Theorie der Kondensierten Materie II SS 2015

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Linked cluster expansion:

Consider the expectation value of the scattering matrix, which appears in the diagrammatic expansion of the Green's function $\langle 0|S|0\rangle$, where

$$S = Te^{-i\int_{-\infty}^{\infty} dt\, V_0(t)} \ .$$

Here $V_0(t)$ is the interaction Hamiltonian in the interaction representation, $V_0 = e^{iH_0t}Ve^{-iH_0t}$ and

$$V = \frac{1}{2}\int dr_1 dr_2\, \Psi^\dagger(r_1)\Psi^\dagger(r_2)V(r_1-r_2)\Psi(r_2)\Psi(r_1) \ .$$

(consider for definiteness spineless bosons)

(a) Consider a scenario in which the interaction was switched adiabatically on around time $t_- = -T/2$ and it is switched adiabatically off around time $t_+ = T/2$. Both switching on and switching off are performed within a time interval of order δT. In order to switch on and off adiabatically, δT must be long enough, but it is still much shorter than T, i.e., $\delta T \ll T$. Relate the energy of the interacting ground state E to the energy of the non-interacting ground state E_0 via $\langle 0|S|0\rangle$.

(b) Work out, using Wick's theorem, the diagrams contributing to $\langle 0|S|0\rangle$ in the first and in the second order in V_0. Classify these into connected and non-connected diagrams. Find all topologically non-equivalent connected diagrams. Determine the multiplicity coefficients, i.e., the number of times these topologically non-equivalent diagrams contribute to $\langle 0|S|0\rangle$.

(c) Argue, based on the result of (a), that the non-connected diagrams could not contribute to $\ln\langle 0|S|0\rangle$.

(d) Prove the "linked cluster theorem", which states that the following expansion holds

$$\ln\langle 0|S|0\rangle = \sum_m C_m \ ,$$

where

$$C_m = \frac{(-i)^m}{m!}\int dt_1\ldots dt_m \langle 0|TV_0(t_1)\ldots V_0(t_m)|0\rangle_{conn} \ .$$

The subscript *conn* means here "connected". Thus, C_m is the sum of all connected diagrams of order m in V_0. Determine the multiplicity factors, with which topologically non-equivalent diagrams contribute to C_m