KARLSRUHE INSTITUTE OF TECHNOLOGY
INSTITUTE FOR THEORETICAL CONDENSED MATTER PHYSICS

Condensed Matter Theory I WS 2022/2023

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Sheet 5
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Category A

1. Heisenberg equations of motion in graphene $(5 + 15 + 10 + 5 + 5 = 40$ Points)

The effective Hamiltonian operator in the vicinity of one of Dirac points in graphene reads

$$\hat{H} = v(\hat{p}_x\hat{\sigma}_x + \hat{p}_y\hat{\sigma}_y).$$

(a) The positive branch energy of the dispersion relation around this point is $\epsilon(\vec{k}) = \hbar v\sqrt{k_x^2 + k_y^2}$. Compute the corresponding group velocity.

(b) Consider the case when an external electromagnetic field $(\phi(\vec{r}, t), \vec{A}(\vec{r}, t))$ is applied. Then,

$$\vec{p} \to \vec{p}_{kin} = \vec{p} - (e/c)\vec{A} \quad , \quad \hat{H} \to \hat{H} + e\phi \ .$$

For this case, derive the Heisenberg equations of motion for the operators $\vec{p}$, $\vec{p}_{kin}$, $\vec{r}$, $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$.

(c) Show that by taking the expectation value of the velocity operator, $\langle\psi|\,\vec{v}\,|\psi\rangle$ where ψ is a combination of eigenstate only in the positive energy branch, you recover the group velocity found from the classical approach (*Hint: First consider the expectation value of an eigenstate of the Hamiltonian*).

(d) Calculate the cyclotron mass m_c in graphene as a function of energy ϵ using the relation $m_c = \dfrac{\hbar^2}{2\pi}\dfrac{\partial S}{\partial \epsilon}$. Here, $S(\epsilon)$ is an area of 2D orbit in k-space encircled by the particle of the energy $\epsilon(\mathbf{k}) = \hbar v\sqrt{k_x^2 + k_y^2}$.

(e) Show that the energies of Landau levels in graphene scale as $E_n \propto \sqrt{n}$ in the limit $n \gg 1$. To show it, use the relation $S_n = \dfrac{A_n}{l_{\mathrm{B}}^4}$ between the areas in k- and r- spaces, S_n and A_n, respectively, where l_{B} is the magnetic length (see lecture). For A_n use semiclassical Bohr-Sommerfeld quantization condition.

Category B

2. Cyclotron mass of an anisotropic parabolic dispersion relation (10 points)

Let us consider such a dispersion relation:

$$\epsilon(\mathbf{k}) = \frac{\hbar^2}{2}\left(\frac{k_x^2}{m_x} + \frac{k_y^2}{m_y} + \frac{k_z^2}{m_z}\right)$$

Assume now there is a magnetic field along the z-direction $\vec{B} = B\vec{e_z}$. Compute the corresponding cyclotron mass.