Karlsruhe Institute of Technology
Institute for Theoretical Condensed Matter physics

Condensed Matter Theory I WS 2022/2023

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Sheet 0
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1. Boson and fermion operators

Let $b^\dagger$ and $f^\dagger$ be the creation operators of a boson and a fermion, respectively. We write $[\ldots]$ for the commutator and $\{\ldots\}$ for the anti-commutator. Calculate the following expressions:

(a) $\left[b^\dagger b, b^\dagger\right]$ and $\left\{f^\dagger f, f^\dagger\right\}$

(b) $\left[b^\dagger b, b\right]$ and $\left\{f^\dagger f, f\right\}$

(c) $\left[e^{-b^\dagger b}, b\right]$ *(hint: expand exponential in power series)*

(d) $\left\{e^{-f^\dagger f}, f\right\}$

2. Many-particle quantum states

We consider a system where the (normalized) single-particle orbitals $\phi_\lambda(\mathbf{r})$ are known, where $\mathbf{r}$ denotes the position coordinate.

(a) Let the system be populated by three identical spinless bosons in the states λ_1, λ_2, λ_3. Write down the normalized three-particle wave-function $\psi_{\lambda_1\lambda_2\lambda_3}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ with proper bosonic symmetry! Beware that $\lambda_1, \lambda_2, \lambda_3$ are not necessarily different and distinguish all possible cases.

(b) Now assume that there is only one spatial orbital $\phi(\mathbf{r})$, but we fill the system with three bosons of spin $S = 2$. How many independent three-particle states exist in this system?

(c) How many spin-$\frac{1}{2}$ fermions or spin-$\frac{3}{2}$ fermions could one place in one spatial orbital $\phi(\mathbf{r})$? Write down the Slater determinant for the second case. State explicitly the normalization factor.