Karlsruher Institut für Technologie | Institut für Theorie der Kondensierten Materie

Theorie der Kondensierten Materie II SS 2015

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1. Shallow well:

A "shallow well" is a potential well with the depth $U_0 \ll \hbar^2/(2ma^2)$, where a is the width (or radius) of the well. In such a potential, the energy of a bound state is much smaller than the well depth U_0, while its wave function extends over distances much greater than the well radius a.

Consider a schallow well in a D-dimensional space and find out in which case do the bound states exist.

(a) Show, that the energy of each bound state corresponds to a pole of the scattering amplitude $F(\vec{k}_1, \vec{k}_2)$ as a function of energy.

(b) Show, that bound states in shallow wells exist only for $D \leqslant 2$.

(c) Compare the results with the standard quantum-mechanical expressions.

Hints: Use the equation for $F(\vec{k}_1, \vec{k}_2)$ derived in the previous exercise. In D-dimensional space the integration measure becomes $d^D q/(2\pi)^D$. To simplify the calculations, you may replace the well potential in the equation for $F(\vec{k}_1, \vec{k}_2)$ by a δ-function.

2. Fermionic Green's functions

(a) Express the particle and current densities of a Fermi gas in terms of its single-particle Green's function.

(b) For a free Fermi gas use the expression for the particle density to derive the relation between the density n and the Fermi momentum p_F. Consider the cases of three- and two-dimensional gases.

3. Friedel oscillations

(a) For free fermions in a one-dimensional space (i.e., moving on a line) find the explicit expression for the Green's function $G_{\alpha\beta}(\epsilon; x, x')$.

(b) Repeat the calculation for the half-line $x > 0$ with the hard-wall boundary condition $\psi(x = 0) = 0$.

(c) In the latter case, show that the fermion density $n(x)$ oscillates as a function of the distance x from the boundary (the so-called Friedel oscillations). What is the period of the oscillations? Plot the resulting density $n(x)$.