Karlsruher Institut für Technologie

Einführung in Theoretische Teilchenphysik

Lecture: Prof. Dr. M. M. Mühlleitner – Exercises: M.Sc. Martin Gabelmann, Dr. Sophie Williamson

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Exercise Sheet 0

Discussion: Tuesday 10.11.20, Thursday 12.11.20

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1. **Applications of invariance**: When two particles of mass M, having 3-momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ respectively, collide in the centre-of-mass frame, their 3-momenta are related by $\mathbf{p}_1 = -\mathbf{p}_2$.

   (a) Show how the Mandelstam variable, $s \equiv (p_1 + p_2)^2$, is related to the energy in the centre-of-mass frame, where p_1 and p_2 denote 4-momenta.

   (b) Using the equations for De-Broglie wavelength and the mass-energy equivalence, estimate the typical length scales investigated at
   (i) the Large Electron-Positron Collider (LEP) ($\sqrt{s} = 45 - 209$ GeV)
   (ii) Run II of the Large Hadron Collider ($\sqrt{s} = 13$ TeV)

2. Given the 3-dimensional Levi-Cevita tensor,

$$\epsilon_{ijk}\epsilon_{lmn} = \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix},$$

   calculate:

   (a) $\epsilon_{123}\epsilon_{123}$

   (b) $\epsilon_{123}\epsilon_{321}$

3. **Nonrelativistic limit of the Klein-Gordon equation** Taking a system with only one spatial coordinate, (t, x), and writing the scalar field as

$$\phi(t, x) = \psi(t, x)e^{-imt},$$

   (a) Find an equation for $\psi(t, x)$ by plugging $\phi(t, x)$ into the Klein-Gordon equation.

   (b) Using the energy operator, $E \to \hat{E} = i\partial_t$, show that for a system with kinetic energy E_k and total energy $E = E_k + m$, we have

$$\frac{i}{m}\partial_t\psi = \frac{E_k}{m}\psi, \qquad -\frac{1}{m^2}\partial_t^2\psi = \frac{E_k^2}{m^2}\psi\,.$$