(1)

$$\vec{j}(\vec{r}) = -e\int d\epsilon\, N(\epsilon) \int \frac{d\hat{p}}{4\pi}\, \vec{v}(\hat{p})\, f(\vec{r},\hat{p},\epsilon)$$

$$f(\vec{r},\hat{p},\epsilon) = f_0(\vec{r},\epsilon) + \vec{\delta f}(\vec{r},\epsilon)\cdot\hat{p}$$

$$\int \frac{d\hat{p}}{4\pi}\, \vec{v}(\hat{p})\, f_0(\vec{r},\epsilon) = 0$$

$$\int \frac{d\hat{p}}{4\pi}\, \hat{v}(\hat{p})\,\vec{\delta f}\,\hat{p} = \frac{1}{3} v\, \vec{\delta f}$$

von Blatt 6: $v \nabla f_0 = -\frac{1}{\tau}\vec{\delta f}$

$$\Rightarrow \vec{\delta f} = -\tau v \nabla f_0$$

$$\Rightarrow \int \frac{d\hat{p}}{4\pi}\, v\, \vec{\delta f}\, \hat{p} = -\underbrace{\frac{1}{2} v^2 \tau}_{D} \nabla f_0$$

$$\Rightarrow \vec{j} = e\int d\epsilon\, N(\epsilon)\, \cancel{\int \frac{d\hat{p}}{4\pi}}\, D\, \nabla f_0$$

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~~allg.~~ Lösung von $D\,\partial_x^2 f(x) = 0$

$$f(x) = ax + b$$

$$f(0) = b = f_L \qquad f(L) = aL + f_L = f_R$$

$$\Rightarrow \cancel{a = f_L\, f_R / L}$$

$$a = (f_R - f_L)/L$$

$$\Rightarrow f(x) = \frac{f_R - f_L}{L} x + f_L$$

$$= f_R \frac{x}{L} + f_L \left(1 - \frac{x}{L}\right)$$

$$\partial_x f(x) = \frac{1}{L}\left(f_R - f_L\right)$$

$$\Rightarrow j = e\nu D \int d\epsilon\, \partial_x f(x)$$

$$= e\nu D \int d\epsilon\, \frac{1}{L}(f_R - f_L)$$

$$= \frac{e^2 \nu D}{L} V = \frac{\sigma}{L} V$$

$$\sigma = e^2 \nu D$$

(3)

Ansatz

$$f_1(x) = \left(1 + \frac{x}{L}\right) f_0 - \frac{x}{L} f_L \qquad -L < x < 0$$

$$f_2(x) = \left(1 - \frac{x}{L}\right) f_0 + \frac{x}{L} f_R \qquad 0 < x < L$$

$$\cancel{j_L = \frac{eDN}{L}(\mu_0 - \mu_L)}$$

$$j_L = e N_L D_L \int dE\, \partial_x f_1$$

$$= e N_L D_L \int dE\, (f_0 - f_L)$$

$$= e N_L D_L (\mu_0 - \mu_L) \overset{!}{=} j_R$$

$$= e N_R D_R (\mu_R - \mu_0)$$

$$(e N_L D_L + e N_R D_R)\, \mu_0 = e N_L D_L \mu_L + e N_R D_R \mu_R$$

$$\Rightarrow \quad \mu_0 = \frac{N_L D_L \mu_L + N_R D_R \mu_R}{N_L D_L + N_R D_R}$$

4

a) In einer Dimension:

$$\cancel{\partial_x (\mu')_\sigma}\, \partial_x^2 f_{\sigma} = \frac{f_{\bar\sigma} - f_\sigma}{2\tau_\sigma}$$

$$f_\sigma = \frac{1}{e^{\beta(\epsilon - \mu_\sigma)} + 1} \qquad \mu_\sigma = \mu_\sigma(x) \qquad \tilde\epsilon = \epsilon - \mu_\sigma$$

$$\partial_x^2 f_\sigma = \frac{2\beta^2 e^{2\beta\tilde\epsilon} \mu_\sigma'^2}{(1+e^{\beta\tilde\epsilon})^3} - \frac{\beta^2 e^{\beta\tilde\epsilon} \mu_\sigma'^2}{(1+e^{\beta\tilde\epsilon})^2} + \frac{\beta e^{\beta\tilde\epsilon} \mu_\sigma''}{(1+e^{\beta\tilde\epsilon})^2}$$

$$= \frac{2\beta^2 e^{\frac{1}{2}\beta\tilde\epsilon} \mu_\sigma'^2}{(e^{-\frac{\beta}{2}\tilde\epsilon} + e^{\frac{\beta}{2}\tilde\epsilon})^3} - \frac{\beta^2 \mu_\sigma'^2}{(e^{-\frac{\beta}{2}\tilde\epsilon} + e^{\frac{\beta}{2}\tilde\epsilon})^2} + \frac{\beta e^{\beta\tilde\epsilon} \mu_\sigma''}{(1+e^{\beta\tilde\epsilon})^2}$$

$$= \frac{2e^{\frac{1}{2}\beta\tilde\epsilon} - e^{-\frac{\beta}{2}\tilde\epsilon} - e^{\frac{\beta}{2}\tilde\epsilon}}{8\cosh[\beta\tilde\epsilon/2]} \beta^2 \mu_\sigma'^2 + \dots$$

$$= \underbrace{\frac{\sinh(\beta\tilde\epsilon/2)}{4\cosh(\beta\tilde\epsilon/2)} \beta^2 \mu_\sigma'^2}_{\text{assymetrisch in } \tilde\epsilon} + \frac{\beta\, \mu_\sigma''}{4\cosh(\beta\tilde\epsilon/2)}$$

$$\Rightarrow \int d\tilde\epsilon\, \partial_x^2 f_\sigma = \partial_x^2 \mu_\sigma \int_{-\infty}^{\infty} d\tilde\epsilon\, \frac{\beta}{4\cosh(\beta\tilde\epsilon/2)}$$

$$\int_{-\infty}^{\infty} d\tilde{\epsilon} \, \frac{\beta}{4\cosh(\beta\tilde{\epsilon}/2)^2} = \int_{-\infty}^{\infty} dx \, \frac{1}{\cosh(x/2)^2}$$

$$= \int_{-\infty}^{\infty} dx \, \frac{2e^{x}}{(1+e^{x})^2} = \int_{0}^{\infty} dy \, \frac{1}{y} \, \frac{2y}{(1+y)^2}$$

$$y = e^x \qquad \frac{dy}{dx} = y$$

$$= \int_0^\infty dy \, \frac{2}{(1+y)^2} = \left[-\frac{1}{1+y} \right]_0^\infty = 1$$

$$\Rightarrow D_\sigma \partial_x^2 \mu_\sigma = \frac{\mu_\sigma - \mu_{\bar\sigma}}{2\tau^\sigma}$$

$$\Rightarrow \partial_x^2 \mu_\sigma = \frac{\mu_\sigma - \mu_{\bar\sigma}}{2 C_\sigma^2}$$

$$C_\sigma = \sqrt{D_\sigma \tau^\sigma}$$

~~b)~~

$$\partial_x f_\sigma = \cancel{\frac{\beta}{\sqrt{1+e^{\beta(\epsilon-\mu)}}}} \frac{\beta}{4\cosh(\beta\tilde{\epsilon}/2)} \mu_\sigma^{\neq 1}$$

$$\Rightarrow j = e \, \mathbb{N}_\sigma \, \mathbb{D}_\sigma \, \cancel{\partial} \int d\epsilon \, \cancel{d\ldots} \, \partial_x f_\sigma$$

$$= e \, \mathbb{N}_\sigma \, \mathbb{D}_\sigma \, \partial_x \mu_\sigma \int d\tilde{\epsilon} \, \frac{\beta}{4\cosh(\beta\tilde{\epsilon}/2)}$$

$$= e \, \mathbb{N}_\sigma \, \mathbb{D}_\sigma \, \partial_x \mu_\sigma$$

b) Gegeben:

$$\mu_{\uparrow/\downarrow} = \frac{c_\downarrow^2 (C_1 + C_2 x) + c_\uparrow^2 (C_3 + C_4 x)}{c_{tot}^2}$$

$$\pm \frac{c_{\downarrow\uparrow}^2}{c_{tot}^2} \left[(C_3 - C_1) \cosh(x/l) + l (C_4 - C_2) \sinh(x/l) \right]$$

$$\mu_{\uparrow} - \mu_{\downarrow} = \underbrace{\frac{2(c_\downarrow^2 + c_\uparrow^2)}{c_{tot}^2}}_{=1} \left[(C_3 - C_1) \cosh(x/l) + l (C_4 - C_2) \sinh(x/l) \right]$$

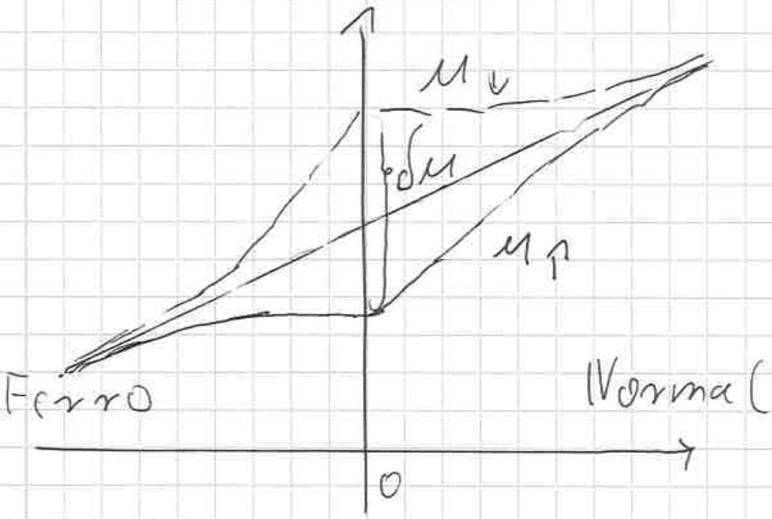
$$\partial_x^2 \mu_{\uparrow\downarrow} = \pm \frac{1}{l^2} \frac{c_{\downarrow\uparrow}^2}{c_{tot}^2} \left((C_3 - C_1) \cosh(x/l) + l (C_4 - C_2) \sinh(x/l) \right)$$

$$= \pm \frac{1}{2 c_{\uparrow\downarrow}} \left((C_3 - C_1) \cosh(x/l) + l (C_4 - C_2) \sinh(x/l) \right)$$

$$\Rightarrow \quad \partial_x^2 \mu_{\uparrow/\downarrow} = \frac{\mu_\uparrow - \mu_\downarrow}{2 l_\sigma}$$

c)

Skizze für $\sigma_F = \sigma_N$:



$$c_\uparrow^2 \partial_x \mu_\uparrow + c_\downarrow^2 \partial_x \mu_\downarrow$$

$$= (c_\uparrow^2 + c_\downarrow^2)\frac{ej}{2\sigma_F} + \underbrace{\frac{c_\uparrow^2 + c_\downarrow^2}{c} e^{x/c} d\mu \frac{c_\downarrow^2 - c_\uparrow^2}{2c_{\cancel{tot}}^2} + \frac{c_\uparrow^2 - c_\downarrow^2}{c} \frac{d\mu}{2} e^{x/c}}_{0}$$

$$= \frac{c_\uparrow^2 + c_\downarrow^2}{2\sigma_F} ej = \frac{c_\uparrow^2 + c_\downarrow^2}{\cancel{2} e^2(\nu_\uparrow D_\uparrow + \nu_\downarrow D_\downarrow)} ej = \frac{j}{e\sigma}$$