Karlsruher Institut für Technologie

Einführung in Theoretische Teilchenphysik

Lecture: Prof. Dr. M. M. Mühlleitner – Exercises: M.Sc. Martin Gabelmann, Dr. Sophie Williamson

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Exercise Sheet 1

Hand-in Deadline: Friday 13.11.20, 14:00.
Discussion: Tuesday 17.11.20, Thursday 19.11.20.

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1. **[7 points] Natural Units.** In natural units, all quantities are expressed in terms of their mass/energy units.

   (a) [4 points] Ultraviolet radiation, such as that emitted by the sun, has a wavelength between $10-400\,\text{nm}$. Express a ray of 200 nm in eV.
   Hint: Use that $c = 3\times 10^8\,\frac{\text{m}}{\text{s}}$, $\hbar = 1.1\times 10^{-34}\,\frac{\text{kgm}^2}{\text{s}}$ and $\text{eV} = 1.6\times 10^{-19}\,\frac{\text{kg m}^2}{\text{s}^2}$. Start by finding the equivalence between m and kg.

   (b) [3 points] The dark energy density of the universe is roughly $(2\text{meV})^4$. Express this in $\text{kg}\,\text{m}^{-3}$.

2. **[5 points] Conservation of probability.** Using the non-relativistic Schrödinger equation,

$$i\partial_t\psi(\mathbf{x},t) = -\frac{\nabla^2}{2m}\psi(\mathbf{x},t) + V\psi(\mathbf{x},t),$$

   and its complex conjugate, derive the continuity equation for the probability density

$$\frac{\partial\rho(\mathbf{x},t)}{\partial t} + \nabla\cdot\mathbf{j}(\mathbf{x},t) = 0\,,$$

   with

$$\rho(\mathbf{x},t) = |\psi(\mathbf{x},t)|^2, \qquad \mathbf{j}(\mathbf{x},t) = -\frac{i}{2m}\left(\psi^*(\mathbf{x},t)\nabla\psi(\mathbf{x},t) - \psi(\mathbf{x},t)\nabla\psi^*(\mathbf{x},t)\right)\,.$$

3. **[8 points] Relativistic wave equation for a scalar particle**

   (a) [1 points] Give reasons why the Schrödinger equation is not suitable for the relativistic formulation of quantum mechanics.

   (b) [2 points] Give two reasons why the Klein-Gordon equation is not an alternative.

   (c) For a scalar field, the Klein-Gordon equation is given by

$$\left(-\frac{\hbar^2}{c^2}\frac{\partial^2}{\partial t^2} + \hbar^2\nabla^2 - m^2c^2\right)\phi(\mathbf{x},t) = 0 \tag{1}$$

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i. [2 points] Using

$$-i\hbar\nabla \to -i\hbar\nabla + \frac{q}{c}\mathbf{A}\,, \tag{2}$$

derive the equation describing a scalar particle in a classical magnetic field.

ii. [2 points] Using the ansatz

$$\phi(x) = \psi(\mathbf{x}, t)e^{\left(-\frac{i}{\hbar}mc^2 t\right)}\,, \tag{3}$$

where $\psi(\mathbf{x}, t)$ contains the non-relativistic part, discuss the non-relativistic limit.
Hint: Start by expanding the relativistic energy-momentum equation, dropping terms of $\mathcal{O}(p^4)$, and then using the ansatz to determine an expression for your equation in the non-relativistic limit. In the last step, use your expression from part i.

iii. [1 point] Show that the result in (ii) is equivalent to using substitution (2) in the Schrödinger equation for a free field,

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x}, t) = i\hbar\frac{\partial\psi(\mathbf{x}, t)}{\partial t}\,.$$