Theorie der Kondensierten Materie II SS 2017

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1. Scattering in 2D: the logarithm and the renormalization group (40 Punkte)

Consider scattering off a short-ranged potential in two dimensions. As an example of such a potential one can take a potential well. We are interested in the energy dependence of the scattering amplitude in the "low energy" limit, i.e. for energies

$$\epsilon \ll \epsilon_a = \frac{\hbar^2}{2ma^2},$$

where a is the width of the well.

Similarly to the problem of the "shallow well" (see the exercise sheet 1), the energy dependence appears after integrating over small momenta $q \ll 1/a$. Therefore in that problem it was sufficient to approximate the well by a δ-function potential. In that case, the momentum integrals diverge logarithmically and have to be cut off at $q \approx 1/a$. This solution yields a qualitatively correct result, but compeltely neglects deep levels in the well if they exist. Let us use a better method.

(a) Consider the perturbative (diagrammatic) expansion for the scattering amplitude (again, see the exercise sheet 1). Using this expansion, calculate the derivative $\partial F(\epsilon)/\partial\epsilon$ by differentiating each individual diagram.

(b) Sum up the resulting series and show that in the limit $|\epsilon| \ll \epsilon_a$ the scattering amplitude satisfies the so-called renormalization group equation

$$\frac{\partial F(\epsilon)}{\partial \epsilon} = \frac{m}{2\pi\epsilon} F^2(\epsilon).$$

(c) Solving this equation, connect the logarithmic cut-off with the scattering amplitude at high energies.

2. Polarizability of a ground state: (60 Punkte)

Consider a charged particle in a two-dimensional "shallow well". Suppose the particle is in the ground state. What is its polarizability in a weak external field?

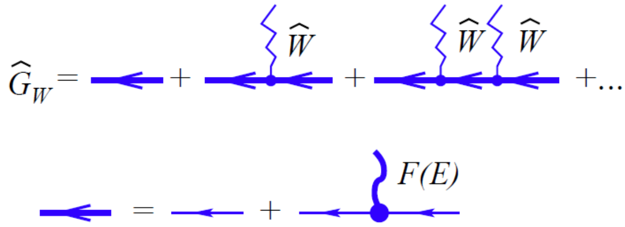
(a) Using the results of the previous exercise, write down the Green's function in the momentum representation. Define its infinitesimal imaginary part (defining the integration contour around the poles) by the requirement that the bound state of the well is occupied, while the states of the continuous spectrum are empty.

(b) Express the dipole moment of the system in terms of the exact Green's function in the momentum representation.

(c) Consider the Green's function in the applied electric field. The operator describing the field can be written as

$$W = -eEx.$$

Show, that the Green's function can be represented as a series corresponding to the diagrams in the figure.



(d) Consider now the correction to the Green's function that is linear in the electric field. Which diagrams vanish after the energy and momentum integration? Use the remaining diagrams to find a graphical representation for the polarizability and evaluate the corresponding expression.

Hint: In this problem you will need to consider carefully the analytic properties of the scattering amplitude derived in the previous problem: the final result will be expressed in terms of the residue of the scattering amplitude.