Karlsruher Institut für Technologie

Institut für Theorie der Kondensierten Materie

Theorie der Kondensierten Materie II SS 2015

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1. Ruderman-Kittel effet:

Consider a localized spin immersed into a free Fermi-gas. The spin interacts with the local electronic spin density by means of the Hamiltonian

$$\hat{H}_{int} = J\hat{S}^i\hat{\sigma}^i(\vec{r}=0),$$

where the local spin density is given by

$$\hat{\sigma}^i(\vec{r}) = \psi^\dagger_\alpha(\vec{r})\sigma^i_{\alpha\beta}\psi_\beta(\vec{r}),$$

and $\sigma^{x,y,z}_{\alpha\beta}$ are the Pauli matrices.

Find the average spin polarization in the electronic system

$$\sigma^i(\vec{r}) = \langle\hat{\sigma}^i(\vec{r})\rangle,$$

at large distances awayt from the impurity spin (i.e., for $k_F r \gg 1$).

Show that the polarization oscillates as a function of r and find the oscillation period. J can be assumed to be small.

Hint: Use the coordinate representation for the electronic Green's functions.

2. Dynamical spin susceptibility:

Find the paramagnetic contribution to the electronic spin susceptibility $\chi(\omega, k)$ at $T = 0$. The spin susceptibility describes the response of the electronic system to an external magnetic field. Consider the limit $\omega \ll E_F$, $k \ll k_F$.

Verify that in the limit $\omega/k \to 0$, $k \to 0$ you recover the Pauli susceptibility. Discuss the importance of the proper limiting procedure and the order of limits.

Solve the problem in two ways - first, by a direct evaluation of the corresponding diagram, and second, by finding the imaginary part of $\chi(\omega, k)$ first, and then restoring the real part using the Kramres-Kronig relations.