Theorie der Kondensierten Materie I WS 2015/16

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1. d-wave superconductivity

(5 + 5 + 10 + 10 = 30 Punkte)

In this exercise discuss BCS theory of d-wave superconductors. We start from the Hamiltonian

$$H = \sum_k (\epsilon_k - \mu) c^+_{k\sigma} c_{k\sigma} - \frac{1}{V} \sum_{k,k'} g(\hat{k} \cdot \hat{k}') c^+_{k',\uparrow} c^+_{-k',\downarrow} . c_{-k,\downarrow} c^+_{k,\uparrow} \tag{1}$$

Here, $\hat{k}$ and $\hat{k}'$ are the unit vectors in the direction of k and k' and $\hat{k} \cdot \hat{k}' \equiv \cos\theta_{k,k'}$ stands for their scalar product. The summations in the interaction term run over a narrow momentum shel near the fermi surface

$$\mu - \omega_D < \epsilon_k, \epsilon'_k < \mu + \omega_D. \tag{2}$$

As compared to the standard BCS, we allow the interaction between the particles to be momentum dependent. The form of the momentum dependence, $g(k, k') = g(\hat{k} \cdot \hat{k}')$, is dictated by the rotation invariance: the interaction does not change under simultaneous rotation of k and k'.

(a) Show that the fermionic statistics of the particle imposes the following requirement on the interaction $g(\hat{k} \cdot \hat{k}')$

$$g(k, -k') \equiv g(-\hat{k} \cdot \hat{k}') = g(\hat{k} \cdot \hat{k}') \equiv g(k, k'). \tag{3}$$

The function $g(k, k') = g(\cos\theta_{k,k'})$ can be expanded in Legendre polynomials, or, equivalently, in spherical harmonics

$$g(\hat{k} \cdot \hat{k}') = \sum_{l=0}^{\infty} g_l P_l(\cos\theta_{k,k'}) \equiv \sum_{l=0}^{\infty} \frac{4\pi g_l}{2l+1} \sum_{m=-l}^{l} Y^*_{lm}(\hat{k}') Y_{lm}(\hat{k}) \tag{4}$$

Show, that the condition (3) is equivalent to $g_l = 0$ for odd l.

In the case of momentum-independent interaction constant ($g_0 > 0$ and $g_{l>0} = 0$) our Hamiltonian reduces to the usual BCS Hamiltonian describing s-wave superconductivity. In the following we are going to study the first possible non-trivial singlet superconducting pairing: Hamiltonian (1) with $g_0 = 0$, $g_2 > 0$ and $g_{l>2} = 0$. This is the case of d-wave superconductivity.

(b) We now apply the BCS variational approach to the Hamiltonian (1). We start from the BCS wave function

$$|BCS\rangle = \prod_k (u_k + v_k c^+_{k,\uparrow} c^+_{-k,\downarrow}) |0\rangle. \tag{5}$$

Here $u_k = \cos\phi_k$ and $v_k = \sin\phi_k$. Use the expectation values for the operators $c^+_{k,\sigma}c_{k,\sigma}$ and $c^+_{k',\uparrow}c^+_{-k',\downarrow}.c_{-k,\downarrow}c^+_{k,\uparrow}$ from the previous exercise sheet to compute the expectation value of the Hamiltonian (1) in a BCS state. Show that in order to minimise the Hamiltonian expectation value the rotation angle ϕ_k should satisfy

$$\tan 2\phi_k = \frac{\Delta_k}{\epsilon_k - \mu} \tag{6}$$

where Δ_k satisfies the self-consistency equation

$$\Delta_k = \frac{1}{V}\sum_{k'} g(k,k')\sin 2\phi_{k'} = \frac{1}{V}\sum_{k'} \frac{g(k,k')\Delta_{k'}}{\sqrt{(\epsilon'_k - \mu)^2 + \Delta^2_{k'}}}. \tag{7}$$

(c) Equation (7) applies to any superconducting state with singlet pairing. Let us now focus specifically on the case of d-wave superconductivity, $g_0 = 0$, $g_2 > 0$, $g_{l>2} = 0$. Write down the explicit form of the self-consistency equation for this case. Switch from the summation over momentum to the energy and angle integration. Discuss the dependence of Δ_k on the direction of k.

(d) Show that the energies of single particle excitations in a d-wave superconductor are given by $E_k = \sqrt{\xi_k^2 + \Delta_k^2}$. Compare the low-temperatures heat capacitance for s-wave and d-wave superconductors.