Karlsruher Institut für Technologie   Institute for Condensed Matter Theory
Institute for Quantum Materials and Technologies

Condensed Matter Theory II: Many-Body Theory (TKM II)   SoSe 2023

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Homework assignment 1
Deadline: 28 April 2023

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1. Real-space Green's functions (5 + 5 = 10 points)

Starting from the momentum-space retarded Green's function

$$G^{\mathrm{R}}(\mathbf{k}, E) = \frac{1}{E - \frac{\hbar^2}{2m}\mathbf{k}^2 + \mathrm{i}\delta}, \tag{1}$$

where $\mathbf{k}$ is a d-dimensional momentum, calculate the real-space retarded Green's function $G^{\mathrm{R}}(\mathbf{r} - \mathbf{r}', E)$

(a) in one dimension, $d = 1$

(b) in three dimensions, $d = 3$.

Hint: Integrals can usually be solved using contour integration, as practiced on sheet 0.

2. Green's function in graphene (10 points)

Starting with the effective Hamiltonian of electrons in one valley of graphene,

$$\widehat{H} = v(\sigma_x \hat{p}_x + \sigma_y \hat{p}_y)$$

(here σ_x and σ_y are the Pauli matrices in the sublattice space), find the retarded Green's function $G^R(\varepsilon, \mathbf{p})$ for free electrons as a 2×2 matrix in the sublattice space.

3. Friedel oscillations around a barrier in a 1D system (8 + 8 + 8 + 6 = 30 points)

(a) Express the density $n(x)$ of non-interacting fermions in $D = 1$ spatial dimension at zero temperature in terms of the imaginary part of the retarded Green's function $G^R(\varepsilon; x, x')$, starting from the zero temperature expression

$$n(x) = \sum_{\alpha \in \text{occupied}} |\psi_\alpha(x)|^2, \tag{2}$$

where ψ_α are the single particle wave functions and the sum is taken over all states with energy $\varepsilon < \varepsilon_{\mathrm{F}} := 0$.

(b) Find $G^R(\varepsilon; x, x')$ for non-interacting one-dimensional fermions with parabolic dispersion $E = \frac{\hbar^2 k^2}{2m}$ in the presence of a δ-barrier $V(x) = V_0 a \delta(x)$.

(c) Using the relation between the scattering amplitude $f(\mathbf{k}, k\mathbf{n})$ and the Green's function (see lecture notes for the relation), calculate the reflection and transmission amplitudes for the δ-barrier. *Hint: How are the reflection and transmission amplitudes related to the wave functions before and after scattering?*

(d) Calculate $n(x)$ around this barrier at zero temperature. (You can use the formula derived in part (a)).