Karlsruher Institut für Technologie

Institut für Theorie der Kondensierten Materie

Theorie der Kondensierten Materie II SS 2015

Prof. Dr. A. Shnirman

Dr. B. Narozhny

Blatt 1

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1. Scattering amplitude:

Recall the scattering problem in quantum mechanics. Scattering of a plane wave $e^{i\vec{k}\vec{r}}$ on a static, one-particle potential $V(\vec{r})$ is described by the wave function

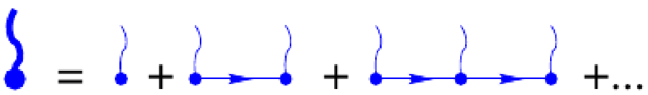
$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\vec{r}} + \chi_{\vec{k}}(\vec{r}),$$

where $\chi_{\vec{k}}(\vec{r})$ has the form of a spherical wave

$$\chi_{\vec{k}}(\vec{r}) = f(\vec{k}, k\vec{n})\frac{e^{ik|\vec{r}|}}{|\vec{r}|}, \quad |\vec{r}| \to \infty, \quad \vec{n} = \frac{\vec{r}}{|\vec{r}|}, \quad k = |\vec{k}|.$$

The function $f(\vec{k}, k\vec{n})$ is known as the scattering amplitude.

(a) Show that the scattering amplitude can be represented by a series of diagrams shown in the Figure.



Which expressions correspond to the elements of the graphs?
Hint: use the momentum representation.

(b) Derive the relation

$$f(\vec{k}_1, \vec{k}_2) = -\frac{m}{2\pi\hbar^2}F(\vec{k}_1, \vec{k}_2); \quad F(\vec{k}_1, \vec{k}_2) = V(\vec{k}_2 - \vec{k}_1) + \int \frac{d^3q}{(2\pi)^3}\frac{V(\vec{k}_2 - \vec{q})F(\vec{k}_1, \vec{q})}{\epsilon - \hbar^2 q^2/(2m) + i\delta}.$$

2. Second Quantization:

Consider an ideal Fermi gas of N particles in volume V in the ground state.

(a) Find the mean particle density and the mean particle number in some volume v.

(b) Consider the correlation of particle number densities with definite values of spin projection s_z at different points in space: find $\overline{n(\vec{r}_1, s_{z1})n(\vec{r}_2, s_{z2})}$ and compare to the product $\overline{n(\vec{r}_1, s_{z1})} \cdot \overline{n(\vec{r}_2, s_{z2})}$. Consider the cases of different and identical values of s_{z1} and s_{z2}.