Einführung in Theoretische Teilchenphysik

Lectures: Prof. Dr. M. M. Mühlleitner – Exercises: M.Sc. Martin Gabelmann, Dr. Sophie Williamson

Exercise Sheet 8

Hand-in Deadline: Friday 22.01.21, 14:00.
Discussion: Tuesday 26.01.21, Thursday 28.01.21.

1. **[15 points] Unitarity in $\nu\bar{\nu} \to W_L^+ + W_L^-$ scattering:**

Consider the production of longitudinally polarised W-bosons via $\nu\bar{\nu}$-scattering,

$$\nu_e(p_1) + \bar{\nu}_e(p_2) \to W_L^+(q_1) + W_L^-(q_2)$$

(a) **[1 point]** Draw the two Feynman diagrams describing this process at tree-level in the Standard Model.

(b) **[3 points]** Determine the corresponding scattering amplitude using the Feynman rules given below, and show that

$$\mathcal{M}_t = \frac{-\,e^2}{4\sin^2\theta_w}\frac{1}{t}\left[\bar{v}(p_2)\,\gamma^\nu\,(1-\gamma_5)\,(\not{p}_1 - \not{q}_1)\,\gamma^\mu\,u(p_1)\right]\,\epsilon^*_\mu(q_1)\,\epsilon^*_\nu(q_2)\,,$$
$$\mathcal{M}_s = \frac{e^2}{4\sin^2\theta_w}\frac{1}{s-m_Z^2}\left[\bar{v}(p_2)\,\gamma_\alpha\,(1-\gamma_5)\,\Gamma^{\mu\nu\alpha}(q_1,q_2,s)\,u(p_1)\,\right]\,\epsilon^*_\mu(q_1)\,\epsilon^*_\nu(q_2)\,,$$

where s, t denote Mandelstam variables, and the electron is assumed to be massless. Calculate the explicit expression of the form factor $\Gamma^{\mu\nu\alpha}(q_1,q_2,s)$ as a linear combination of 4-momenta and metric tensors.

(c) **[4 points]** The longitudinal polarization vector of a massive gauge boson with mass m can be written generically as

$$\epsilon_L^\mu(k) = \gamma\left(|\vec{\beta}|, \hat{\vec{\beta}}\right)\,,$$

with

$$\vec{\beta} \equiv \frac{\vec{k}}{k^0} \qquad \gamma \equiv (1-\beta^2)^{-1/2} \qquad \hat{\vec{\beta}} \equiv \frac{\vec{\beta}}{|\vec{\beta}|}.$$

Show that in the centre-of-mass frame, the longitudinally polarised W-bosons are given by

$$\epsilon_L^\mu(q_1) = \frac{\sqrt{s}}{2m_W}\left(\sqrt{1-\frac{4m_W^2}{s}}, \sin\theta, 0, \cos\theta\right)\,,$$
$$\epsilon_L^\mu(q_2) = \frac{\sqrt{s}}{2m_W}\left(\sqrt{1-\frac{4m_W^2}{s}}, -\sin\theta, 0, -\cos\theta\right)\,.$$

Here, θ is defined as the scattering angle between the incoming neutrino ν_e and the outgoing W^+ in the centre-of-mass frame.

(d) **[5 points]** In the Dirac representation, Dirac fermions are written as

$$u^+(p) = \frac{\not{p}+m}{\sqrt{(p^0+m)}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad u^-(p) = \frac{\not{p}+m}{\sqrt{(p^0+m)}}\begin{pmatrix}0\\1\\0\\0\end{pmatrix},$$

$$v^+(p) = \frac{\not{p}-m}{\sqrt{(p^0+m)}}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \quad v^-(p) = \frac{\not{p}-m}{\sqrt{(p^0+m)}}\begin{pmatrix}0\\0\\1\\0\end{pmatrix},$$

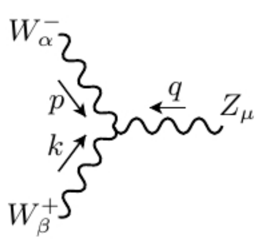
with the gamma matrices given by

$$\gamma^0 = \begin{pmatrix}\mathbb{1}_{2\times 2} & 0\\ 0 & -\mathbb{1}_{2\times 2}\end{pmatrix}, \qquad \gamma^k = \begin{pmatrix}0 & \sigma^k\\ -\sigma^k & 0\end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix}0 & \mathbb{1}_{2\times 2}\\ \mathbb{1}_{2\times 2} & 0\end{pmatrix},$$

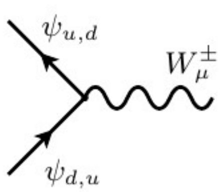
where σ^k are the Pauli-matrices. Using this representation explicitly, check that the two scattering amplitudes $M_{s,t}$ behave like $\mathcal{M}_{s,t} \sim s$ in the limit $s \to \infty$.

(e) **[2 points]** How does the total amplitude $\mathcal{M}_t + \mathcal{M}_s$ change in the high energy limit? Interpret your result.

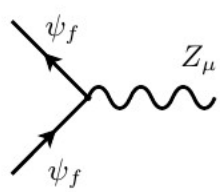
Feynman Rules:



$$ig\cos\theta_W\left[g_{\alpha\beta}(p-k)_\mu + g_{\beta\mu}(k-q)_\alpha + g_{\mu\alpha}(q-p)_\beta\right]$$



$$i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2}$$



$$i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f - g_A^f\gamma_5\right)$$

Figure 1: Feynman rules for the relevant electroweak vertices, showing a) the WWZ vertex and b) ffV vertices, with $g_V^f = \frac{1}{2}I_3^f - Q_f\sin^2\theta_w \qquad g_A^f = \frac{1}{2}I_3^f$.

2. **[5 points] The Glashow-Iliopoulos-Maiani Mechanism:**

In the following, consider only the 3 lightest quarks u, d, s of the Standard Model. Their electroweak interactions are described by the following Lagrangian density:

$$\mathcal{L} = -\frac{g}{2\sqrt{2}}W_\mu^+ J_{\rm CC}^\mu - \frac{g}{2\cos\theta_W}Z_\mu J_{\rm NC}^\mu,$$

$$J_{\rm CC}^\mu = \overline{u}\,\gamma^\mu(1-\gamma_5)\,d', \qquad J_{\rm NC}^\mu = \overline{u}\,\gamma^\mu(g_v - g_a\gamma_5)\,u + \overline{d}'\,\gamma^\mu(g_v - g_a\gamma_5)\,d',$$

where $d' = d\,\cos\theta_C + s\,\sin\theta_C$, with s, d representing the physical eigenstates of the down quark and strange quark respectively, and θ_C denoting the Cabbibo angle.

(a) **[2 points]** Show that both the charged and the neutral currents would lead to flavour-changing processes, e.g. ones which alter strangeness by $\Delta S = 1$.

(b) **[3 points]** As there was was no experimental evidence for flavour-changing processes, in 1973, Glashow, Iliopoulos, and Maiani postulated the charm quark to prohibit these direct flavour changing neutral currents (FCNCs) in the Standard Model.

Check whether these FCNCs are present when a second quark-doublet is introduced,

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix} \qquad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ d\,X + s\,Y \end{pmatrix} .$$

Determine the appropriate values for X and Y and calculate the unitarity of the mixing matrix $V_{dd',ss'}$

$$V_{dd',ss'} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ X & Y \end{pmatrix} .$$