Karlsruhe Institute of Technology — Institute for Theoretical Condensed Matter physics

Condensed Matter Theory I   WS 2022/2023

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Category A

1. Landau levels in graphene $(10 + 10 + 10 = 30$ Points$)$

Let us consider Dirac electrons in graphene. If we consider both Dirac points ($K_\pm$-points), we describe quasiparticles in graphene using a Bloch function with 4 components:

$$\Phi = (\phi_{A,K_+}, \phi_{B,K_+}, \phi_{B,K_-}, \phi_{A,K_-}).$$

Here $K_\pm$ denotes the two Dirac points located at the edges of the Brillouin zone and $A(B)$ are the two subgrids. In this basis the effective Hamiltonian operator of an electron is

$$\mathcal{H} = v\boldsymbol{\Sigma}\cdot\boldsymbol{p},$$

where

$$\Sigma_x = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \qquad \Sigma_y = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}.$$

We consider the electron in an external magnetic field with potential vector $\boldsymbol{A}(\boldsymbol{r}, t)$). We use the minimal coupling

$$\boldsymbol{p} \to \boldsymbol{p}_{kin} = \boldsymbol{p} - (e/c)\boldsymbol{A} \quad ,$$

and consider the gauge:

$$\boldsymbol{A} = (-By, 0).$$

(a) The two valleys ($K_\pm$) are not coupled. First, consider the solutions for the K_+-valley. Here the Schrödinger equation couples the two components ϕ_{A,K_+} und ϕ_{B,K_+}. Write the corresponding equations. Show that it can be express in terms of the operator $\hat{a}$ and its hermitian conjugate $\hat{a}^\dagger$ satisfying the commutation relation $[\hat{a}, \hat{a}^\dagger] = \mathbb{I}$, the so-called ladder operator

(b) Use the known solutions of the oscillator equation to find the Landau levels as well as the eigenstate. Do the same derivation for the valley K_-.

(c) Add a Dirac-mass $m\sigma_z$ to the Hamiltonian and re-calculate the energy of the Landau levels. How would the result differ at the points K_+ and K_- ?

Category B

2. Berry-connection for spin-1/2: (20 Points)

In the lecture you introduced the time-dependent unitary matrix $R(t)$, which gives the basis change to the instantaneous eigenbasis results.

Now calculate the so-called Berry connection

$$i\dot{R}R^{-1},$$

in the case

$$R^{-1}(t) = e^{-i\varphi(t)\sigma_z/2}e^{-i\theta(t)\sigma_y/2}e^{-i\psi(t)\sigma_z/2}.$$