Karlsruher Institut für Technologie

Institut für Theorie
der Kondensierten Materie

Theorie der Kondensierten Materie II SS 2015

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1. Polarons

We consider electrons in the conduction band of a semiconductor. The dispersion relation is $E(\vec{p}) = (\vec{p})^2/2m$, where m is the effective (band) mass and the energy is measured from the bottom of the conduction band. The electronic gas in the conduction band is non-degenerate, i.e., the chemical potential is in the gap between the valence and the conduction bands, i.e., $\mu < 0$.

Consider a situation in which electrons interact only via emission and absorption of virtual phonons (no direct Coulomb interaction). Effectively this means that the "wavy" line in our diagrammatic expansion is now replaced by a phononic line. The latter is proportional to the phonon Green's function:

$$U(\omega, \vec{q}) = g^2 \frac{\omega_0^2(\vec{q})}{\omega^2 - \omega_0^2(\vec{q}) + i0} \ . \tag{1}$$

Only acoustic phonons with the dispersion relation $\omega_0(\vec{q}) = c|\vec{q}|$ and $|\vec{q}| < q_D$ are taken into account. Here c is the sound velocity, q_D is the Debye momentum, and g is the coupling constant (deformation potential).

Thus far we considered only direct instantaneous interaction: $U(\vec{r}_1 - \vec{r}_2, t_1 - t_2) = V(\vec{r}_1 - \vec{r}_2)\delta(t_1 - t_2) \to U(\omega, \vec{q}) = V(\vec{q})$. In contrast the line due to phonons (Eq. (1)) describes interaction with retardation and is ω dependent.

(a) Calculate the lowest order contribution to the self-energy of the electrons, $\Sigma(\epsilon, \vec{p})$. The resulting Green's function describes now polarons (electrons dressed by phonons).

(b) From $\mathrm{Re}\Sigma(\epsilon, \vec{p})$ extract the dispersion relation of the polaron. Find the binding energy and the effective mass of the polaron. *Tip: show that near the mass shell ($\epsilon \approx E(\vec{p}) - \mu$) and for $|\vec{p}| \ll mc$ the self energy reads*

$$\Sigma(\epsilon, \vec{p}) = \epsilon_0 - \alpha_1 \left(\epsilon + \mu - E(\vec{p})\right) - \alpha_2 E(\vec{p}) \ .$$

(c) Consider $\mathrm{Im}\Sigma(\epsilon, \vec{p})$ and find the life-time of a polaron with momentum $\vec{p}$.

2. Fermionic chain (Kitaev model)

Consider spinless fermions on a one-dimensional chain of sites, numbered by an index n. The Hamiltonian reads $H = H_0 + V$, where

$$H_0 = \sum_n \left(t a_n^\dagger a_{n+1} + t a_{n+1}^\dagger a_n - \mu a_n^\dagger a_n \right)$$

and

$$V = \sum_n \left(\Delta a_n a_{n+1} + \Delta a_{n+1}^\dagger a_n^\dagger \right) \ .$$

Here t, Δ and μ are real constants.

(a) Find the Green's function G_0 corresponding to H_0. *Tip: use the Fourier representation.*

(b) Consider the perturbation series for the Green's function G of the full problem. Develop the diagrammatic rules. Sum up the series and determine the dispersion relation of the new excitations.

(c) Could the solution be found without perturbation theory?