

Condensed Matter Theory II: Many-Body Theory (TKM II) SoSe 2022

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Sheet 0

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Live exercise on 21 April 2022

1. Complex contour integrals

(? Bonus points)

Calculate the following integrals using the techniques discussed during class.

(a) $I_1 = \oint_{|z|=2} dz \frac{z}{\exp(iz)-1}$

Hint: Use the residue theorem.

(b) $I_2 = \oint_{|z|=2} dz \exp\left(\frac{z}{z-1}\right)$

Hint: Use the residue theorem, calculating the residue via Laurent expansion.

(c) $I_3 = \int_{-\infty}^{\infty} dx \frac{x \sin(\alpha x)}{x^2 + \beta^2}$, with real numbers $\alpha, \beta > 0$.

Hint: Find a way to rewrite this integral in terms of a complex contour integral. Solve the resulting integral using the residue theorem.

(d) $I_4 = \int_{-1}^1 dx (1+x)^\alpha (1-x)^{1-\alpha}$, $0 < \alpha < 1$

Hint: The integrand contains a branch cut in the complex plane. Find this branch cut, and show that the integral can be expressed as

$$I_4 = \text{const} \cdot \oint_{\mathcal{C}} dz (1+z)^\alpha (z-1)^{1-\alpha} \quad (1)$$

where $\mathcal{C}$ is an appropriately chosen contour enclosing the line $[-1, 1]$. Determine the constant and solve the resulting integral using a substitution $z \rightarrow 1/w$.