Aufgabe 1

$$A(x) = -\pi^{-1} \operatorname{Im} G_R(x) \qquad (2.68)$$

$$A(x) = Z^{-1}(1+\epsilon e^{-\beta x}) \sum_{m,n} e^{-\beta E_m} A_{mn} B_{nm}$$
$$\times \delta(x - E_n + E_m)$$

$$= (1+\epsilon e^{-\beta x}) \int dt\, e^{i(x-E_n+E_m)t}$$
$$\sum_{m,n} \frac{e^{-\beta E_m}}{Z} A_{mn} B_{nm}$$

$$= (1+\epsilon e^{-\beta x}) \int dt\, e^{ixt} \operatorname{Tr}(\rho\, e^{iHt} A e^{-iHt} B)$$

$$= (1+\epsilon e^{-\beta x}) \int dt\, e^{ixt} \langle A(t) B(0) \rangle$$

$$= (1+\epsilon e^{-\beta x}) \langle AB \rangle_x \qquad \begin{array}{l} x \to \omega \\ A \to B \\ \epsilon \to -1 \quad \text{Bosonen} \end{array}$$

$$\operatorname{Im} G_R(\omega) = \pi(1 - e^{-\beta\omega}) \langle BB \rangle_\omega$$

Aufgabe 2a)

$$G(\tau) = -\langle T\, e^{\tau H} a\, e^{-\tau H} a^+(0) \rangle$$

$$\frac{\partial G}{\partial \tau} = -\langle e^{+\tau H}[H,a]e^{-\tau H} a^+ \rangle \Theta(\tau)$$

$$-\langle a^+ e^{+\tau H}[H,a]e^{-\tau H} a \rangle \Theta(-\tau)$$

$$+\langle a(\tau) a^+ \rangle \delta(\tau) + \langle a^+ a(\tau) \rangle \delta(\tau)$$

$$[H,a] = -\Omega a - \sum g_i b_i$$

$$[a^+a, a] = -a \qquad [a^+, a] = -1$$

$$\Rightarrow \frac{\partial G}{\partial \tau} = -\Omega G - \sum g_i F_i - \delta(\tau)$$

$$\left(\frac{\partial}{\partial \tau} + \Omega\right) G = -\delta(\tau) - \sum g_i F_i$$

$$\frac{\partial}{\partial \tau} F_i = \omega_i \langle T\, b_i(\tau) a^+ \rangle - g_i \langle T a(\tau) a^+ \rangle$$

$$\Rightarrow \left(\frac{\partial}{\partial \tau} + \omega_i\right) F_i = -g_i G$$

b) $(-i\omega_n + \Omega)\, G(\omega_n) = -1 - \sum g_i F_i(\omega_n)$

$(-i\omega_n + \omega_i)\, F_i = -g_i G(\omega_n)$

$\Rightarrow F_i = -g_i G(\omega_n) / [-i\omega_n + \omega_i]$

$$\left(-i\omega_n + \Omega + \sum_i \frac{g_i}{[i\omega_n + \omega_i]}\right) G(\omega_n) = -1$$

$$G(\omega_n) = \left(i\omega_n - \Omega - \sum \frac{g_i}{i\omega_n - \omega_i}\right)^{-1}$$