Theorie der Kondensierten Materie II SS 2017

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1. Fermionic chain (Kitaev model) $(15 + 25 + 10 = 40$ Punkte)

Consider spinless fermions on a one-dimensional chain of sites, numbered by an index n. The Hamiltonian reads $H = H_0 + V$, where

$$H_0 = \sum_n \left(t a_n^\dagger a_{n+1} + t a_{n+1}^\dagger a_n - \mu a_n^\dagger a_n \right)$$

and

$$V = \sum_n \left(\Delta a_n a_{n+1} + \Delta a_{n+1}^\dagger a_n^\dagger \right) .$$

Here t, Δ and μ are real constants.

(a) Find the Green's function G_0 corresponding to H_0. *Tip: use the Fourier representation.*

(b) Consider the perturbation series for the Green's function G of the full problem. Develop the diagrammatic rules. Sum up the series and determine the dispersion relation of the new excitations. What is the relation between the imaginary part of the Green's function and the ground state and the occupation of the single-particle states?

(c) Could the solution be found without perturbation theory?

2. Heavy particle in the Fermi-gas: $(5 + 15 + 20 + 20 = 60$ Punkte)

Consider an atom of the mass M interacting with the particles of the surrounding Fermi gas. In the absence of scattering, the Green's function of the atom is given by the usial expression

$$G(\epsilon, \vec{p}) = \frac{1}{\epsilon - p^2/(2M) + i\delta}.$$

We are interested in the effects of interaction between the atom and the fermions. For simplicity, we assume weak, contact interaction

$$U(\vec{r} - \vec{r}') = \lambda \delta(\vec{r} - \vec{r}').$$

(a) Assume that the atom is much heavier than the fermions, $M \gg m$. Characterize the scattering of the fermions on the heavy atom by analyzing the energy and momentum conservation.

Hint What is the maximal momentum transfer and the corresponding transferred energy ϵ_M?

(b) Since the fermions are much faster than the atom, we will use the "adiabatic" approximation. This means that we will exclude ("integrate out") the dynamics of the fermions, which leads to an effective retarded "self-interaction" of the heavy particle. Show that the effective interaction appears in the second order in λ and is desribed by the density-density correlator (the polarization operator) of the Fermi gas.

Hint Recall "integrating out" phonons to obtain the effective BCS interaction in the theory of superconductivity discussed in TKM I.

(c) Find the self-energy of the heavy particle in the leading order in the effective interaction. In particular, consider the limiting cases of low and high energies ($\epsilon \ll \epsilon_M$ and $\epsilon \gg \epsilon_M$, respectively).

Hint Use the geberal analytic properties of the Green's function and the self-energy to regularize the formally divergent integrals.

(d) For not too small λ and $\epsilon \approx \epsilon_M$, the renormalization of the Green's function may be logarithmically large. Using the perturbation theory, sum up the leading (logarithmically large) contributions and show that the full Green's function of the particle in the energy region $\epsilon_M \ll \epsilon \ll E_F$ has the form

$$G(\epsilon, \vec{p}) = \frac{a}{(\epsilon - p^2/(2M) + i\delta)^{1+\alpha}},$$

where the exponent α depends on λ.

Hint Use the renormalization group ideas discussed in the previous exercise sheet.