Condensed Matter Theory II: Many-Body Theory (TKM II)   SoSe 2023

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Homework assignment 13
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1. Conductance of a non-interacting quantum wire (10 + 10 = 20 point)

In Sec. 7.8.1 of the Lecture Notes, an impurity in a quantum wire was described by the following term in the Hamiltonian: $H_{\text{imp}} = \int dx\, \mathcal{U}(x)\psi^\dagger(x)\psi(x)$, where $\psi(x) = \psi_+(x) + \psi_-(x)$ is the total fermionic operator, including right-moving and left-moving contributions. The impurity was then characterized by the forward and backward scattering amplitudes

$$\mathcal{U}_{\text{f}} = \mathcal{U}(k=0) \equiv \int dx\, \mathcal{U}(x), \tag{1}$$

$$\mathcal{U}_{\text{b}} = \mathcal{U}(k=2k_F) \equiv \int dx\, \mathcal{U}(x) e^{-2ik_F x}. \tag{2}$$

Consider a quantum wire connected to two reservoirs biased by the voltage V. The interfaces between the wire and the reservoirs do not lead to the electron backscattering. The conductance g of the wire is given by the ratio of the electrical current $I = \frac{e}{h}\int \mathrm{d}k \frac{\partial \varepsilon_k}{\partial k} n_k$ and the voltage V: $g = I/V$. Calculate the conductance of a non-interacting wire at zero temperature...

(a) in the absence of the impurity potential (clean wire).

(b) in the presence of a single (weak) impurity in the wire.

2. Interaction-induced backscattering (15 points)

In Sec. 7.9.1 of Lecture Notes, interaction-induced backscattering in a spinful Luttinger liquid was introduced. After bosonization, the interaction Hamiltonian is

$$H_{1\perp} = \frac{2g_{1\perp}}{(2\pi\lambda)^2} \int \mathrm{d}x \cos[2\sqrt{2}\phi_\sigma(x)]. \tag{3}$$

Considering the coupling $g_{1\perp}$ as small, derive the RG equation

$$\frac{dg_{1\perp}}{d\ln b} = (2 - 2K_\sigma) g_{1\perp}\,.$$

in the same way as it was done for the impurity-induced backscattering amplitude $\mathcal{U}_{\text{b}}$ in Sec. 7.8.1 of Lecture Notes.

3. Kubo formula for the conductivity (15 points)

In Sec. 8.2 of Lecture Notes, the Kubo formula for the conductivity of non-interacting fermions was derived. Starting from the Matsubara expression for the current-current response function,

$$\mathscr{D}^M_{jj;\mu\nu}(\mathbf{r},\mathbf{r}';\omega_m) = e^2 \frac{1}{\beta} \sum_{\varepsilon_n} \hat{v}_\mu\, \mathscr{G}_{M,0}(\mathbf{r},\mathbf{r}';\varepsilon_n+\omega_m)\, \hat{v}_\nu\, \mathscr{G}_{M,0}(\mathbf{r}',\mathbf{r},\varepsilon_n).$$

derive the retarded response function $\mathscr{D}^R_{jj;\mu\nu}(\mathbf{r},\mathbf{r}';\omega)$ [Eq. (8.31) of Lecture Notes].