Computational Photonics

Basics of grating theories

Grating theories

- rigorous solution of the diffraction of light at a periodic structure, e.g. dielectric gratings, finite photonic crystal slabs, arrays of metallic nano particles etc.

- methods

  A) thin element approximation *(scalar approximation)*

  B) rigorous solutions *(Fourier modal method)*

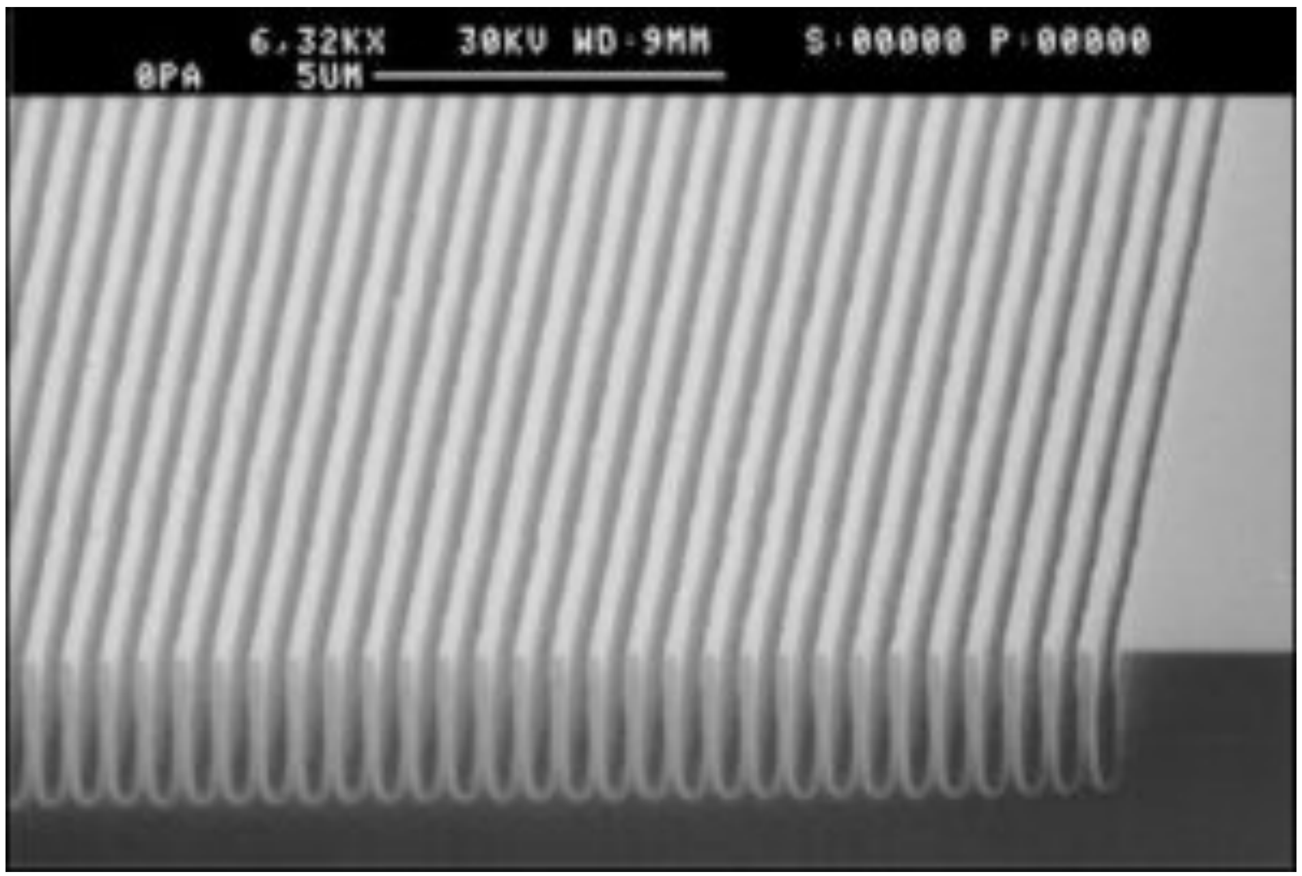
- strategy for rigorous solution

  • finding the eigenmodes sustained in the periodic region

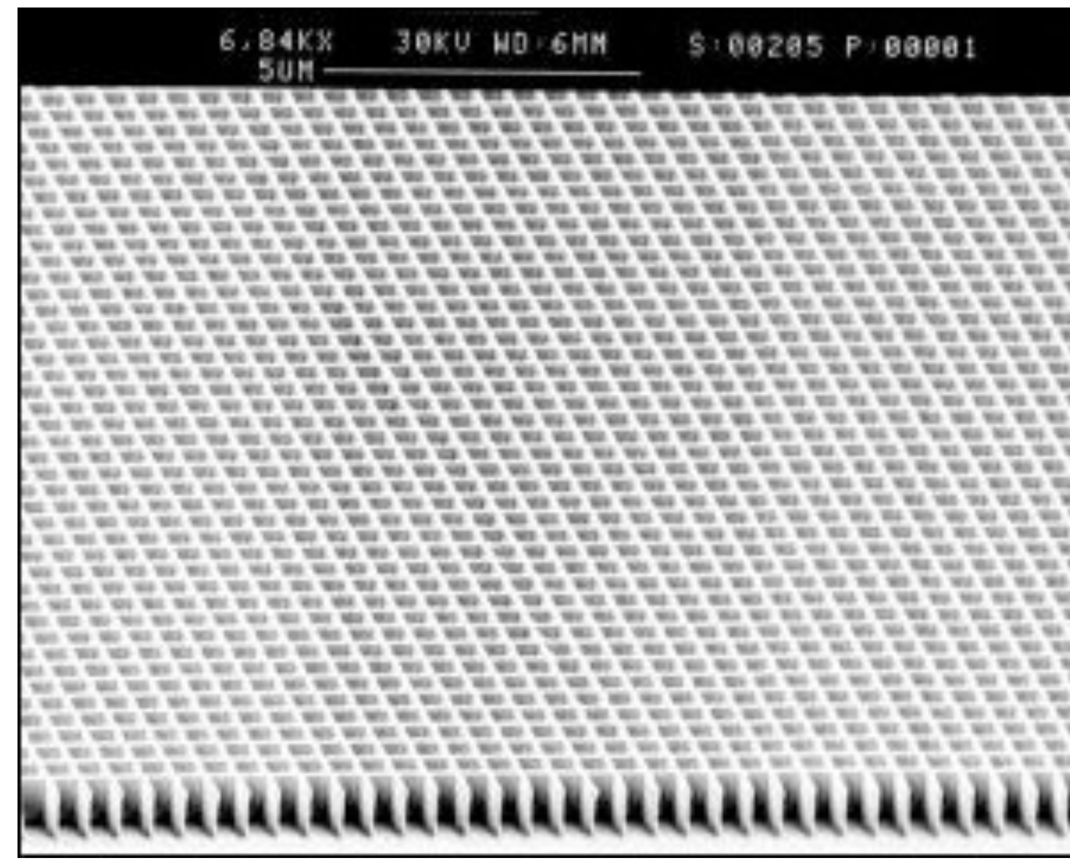
  • finding modal amplitudes by matching boundary condition

Final goal calculating diffraction at a grating

characterised by a periodic variation of the materials in x and y direction



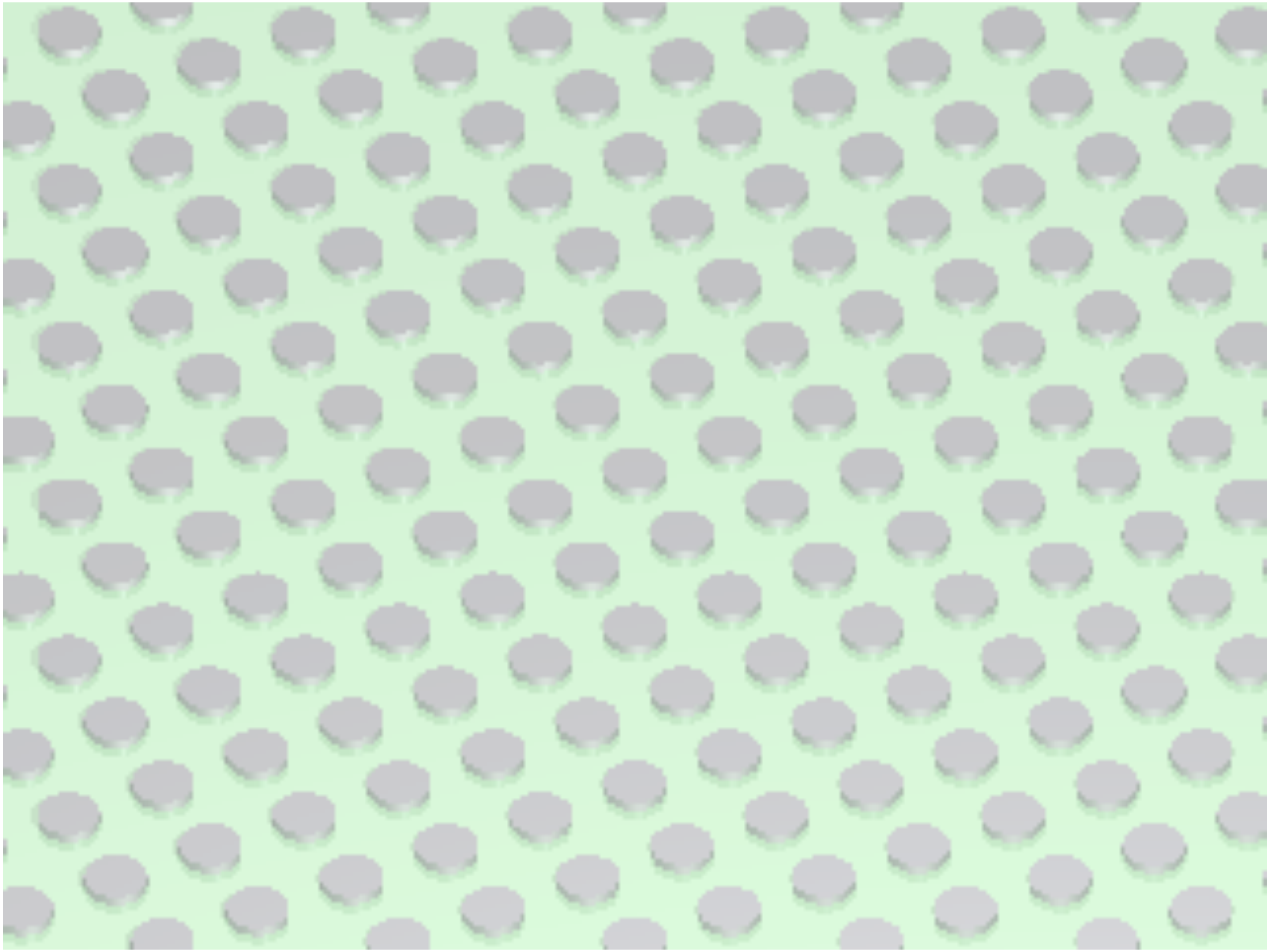
1D grating



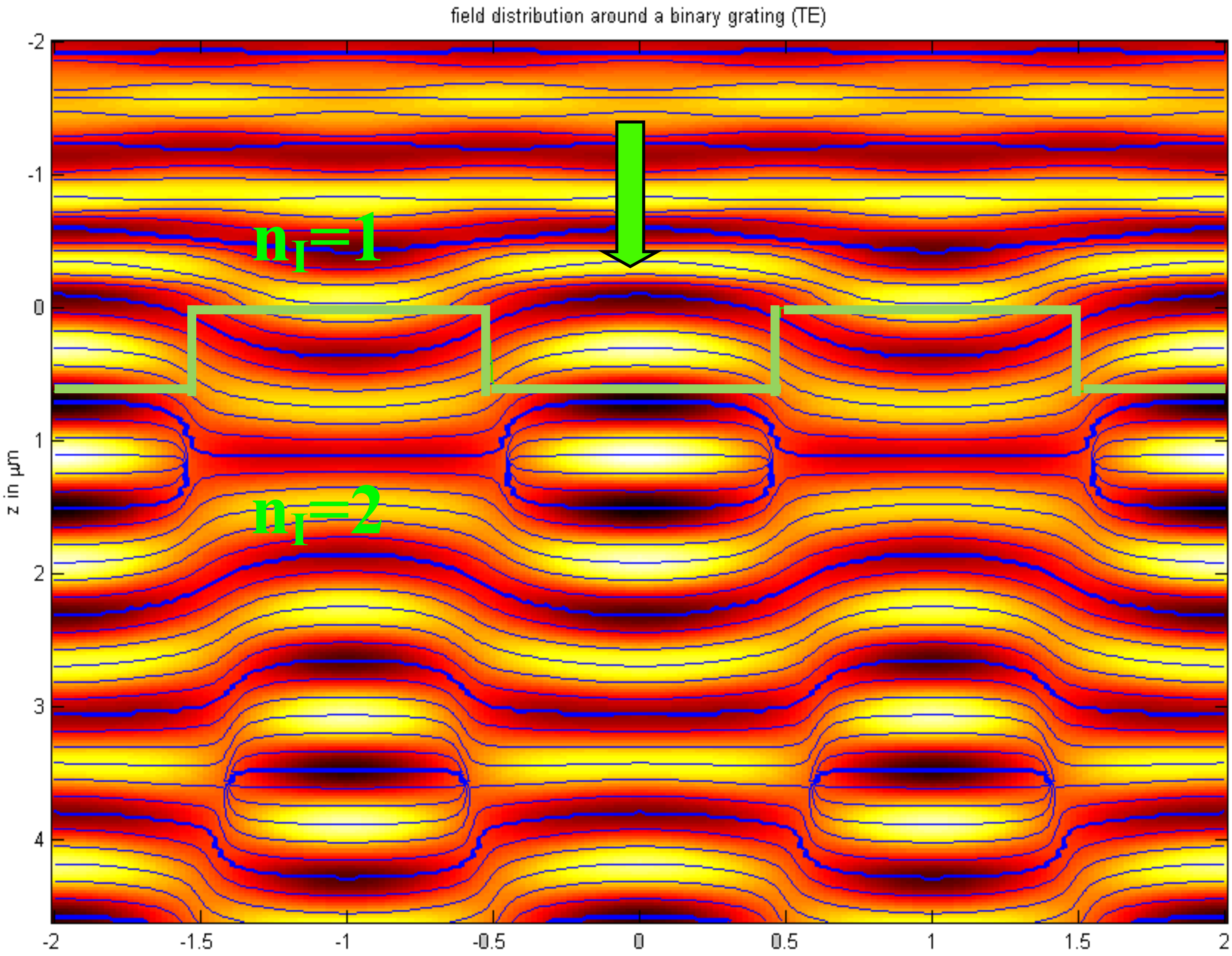
2D grating
(biperiodic)

(Pictures from the IAP Uni Jena, E. B. Kley)

Final goal calculating diffraction at a grating



Statement of the problem in 2D



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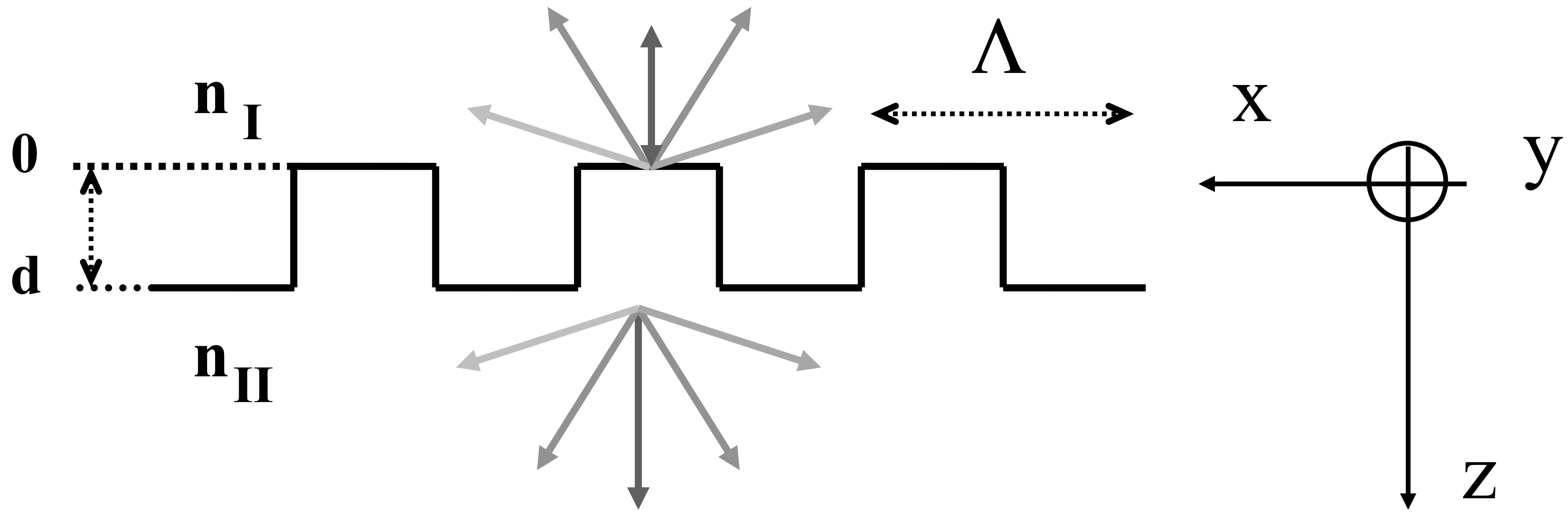

$$E_{\mathrm{inc},y} = \exp\left[ik_0 n_I \left(\sin\theta x + \cos\theta z\right)\right]$$

θ - angle of incidence

TE polarisation

$$E_{\mathrm{I},y} = E_{\mathrm{inc},y} + \sum_i R_i \exp\left[i\left(k_{xi}x - k_{I,zi}z\right)\right] \quad z<0$$

$$E_{\mathrm{II},y} = \sum_i T_i \exp\left[i\left(k_{xi}x + k_{II,zi}(z-d)\right)\right] \quad z>d$$

Direction and amplitude of reflected and transmitted amplitudes?

Ewald sphere

wave number of each wave is given by

$$k = \frac{2\pi}{\lambda_0} n = \sqrt{k_x^2 + k_z^2}$$

incident wave is characterised by

$$k_{zI} = \frac{2\pi}{\lambda_0} n_I \cos\theta$$

$$k_{xI} = \frac{2\pi}{\lambda_0} n_I \sin\theta$$

grating provides a momentum for each diffraction order of

$$k_{xi} = i\frac{2\pi}{\Lambda}$$

i integer

Ewald sphere

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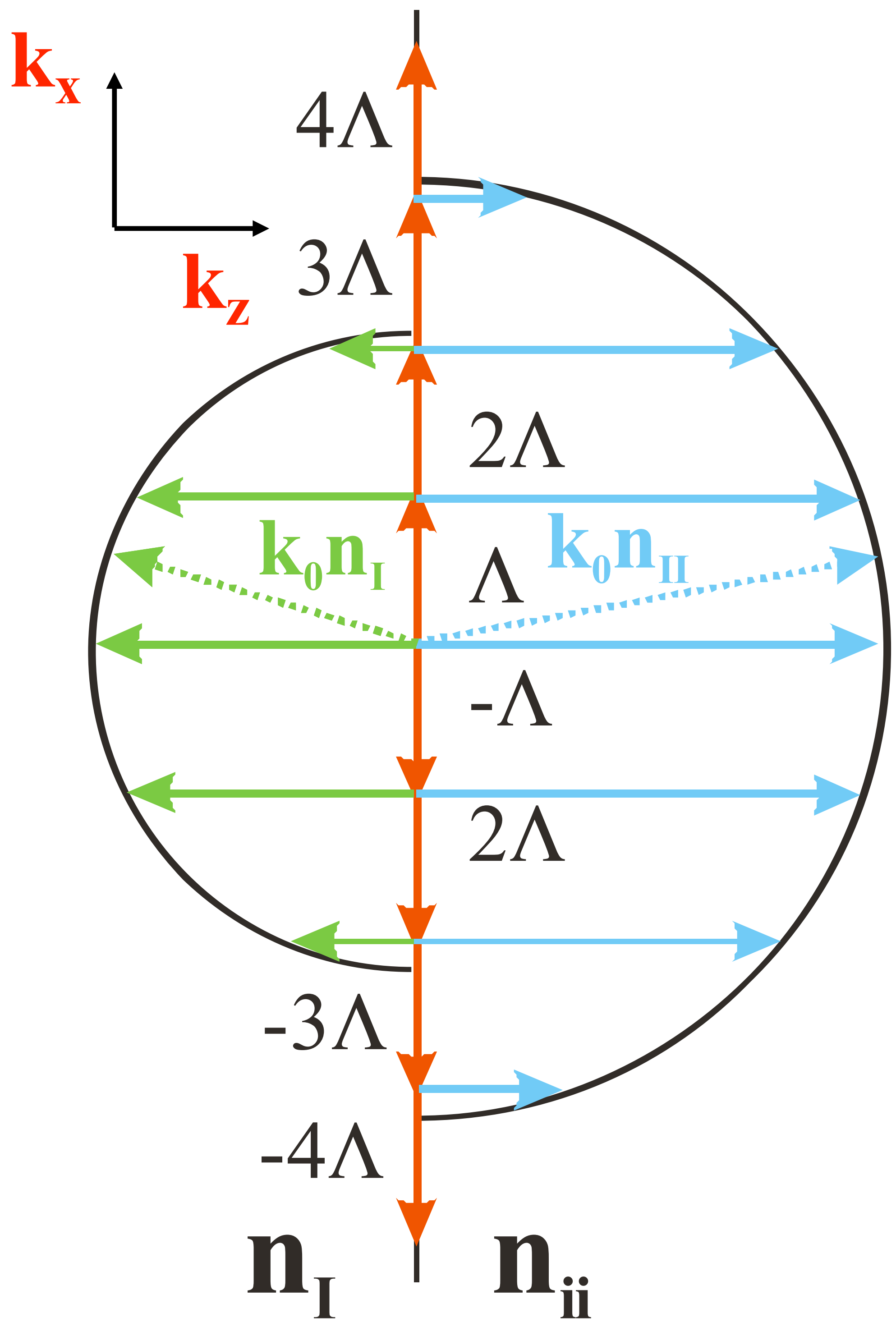
$$k_{xi} = i\frac{2\pi}{\Lambda} + k_0 n_I \sin\theta$$

diffracted waves propagate along

$$k_{I/II,zi} = \sqrt{k_0^2 n_{I/II}^2 - k_{xi}^2}$$

Ewald sphere



$$\mathbf{k_{xi}}$$

$$k_{xi} = i\frac{2\pi}{\Lambda} + k_0 n_I \sin\theta$$

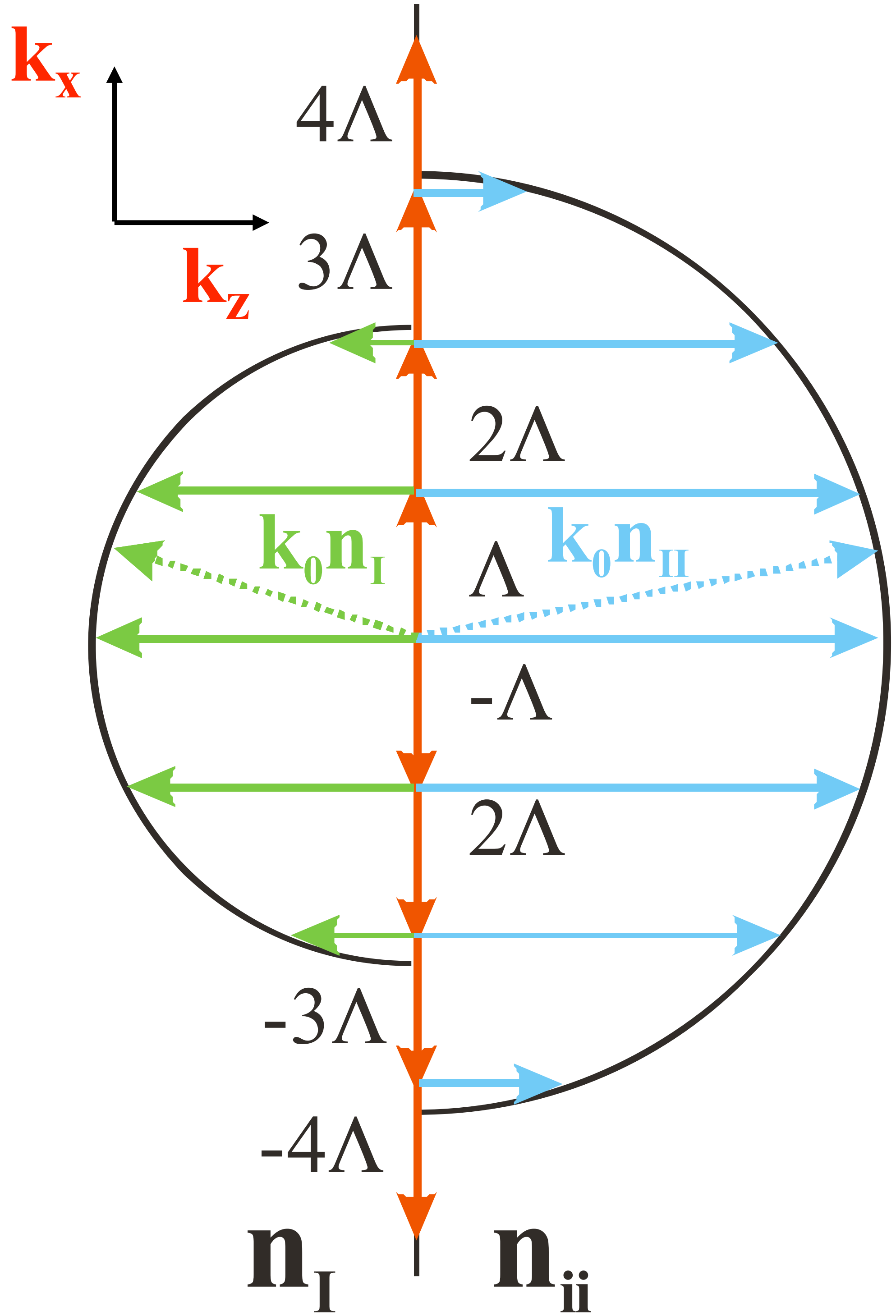
$$\mathbf{k_{I,zi}} \quad \mathbf{k_{II,zi}}$$

$$k_{I/II,zi} = \sqrt{k_0^2 n_{I/II}^2 - k_{xi}^2}$$

propagation direction of each diffracted wave

(applies to all kind of diffraction; X-Ray, grating)

Ewald sphere



$\mathbf{k_{xi}}$

$$k_{xi} = i\frac{2\pi}{\Lambda} + k_0 n_I \sin\theta$$

$\mathbf{k_{I,zi}}$ **$\mathbf{k_{II,zi}}$**

$$k_{I/II,zi} = \sqrt{k_0^2 n_{I/II}^2 - k_{xi}^2}$$

$k_{xi}^2 \leq k_0^2 n_{I/II}^2$ ⟹ **Propagating**

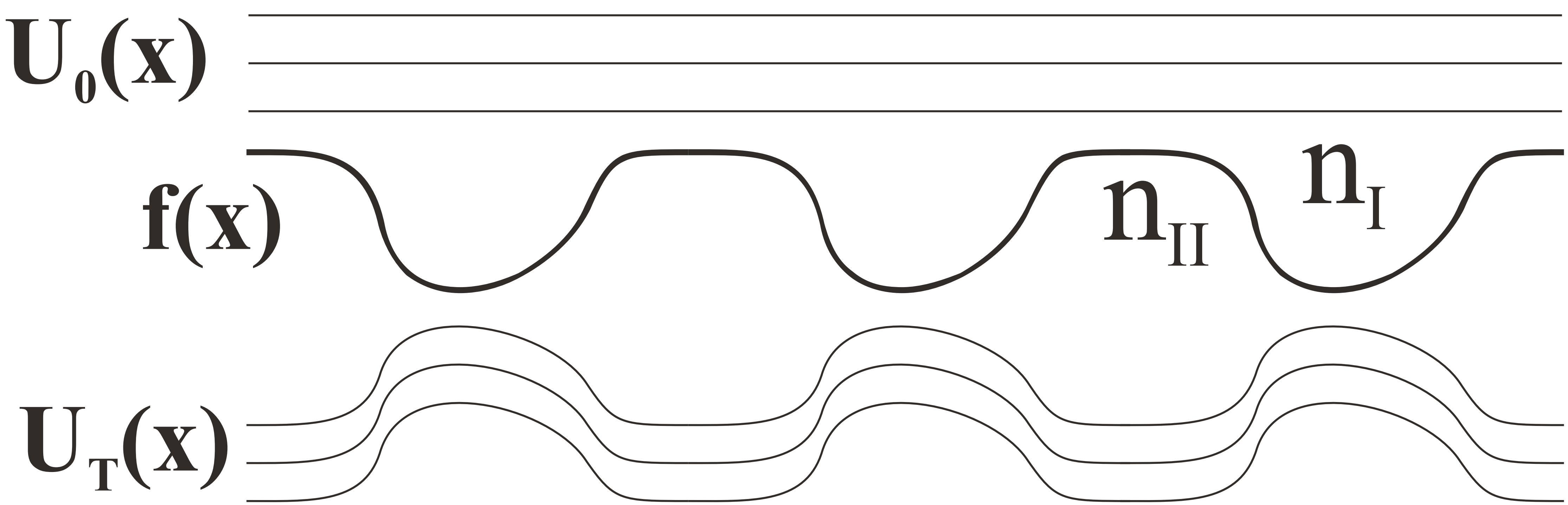
$k_{xi}^2 > k_0^2 n_{I/II}^2$ ⟹ **Evanescent**

Scalar theory and thin element approach

field after the grating is given by the incident field multiplied by the transmission function

$$U_T(x, y) = T(x, y)U_0(x, y) \qquad U = \mathbf{E}_\omega \cdot \mathbf{y}$$

$$T(x, y) = |T(x, y)|e^{\imath\phi(x,y)}$$



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for a phase grating the amplitude transmission function equals unity and the phase is given by

$$\phi(x) = k_0(n_{II} - n_I)f(x)$$

Scalar theory and thin element approach

for normal plane wave illumination the field after the structure is given by

$$e^{\imath\phi(x)}$$

amplitudes of the diffracted waves are given by Fourier series

$$T_n = \int_0^{\Lambda} e^{\imath k_0 (n_{II} - n_I) f(x)} e^{-\imath k_{xn} x} dx$$

$$T_n = \mathrm{FT}\left[e^{i\phi(x)}\right]$$

amplitudes are given by a Fourier-transformation of the transmission function

Diffraction efficiencies

Diffraction efficiency corresponds to the energy transferred into a diffraction order normalised to the incident energy

(given by the Poynting vector)

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

Introducing the plane waves into the equation leads to

$$\eta_{T,n} = \frac{p_I}{p_{II}} Re\left\{\frac{k_{II,zi}}{k_0}\right\} |T_n|^2$$

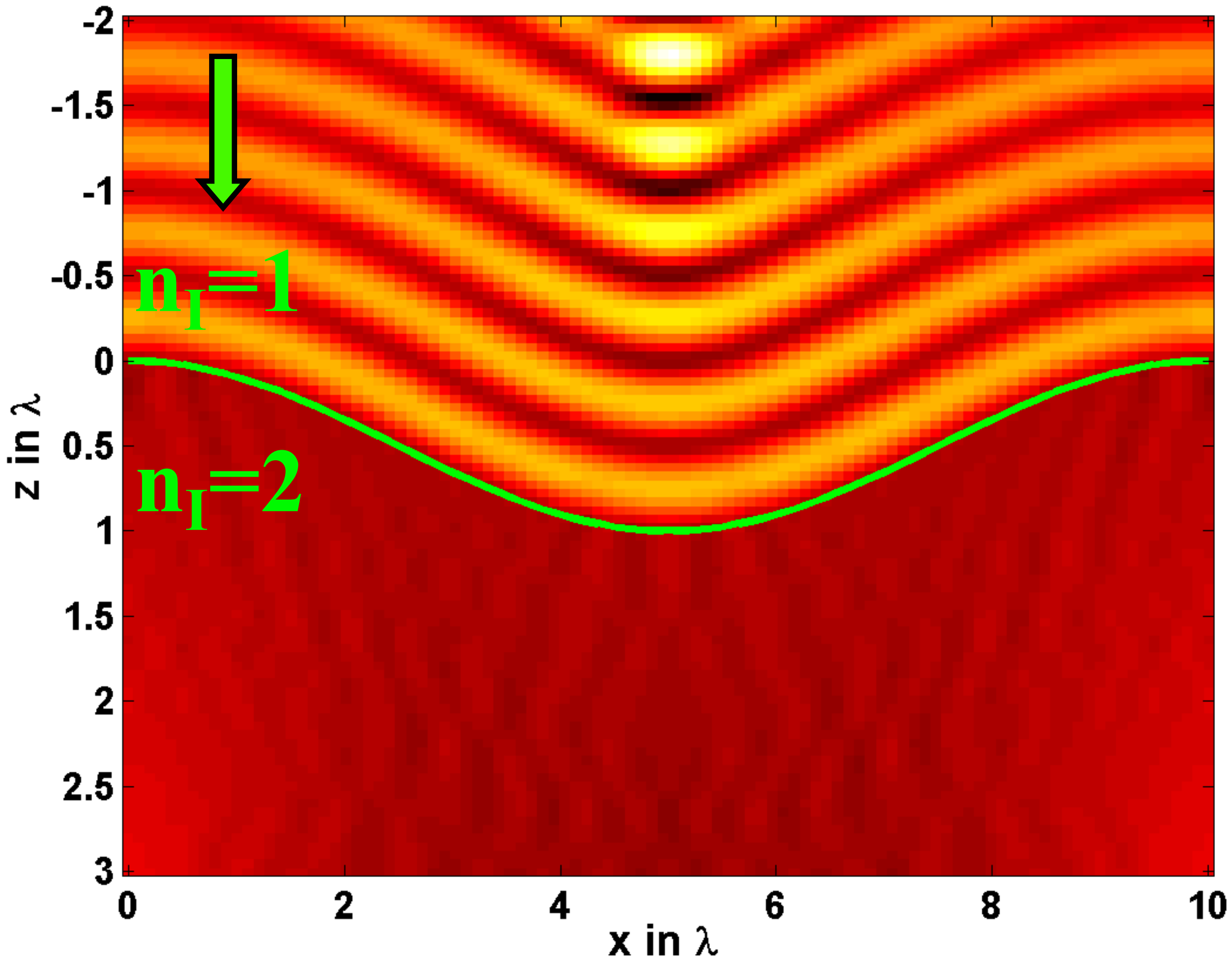
TE: $p_{I,II} = 1$

TM: $p_{I,II} = \epsilon_{I,II}$

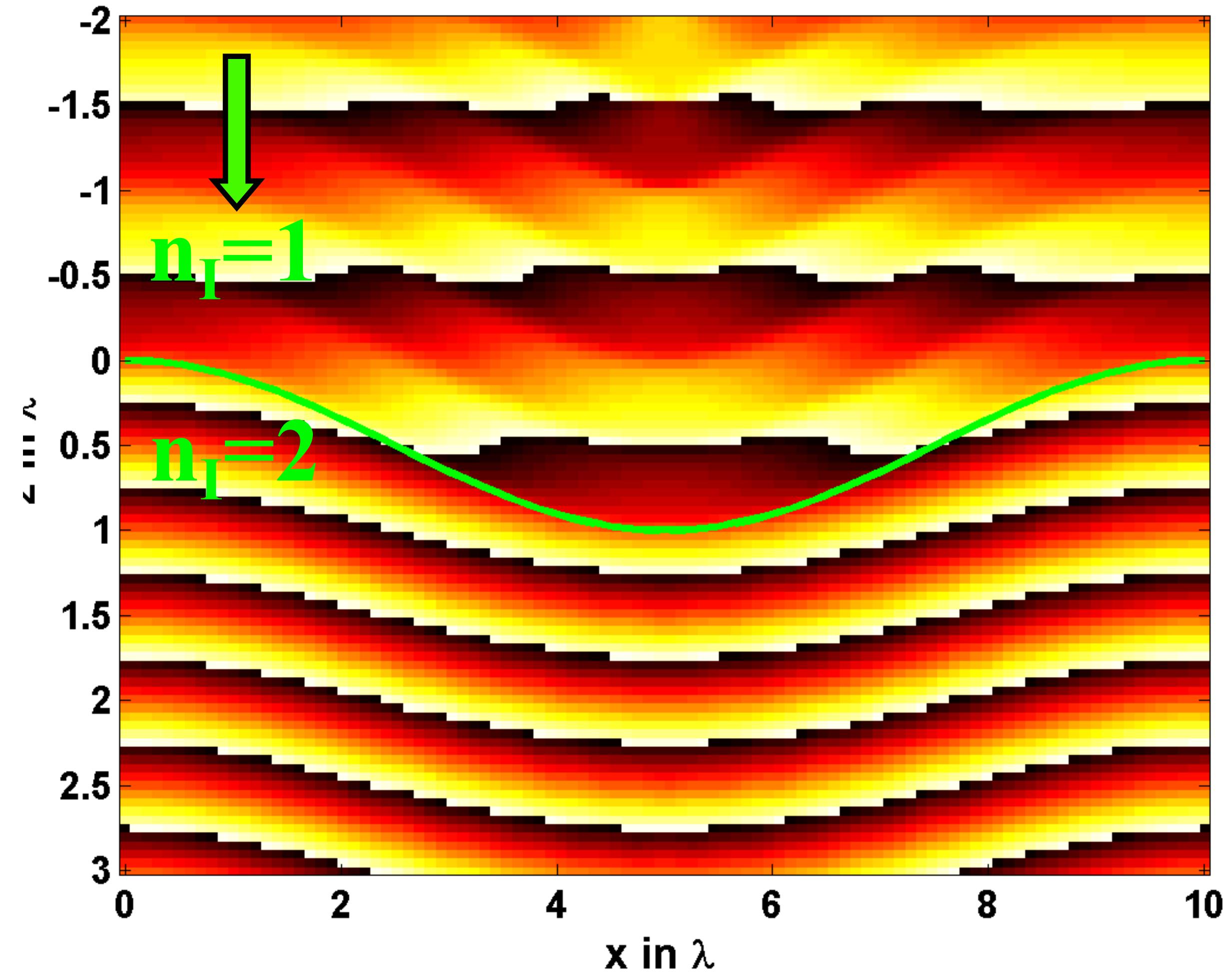
$$\eta_{R,n} = Re\left\{\frac{k_{I,zi}}{k_0}\right\} |R_n|^2$$

Energy conservation for loss-less materials!!

Limitations of the scalar method



Amplitude



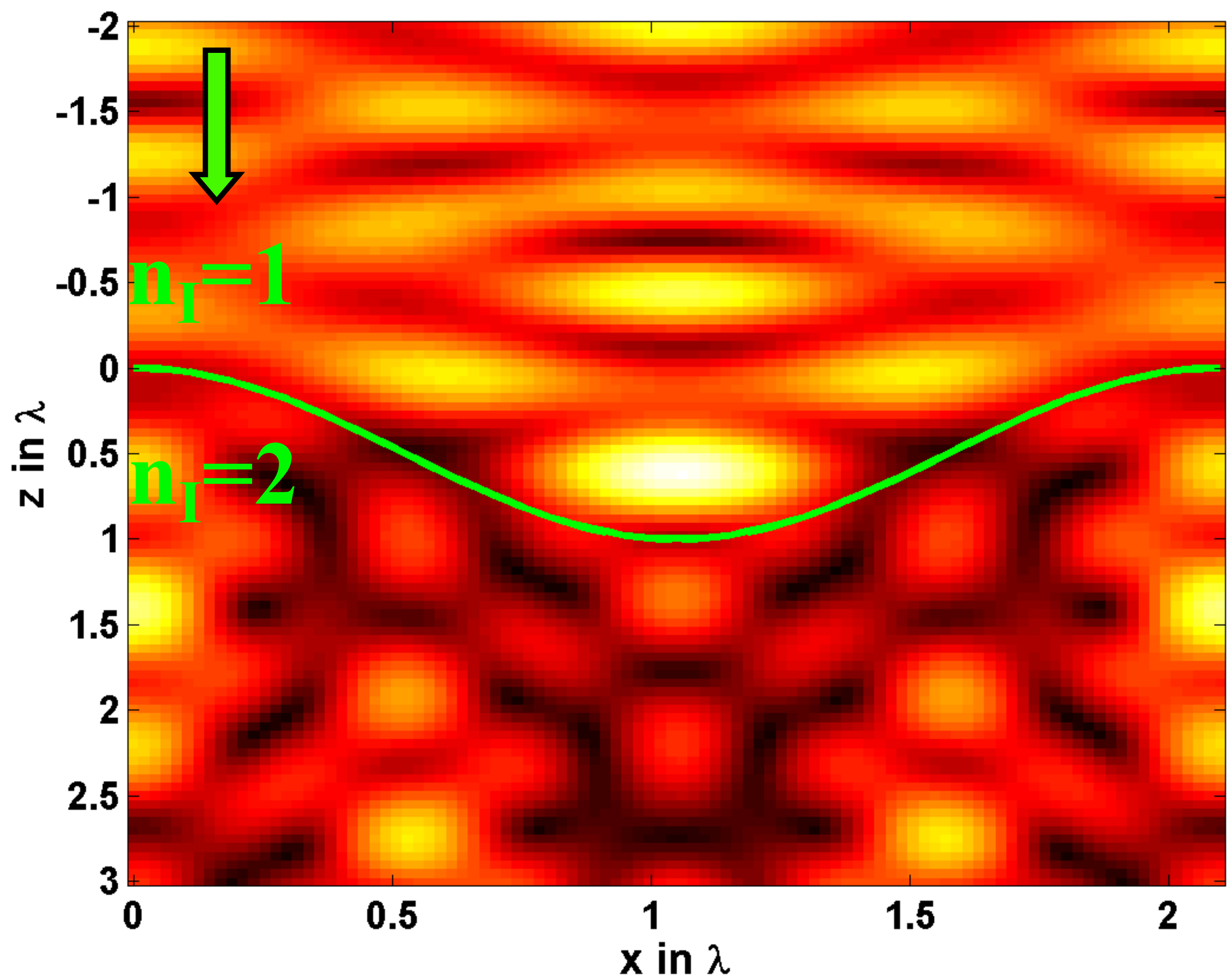
Phase

$\Lambda=10\lambda$

field after illuminating a sinusoidal grating with a plane wave

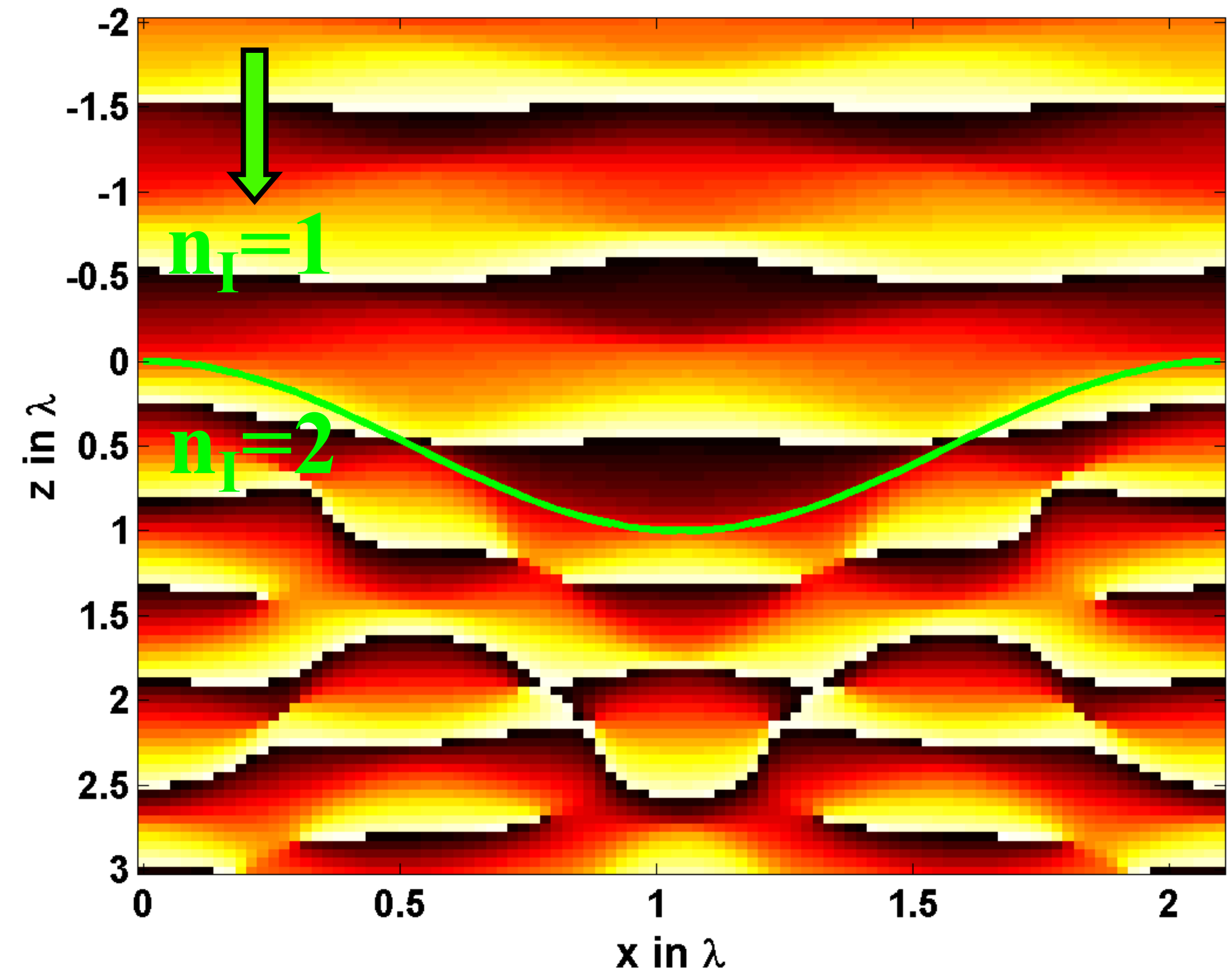
Λ is much larger than λ ⟹ thin element approach justified

Limitations of the scalar method



Amplitude

$\Lambda = 2.1\lambda$



Phase

field after illuminating a sinusoidal grating with a plane wave

Λ is much larger than λ → thin element approach justified

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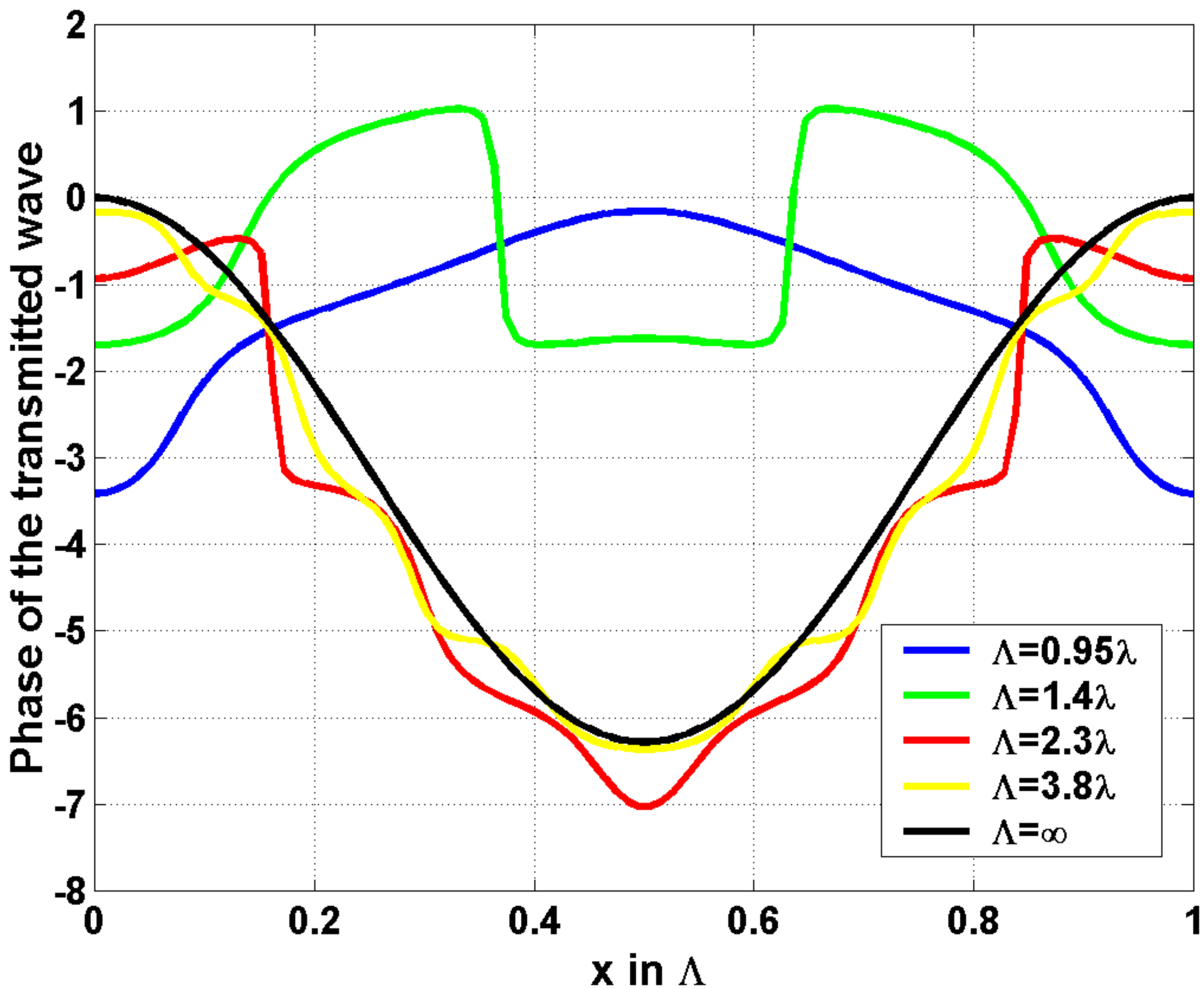

phase of the field directly after the grating
(h=λ, n_I=1, n_{II}=2, TE, sinusoidal)

- scalar theory fails for periods comparable to wavelength and for significant grating thicknesses
- no proper description of the field inside the grating (0<z<d)

⇨ have to solve Maxwell's equations properly also for the region inside the grating

Basics of grating theories