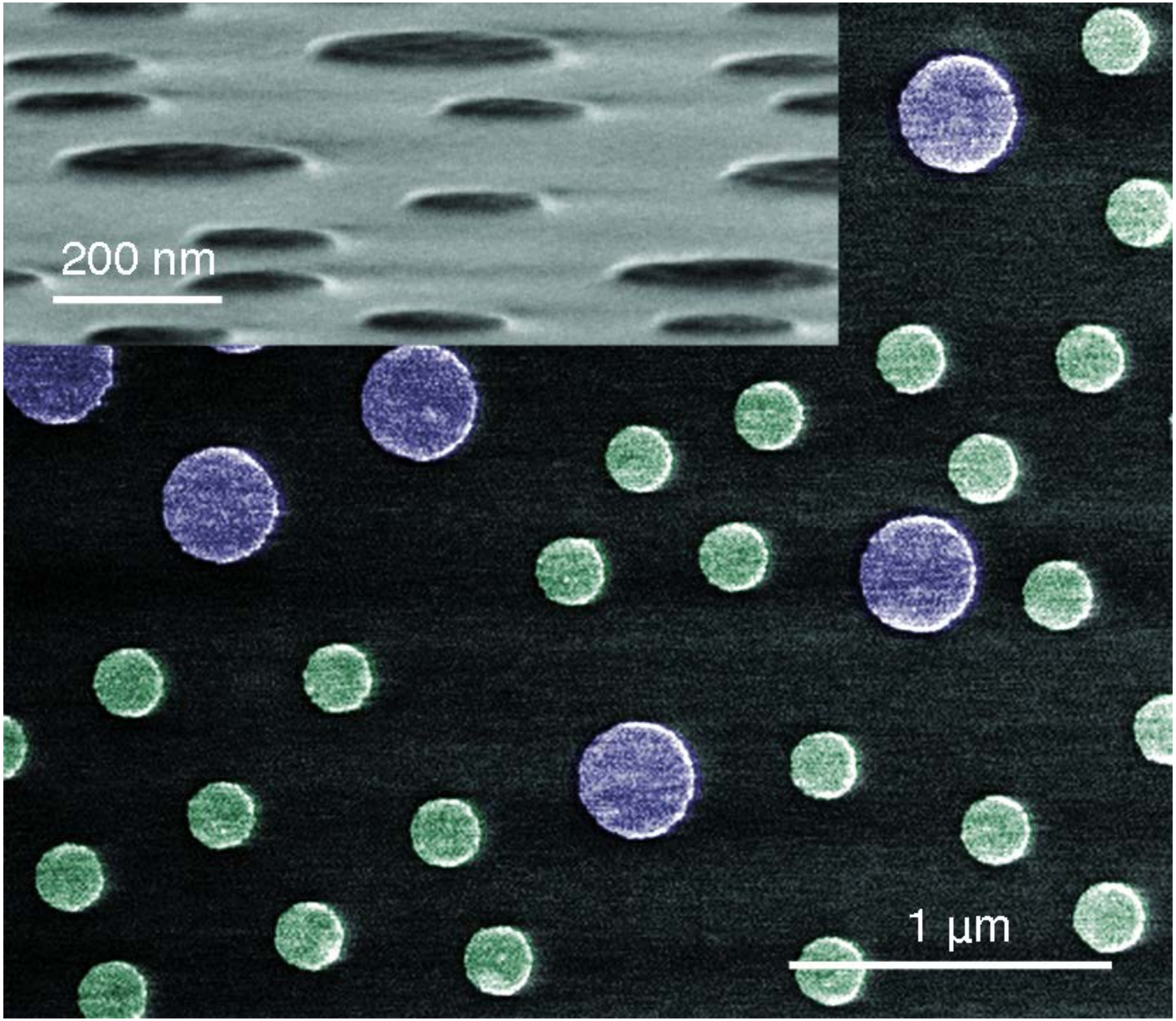


localized plasmon polariton excited in a metallic nanoparticle

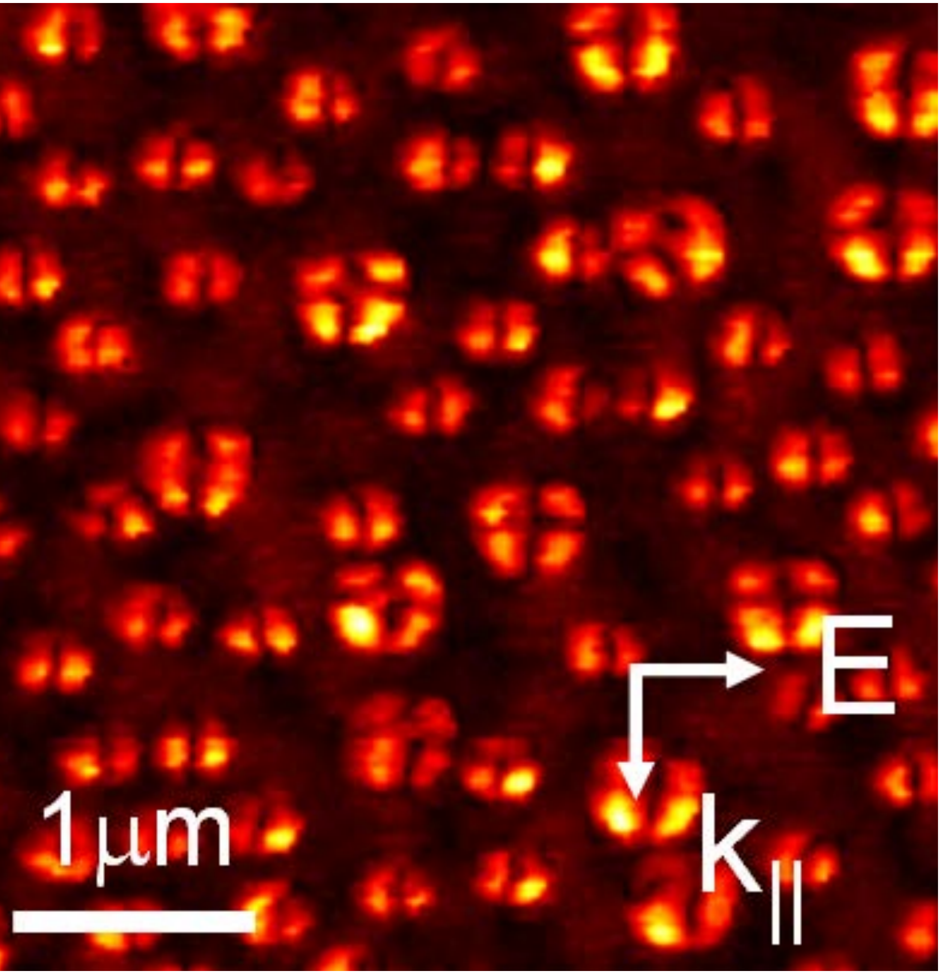
Optical nanoantennas



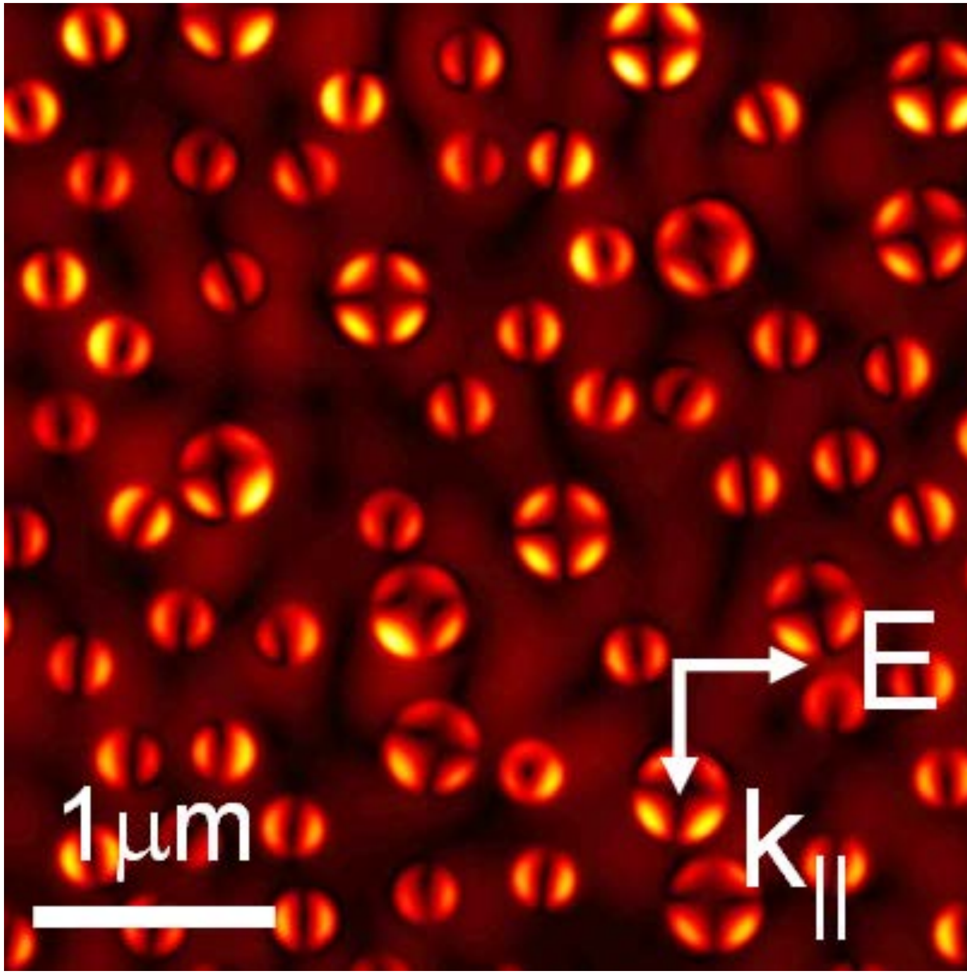
Optical nanoantennas



Optical nanoantennas

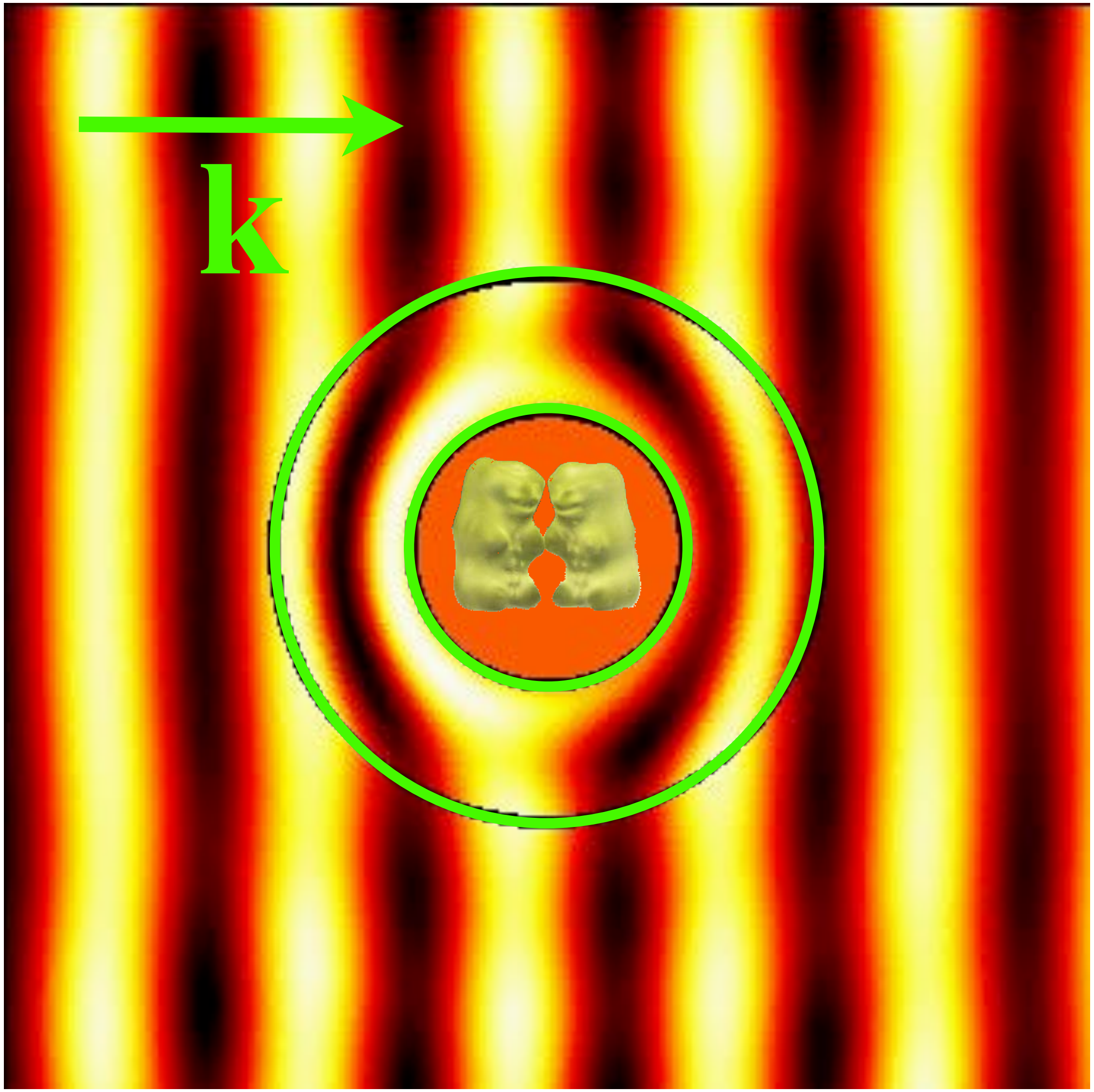


experiment

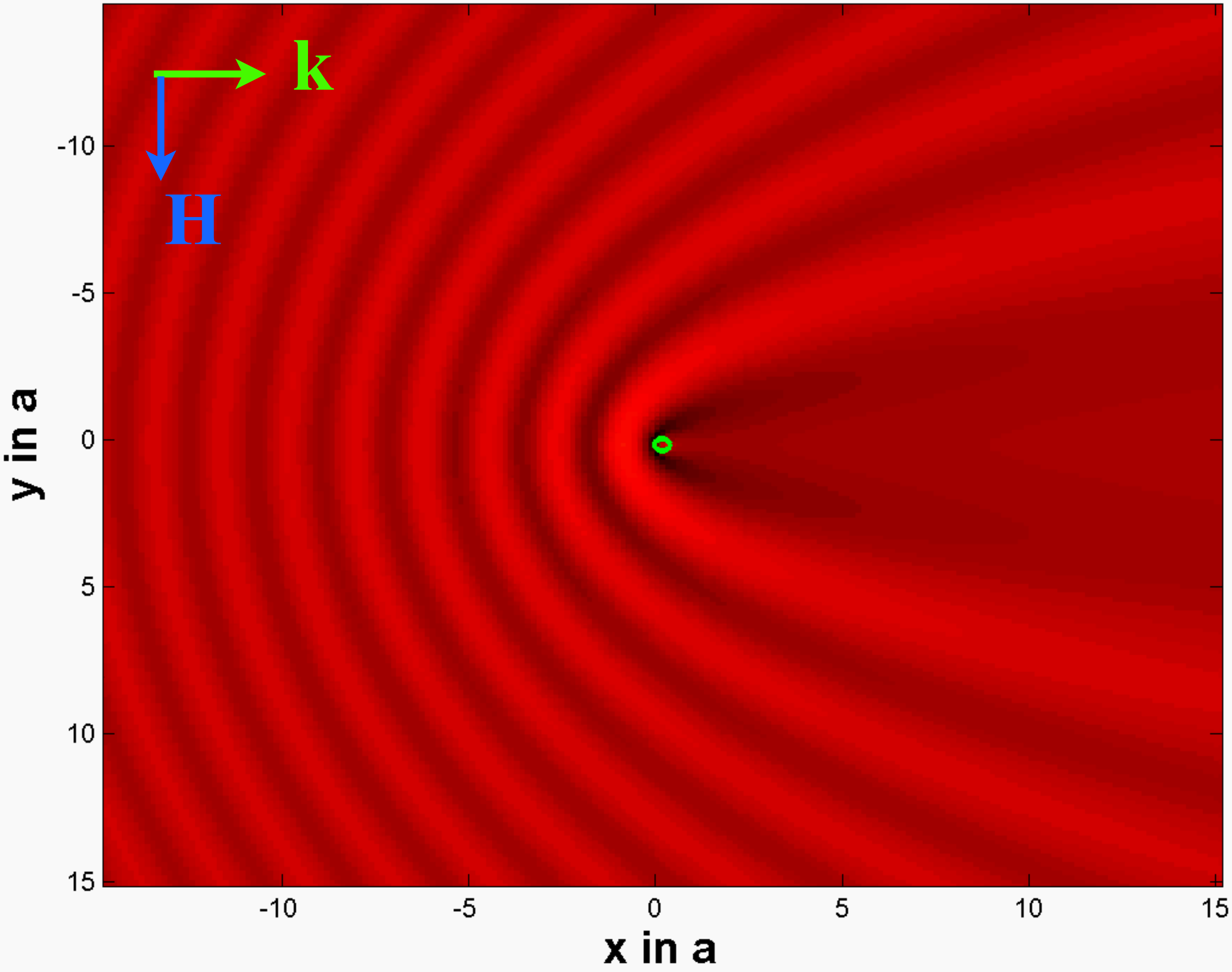


simulation

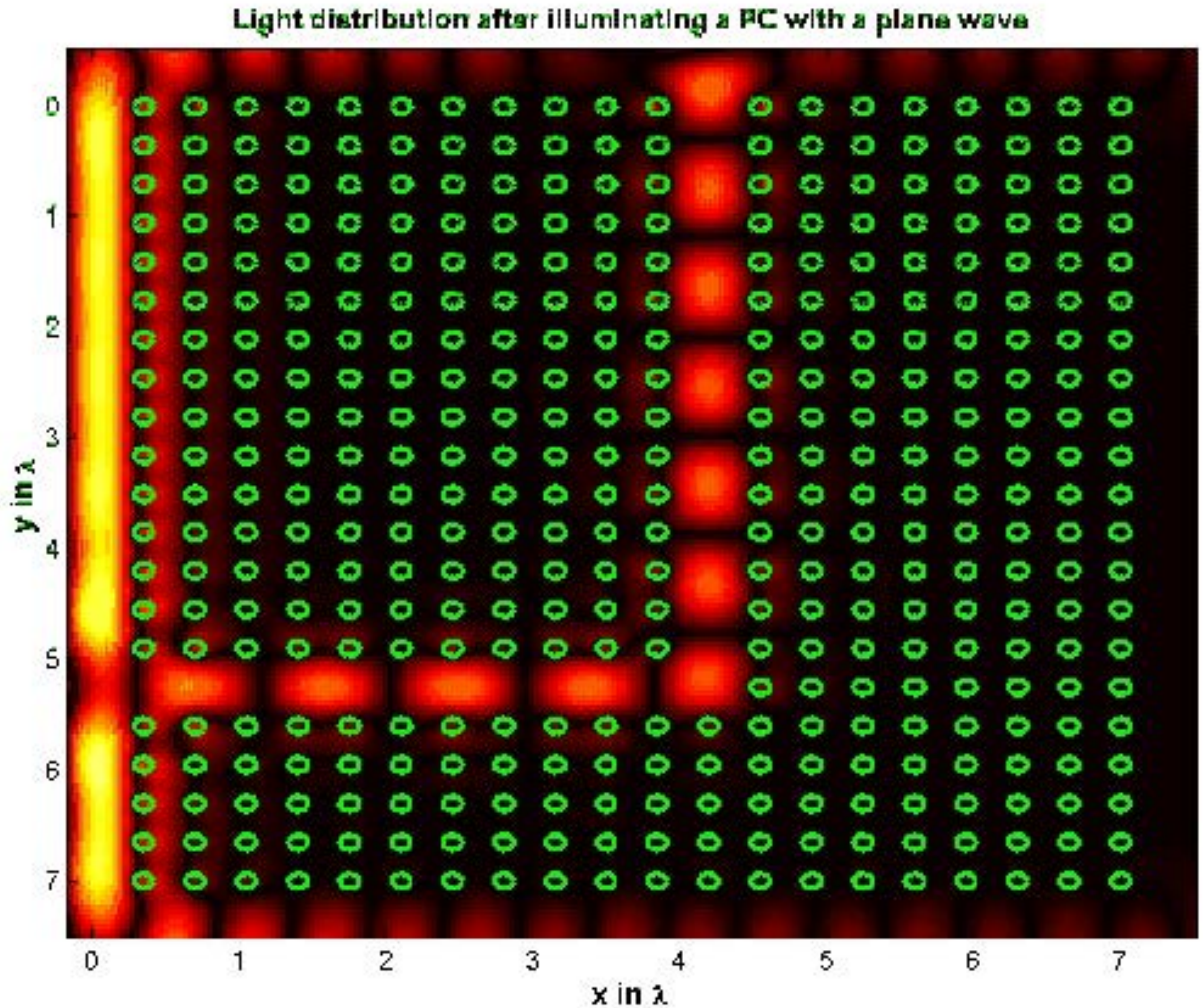
Transformation optics



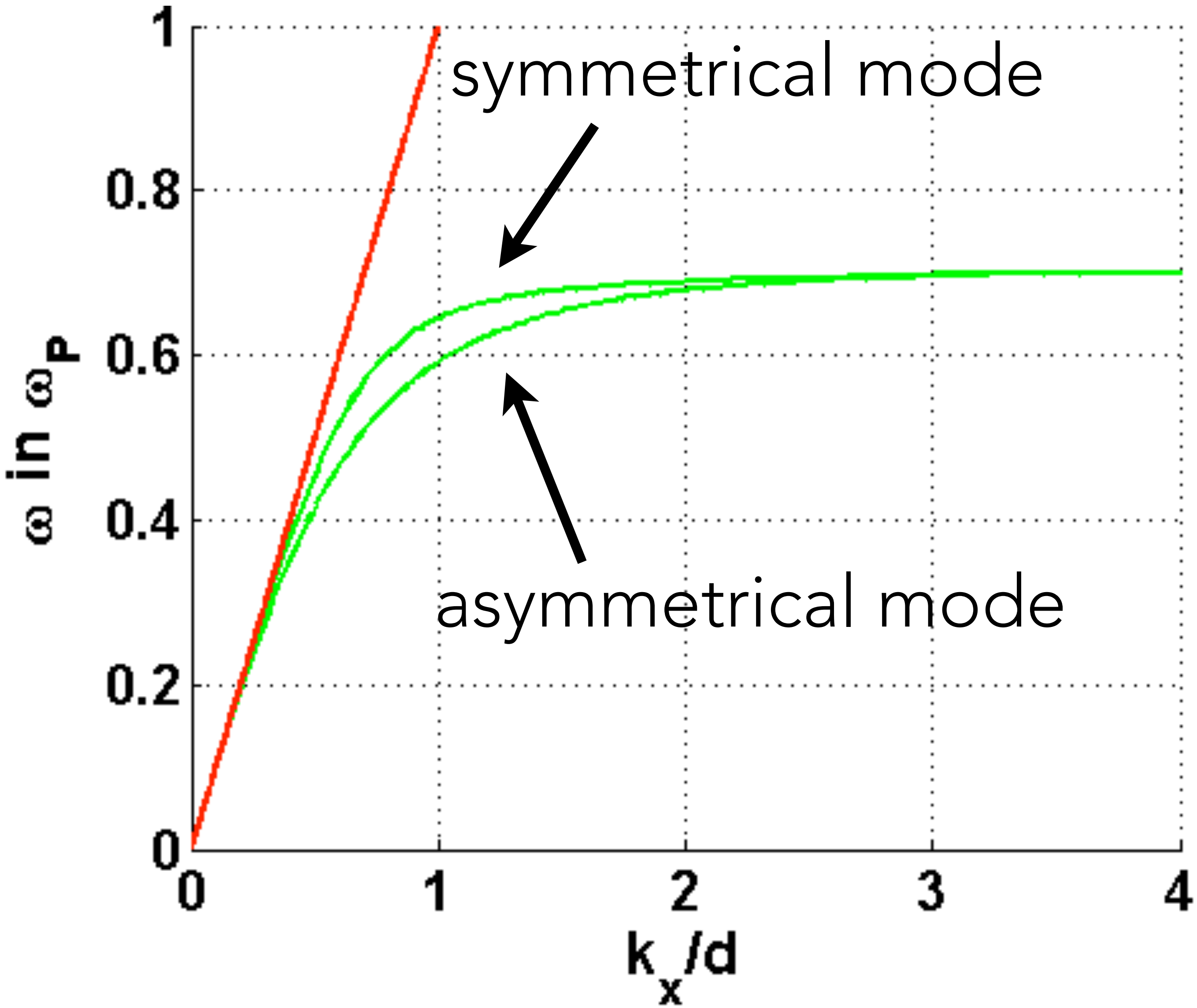
Multiple particle scattering



Photonic crystals

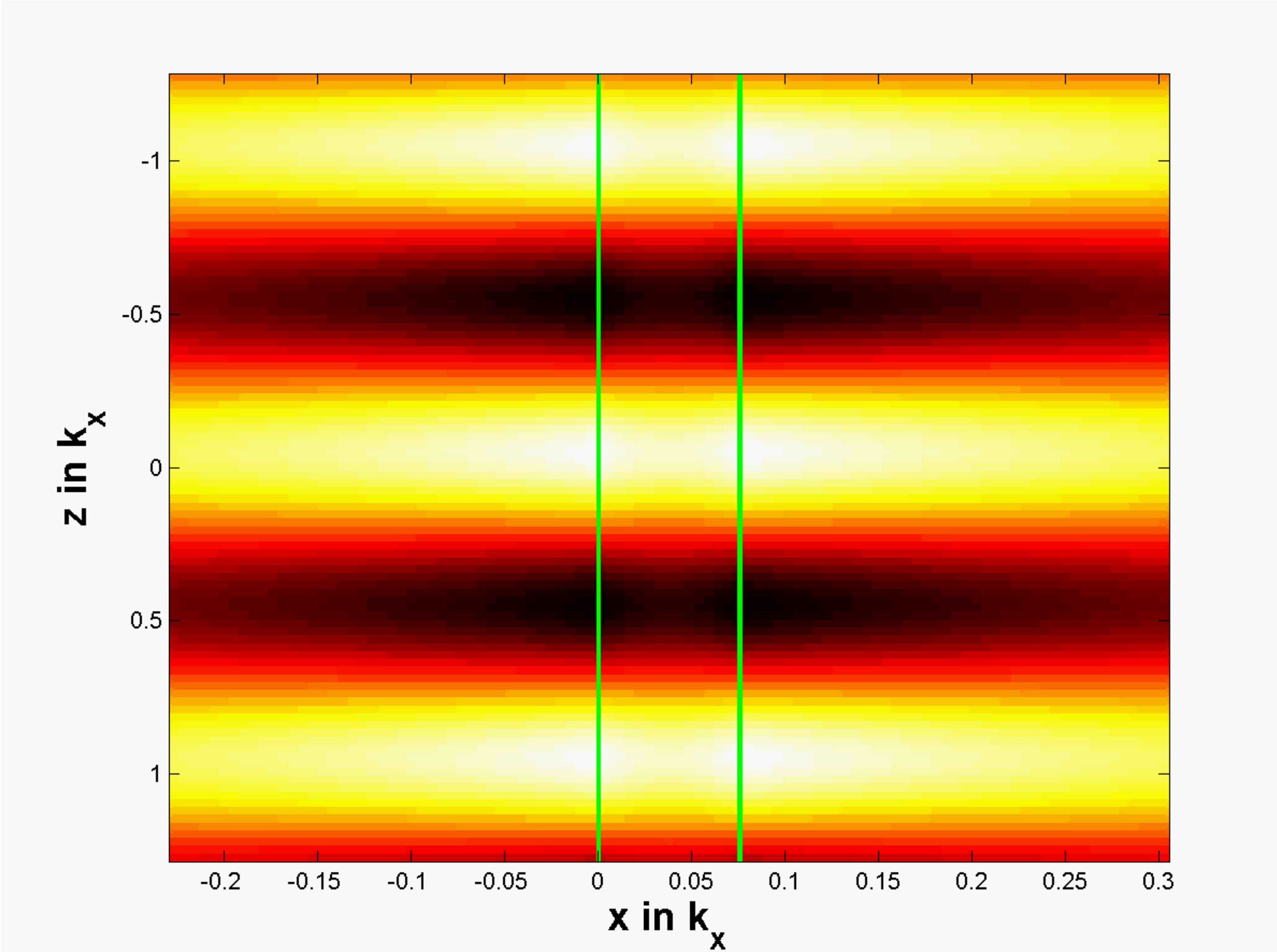


Guided modes



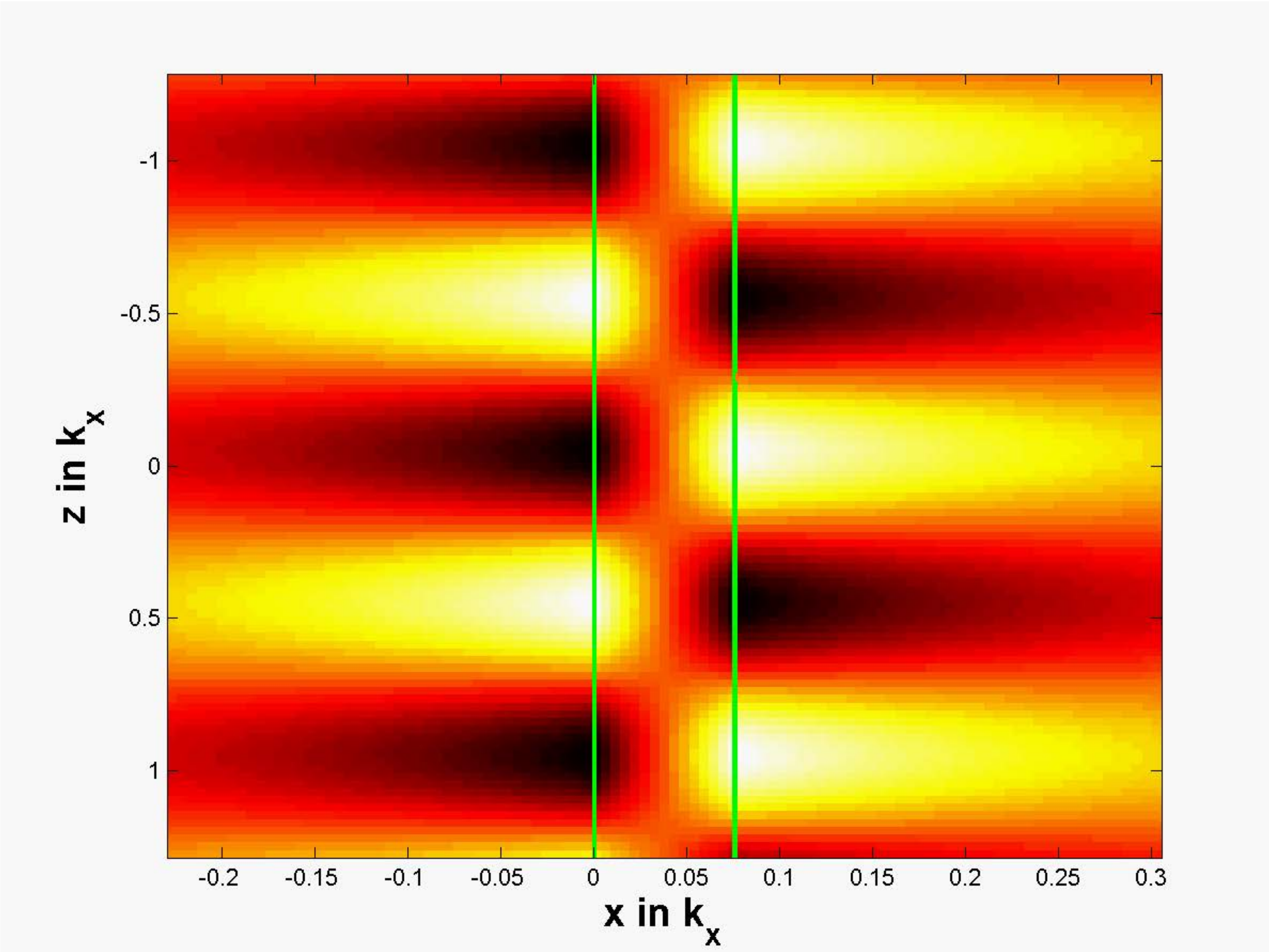
dispersion relation and eigenmodes of a metallic film

Guided modes



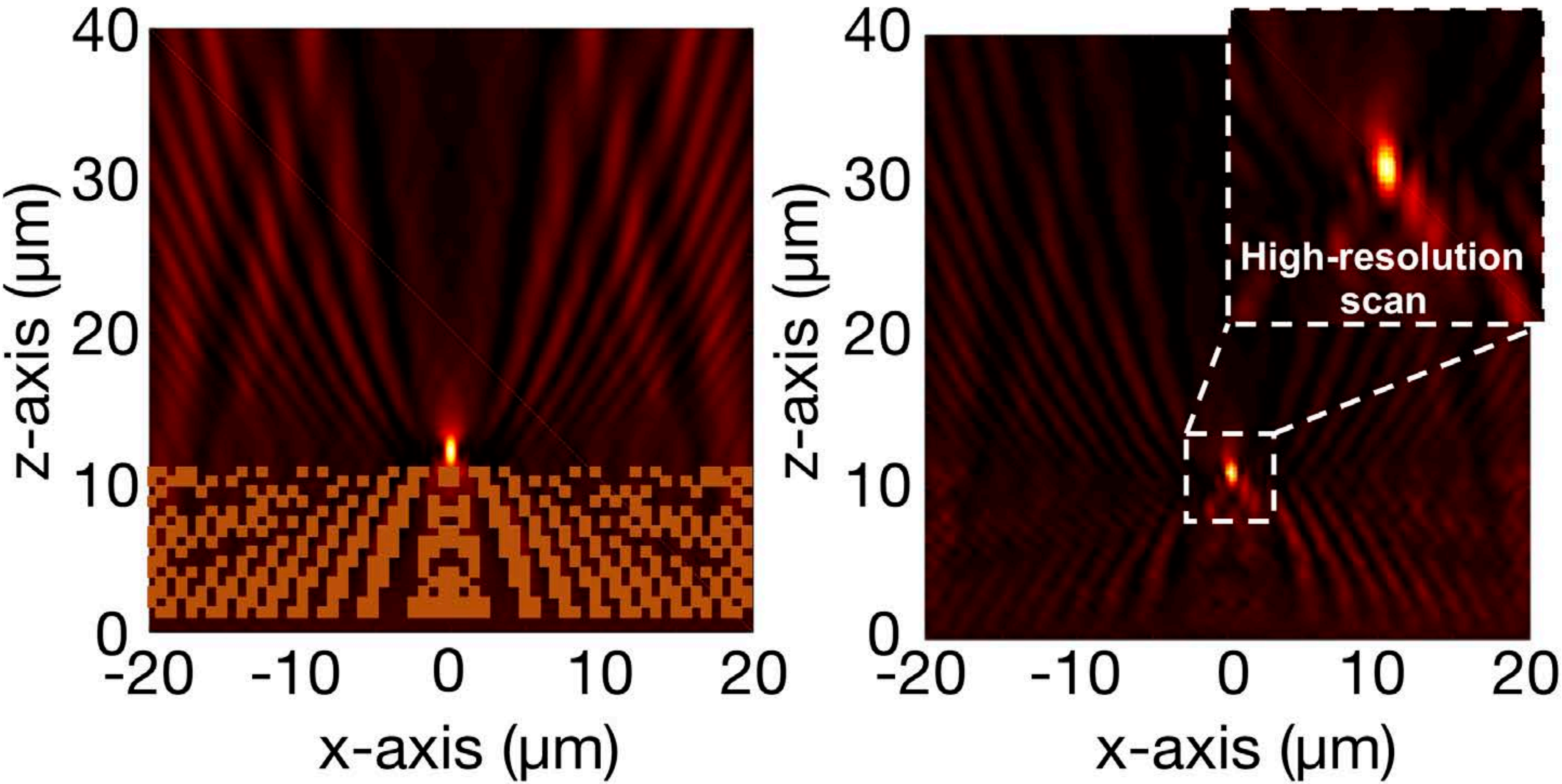
symmetric mode

Guided modes



anti-symmetric mode

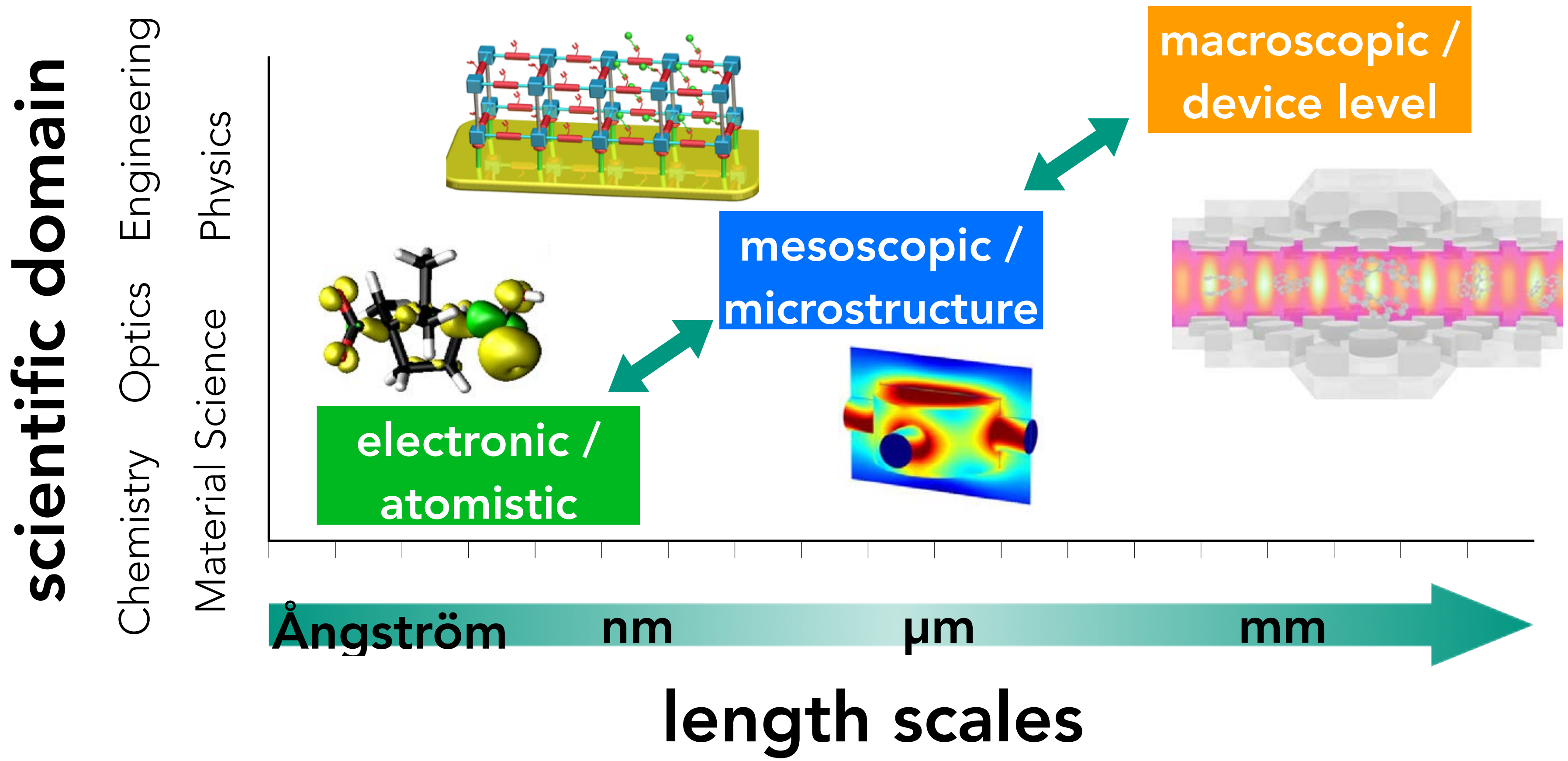
Inverse design of functional elements



focusing Bloch surface waves into tiny spots

Inverse design of functional elements

Multiscale modelling



Computational Photonics

Basics of Maxwell's equations

Maxwell's equations in time domain

$$\mathbf{rot}\mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \qquad \mathbf{div}\mathbf{D}(\mathbf{r},t) = \rho_{\mathrm{ext}}(\mathbf{r},t)$$

$$\mathbf{rot}\mathbf{H}(\mathbf{r},t) = \mathbf{j}_{\mathrm{makr}}(\mathbf{r},t) + \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} \qquad \mathbf{div}\mathbf{B}(\mathbf{r},t) = 0$$

$\mathbf{E}(\mathbf{r},t)$	electric field	$[\mathrm{Vm}^{-1}]$
$\mathbf{H}(\mathbf{r},t)$	magnetic field	$[\mathrm{Am}^{-1}]$
$\mathbf{D}(\mathbf{r},t)$	electric displacement field	$[\mathrm{Asm}^{-2}]$
$\mathbf{B}(\mathbf{r},t)$	magnetic induction	$[\mathrm{Vsm}^{-2}]$
$\rho_{\mathrm{ext}}(\mathbf{r},t)$	external charge density	$[\mathrm{Asm}^{-3}]$
$\mathbf{j}_{\mathrm{makr}}(\mathbf{r},t)$	macroscopic current density	$[\mathrm{Am}^{-2}]$

Material equations in time domain

$$\mathbf{D}(\mathbf{r},t) = \epsilon_0 \mathbf{E}(\mathbf{r},t) + \mathbf{P}(\mathbf{r},t)$$

$$\mathbf{B}(\mathbf{r},t) = \mu_0 \mathbf{H}(\mathbf{r},t) + \mathbf{M}(\mathbf{r},t)$$

$\mathbf{P}(\mathbf{r},t)$ electric polarisation $[\mathrm{Asm}^{-2}]$

$\mathbf{M}(\mathbf{r},t)$ magnetic polarisation $[\mathrm{Vsm}^{-2}]$

ϵ_0 permittivity of vacuum

$$\epsilon_0 = \left(\mu_0 c_0^2\right)^{-1} \approx 8.85 \cdot 10^{-12} \mathrm{As/Vm}$$

μ_0 permeability of vacuum

$$\mu_0 = 4\pi \cdot 10^{-12} \mathrm{Vs/Am}$$

$$\mathbf{P}(\mathbf{r},t) = \epsilon_0 \int_0^\infty R(\mathbf{r},t')\mathbf{E}(\mathbf{r},t-t')dt'$$

linear,local,isotropic medium

$$\mathbf{M}(\mathbf{r},t) = 0$$

usually in optics

Maxwell's equations in frequency domain

using Fourier transformation to transform into frequency space

$$\mathbf{V}(\mathbf{r},t) = \int_{-\infty}^{\infty} \tilde{\mathbf{V}}(\mathbf{r},\omega) e^{-i\omega t} d\omega \qquad \tilde{\mathbf{V}}(\mathbf{r},\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{V}(\mathbf{r},t) e^{i\omega t} dt$$

Maxwell's equations in frequency domain by

$$\frac{\partial}{\partial t} \xrightarrow{\mathrm{FT}} -i\omega$$

$$\mathbf{rot}\tilde{\mathbf{E}}(\mathbf{r},\omega) = i\omega\tilde{\mathbf{B}}(\mathbf{r},\omega) \qquad \mathbf{rot}\tilde{\mathbf{H}}(\mathbf{r},\omega) = -i\omega\tilde{\mathbf{D}}(\mathbf{r},\omega)$$

$$\mathbf{div}\tilde{\mathbf{D}}(\mathbf{r},\omega) = \rho(\mathbf{r},\omega) \qquad \mathbf{div}\tilde{\mathbf{B}}(\mathbf{r},\omega) = 0$$

Matter equations in frequency domain

defining equation for the material's susceptibility

$$\chi(\mathbf{r},\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(\mathbf{r},t)e^{i\omega t}dt$$

linked to the dielectric function

$$\epsilon(\mathbf{r},\omega) = 1 + \chi(\mathbf{r},\omega)$$

$$\tilde{\mathbf{P}}(\mathbf{r},\omega) = \epsilon_0\chi(\mathbf{r},\omega)\tilde{\mathbf{E}}(\mathbf{r},\omega) \quad \tilde{\mathbf{D}}(\mathbf{r},\omega) = \epsilon_0\epsilon(\mathbf{r},\omega)\tilde{\mathbf{E}}(\mathbf{r},\omega)$$

$$\tilde{\mathbf{M}}(\mathbf{r},\omega) = 0 \quad \tilde{\mathbf{B}}(\mathbf{r},\omega) = \mu_0\tilde{\mathbf{H}}(\mathbf{r},\omega)$$

please note; I will largely skip the tilde to keep the notation more light but it should be clear from the arguments what is meant

Wave equations for different fields

e.g. for the electric field:

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r},\omega) = \omega^2 \epsilon_0 \mu_0 \epsilon(\mathbf{r},\omega) \mathbf{E}(\mathbf{r},\omega)$$

e.g. for the electric displacement:

$$\nabla \times \nabla \times \frac{1}{\epsilon_0 \epsilon(\mathbf{r},\omega)} \mathbf{D}(\mathbf{r},\omega) = \omega^2 \mu_0 \mathbf{D}(\mathbf{r},\omega)$$

e.g. for the magnetic field:

$$\nabla \times \frac{1}{\epsilon_0 \epsilon(\mathbf{r},\omega)} \nabla \times \mathbf{H}(\mathbf{r},\omega) = \omega^2 \mu_0 \mathbf{H}(\mathbf{r},\omega)$$

please note, the equations are eigenvalue problems

Special case homogenous space $\epsilon(\mathbf{r},\omega) = \epsilon(\omega)$

divergence of the electric field vanishes

$$\nabla \cdot \epsilon_0 \epsilon(\omega) \mathbf{E}(\mathbf{r},\omega) = \epsilon_0 \epsilon(\omega) \nabla \cdot \mathbf{E}(\mathbf{r},\omega) = 0$$

simplifying the wave equation with

$$\nabla \times \nabla \times = \nabla \left[\nabla \cdot \right] - \triangle$$

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r},\omega) = \nabla \left[\nabla \cdot \mathbf{E}(\mathbf{r},\omega) \right] - \triangle \mathbf{E}(\mathbf{r},\omega) = - \triangle \mathbf{E}(\mathbf{r},\omega)$$

Helmholtz equation from the eigenvalue problem

$$\triangle \mathbf{E}(\mathbf{r},\omega) + \frac{\omega^2}{c^2} \epsilon(\omega) \mathbf{E}(\mathbf{r},\omega) = 0 \qquad c^2 = \frac{1}{\epsilon_0 \mu_0}$$

Solution to the wave equation

many solutions are possible (e.g.depending on the coordinate system)

most prominent is plane wave: $$\mathbf{E}(\mathbf{r},\omega) = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}}$$

(please note, the actual electric field in real space also has an exponential time dependency)

amplitude vector: $$\mathbf{E_0} = \begin{bmatrix} E_x^0 \\ E_y^0 \\ E_z^0 \end{bmatrix}$$ wave vector: $$\mathbf{k} = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

all six parameters are free to a certain extent

⇨ restrictions are imposed by side constraints $$\epsilon_0\epsilon(\mathbf{r},\omega)\nabla\cdot\mathbf{E}(\mathbf{r},\omega) = 0$$

Solution to the wave equation

plug this ansatz into the eigenvalue problem to determine the free parameters

$$\nabla \cdot \mathbf{E}(\mathbf{r},\omega) = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \cdot \begin{bmatrix} E_x^0 \\ E_y^0 \\ E_z^0 \end{bmatrix} e^{i(k_x x + k_y y + k_z z)} = i\mathbf{k} \cdot \mathbf{E}(\mathbf{r},\omega)$$

$$\nabla \cdot (\nabla \cdot \mathbf{E}(\mathbf{r},\omega)) = \triangle \mathbf{E}(\mathbf{r},\omega) = -\mathbf{k}^2 \cdot \mathbf{E}(\mathbf{r},\omega)$$

$$\triangle \mathbf{E}(\mathbf{r},\omega) + \frac{\omega^2}{c^2}\epsilon(\omega)\mathbf{E}(\mathbf{r},\omega) = 0$$

$$\mathbf{k}^2 = \frac{\omega^2}{c^2}\epsilon(\omega)$$

dispersion relation of free space

Types of solution

$$k_z = \pm\sqrt{\frac{\omega^2}{c^2}\epsilon(\omega) - k_x^2 - k_y^2}$$

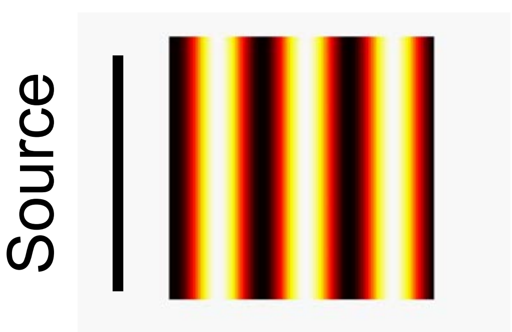
$$\frac{\omega^2}{c^2}\epsilon(\omega) > k_x^2 + k_y^2$$

propagating wave

$$k_z = \sqrt{\frac{\omega^2}{c^2}\epsilon(\omega) - k_x^2 - k_y^2}$$

sign dictates forward / backward wave

$$\mathbf{E_0} e^{i(\mathbf{k}_\perp \cdot \mathbf{r}_\perp)} e^{-i\omega t} e^{ik_z z}$$



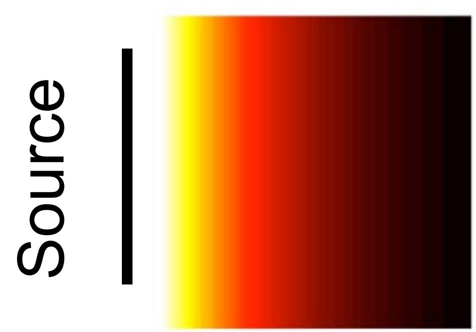
$$\frac{\omega^2}{c^2}\epsilon(\omega) < k_x^2 + k_y^2$$

evanescent wave

$$ik_z = \gamma = -\sqrt{k_x^2 + k_y^2 - \frac{\omega^2}{c^2}\epsilon(\omega)}$$

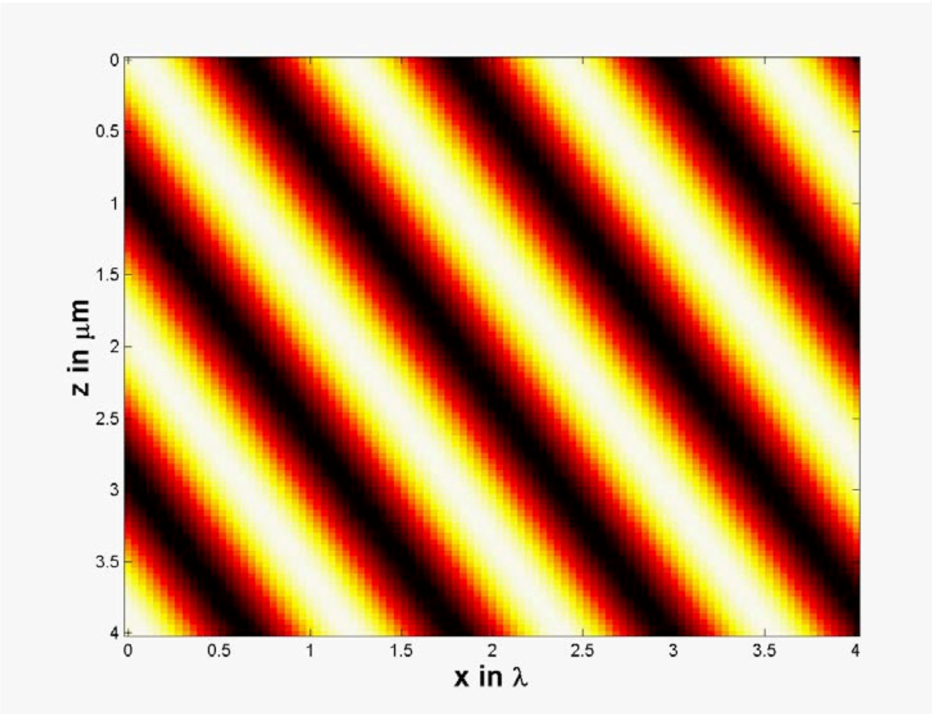
sign has to be chosen to ensure decay

$$\mathbf{E_0} e^{i(\mathbf{k}_\perp \cdot \mathbf{r}_\perp)} e^{-i\omega t} e^{-\gamma z}$$

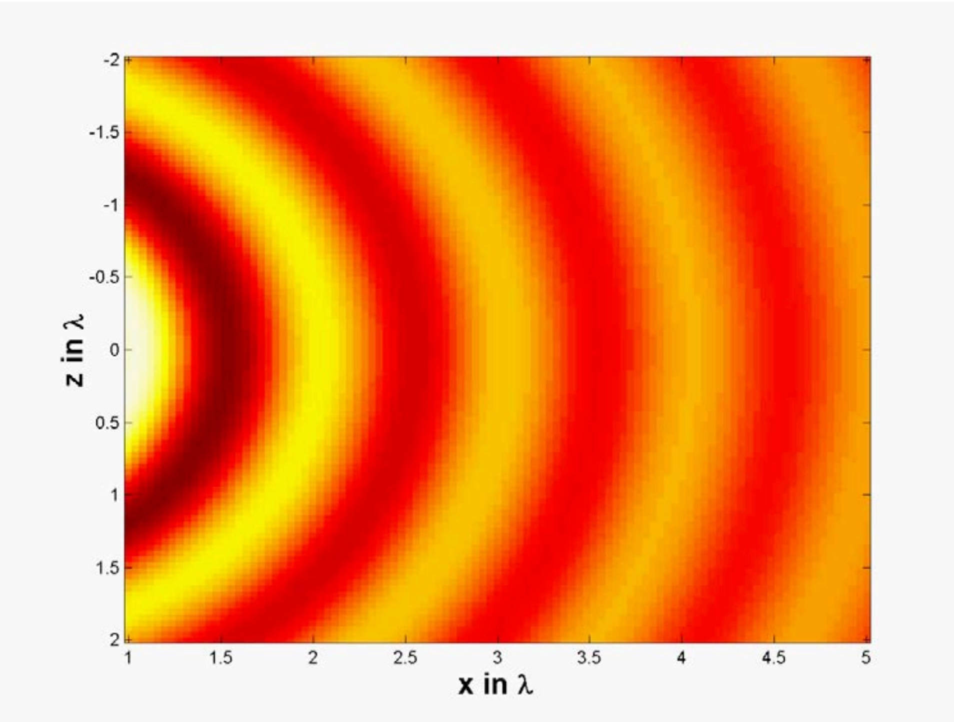


Types of solution

solutions for Maxwell equations in free space?



plane wave



spherical wave

How to find solutions for Maxwell equations for inhomogenous space?