Karlsruher Institut für Technologie

Einführung in Theoretische Teilchenphysik

Lectures: Prof. Dr. M. M. Mühlleitner – Exercises: M.Sc. Martin Gabelmann, Dr. Sophie Williamson

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Exercise Sheet 9

Hand-in Deadline: Friday 29.01.21, 14:00.
Discussion: Tuesday 02.02.21, Thursday 04.02.21.

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1. **[15 points] Higgs Boson Decay at Tree-Level**: In this exercise we want to discuss all $1 \to 2$ Higgs boson decays in the Standard Model of particle physics, which emerge at tree-level. The general formula for the decays is

$$d\Gamma(H \to XY) = \frac{1}{2m_H} d\Phi_2 \overline{\sum}_{\text{spins, colors}} |\mathcal{M}_{H\to XY}|^2 \,,$$

where $d\Phi_2$ is the two-particle phase space and for $X = Y$ an additional symmetry factor $\frac{1}{2}$ has to be added.

From this, show that the Higgs decays to the products detailed below are given by the following:

(a) **[5 points]** a pair of W-bosons, $H \to W^+W^-$,

$$\Gamma(H \to W^+W^-) = \frac{G_F m_H^3}{8\sqrt{2}\pi}\left(1 - \frac{4m_W^2}{m_H^2}\right)^{1/2}\left(1 - \frac{4m_W^2}{m_H^2} + \frac{12m_W^4}{m_H^4}\right);$$

(b) **[2 points]** a pair of Z-bosons, $H \to ZZ$,

$$\Gamma(H \to ZZ) = \frac{G_F M_H^3}{16\sqrt{2}\pi}\left(1 - \frac{4M_Z^2}{M_H^2}\right)^{\frac{1}{2}}\left(1 - \frac{4M_Z^2}{M_H^2} + \frac{12M_Z^4}{M_H^4}\right);$$

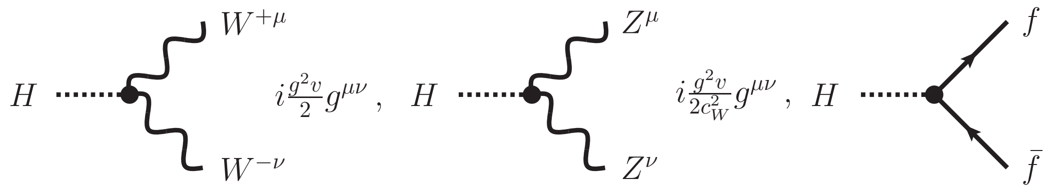
(c) **[5 points]** a pair of fermions, $H \to ff$,

$$\Gamma(H \to f\bar{f}) = N_c \frac{G_F m_f^2 m_H}{4\sqrt{2}\pi}\left(1 - \frac{4m_f^2}{m_H^2}\right)^{3/2} \quad ;$$

with $G_F = \frac{1}{\sqrt{2}v^2} = 1.16638 \cdot 10^{-5}\,\text{GeV}$ and a color factor N_c, which is 3(1) for quarks (leptons). For the decays to gauge-bosons, make use of the polarisation sum for massive gauge bosons with mass m_V, being

$$\sum_{\lambda=1}^{3} \epsilon_\mu(p,\lambda)\epsilon_\nu^*(p,\lambda) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_V^2} \,.$$

The relevant Feynman rules are given by



$$H \to W^{+\mu}W^{-\nu}: \; i\frac{g^2 v}{2} g^{\mu\nu}\,, \quad H \to Z^\mu Z^\nu: \; i\frac{g^2 v}{2c_W^2} g^{\mu\nu}\,, \quad H \to f\bar{f}: \; i\frac{m_f}{v}\,.$$

where we can identify $m_W = \frac{1}{2}gv$ and $m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v = \frac{m_W}{c_W}$ with the cosine of the weak mixing angle $c_W = \frac{m_W}{m_Z}$.

(d) **[3 points]** For each final state, $H \to XY$, we define the branching ratio by the ratio of the partial width to the total width of the Higgs boson, i.e.

$$\mathrm{BR}(H \to XY) = \frac{\Gamma(H \to XY)}{\sum_{AB} \Gamma(H \to AB)}\,.$$

Consider the decays into the two massive gauge bosons as well as into the four heaviest fermions of the Standard Model, which have masses

$$\begin{aligned} &\text{Gauge bosons}: && m_W = 80.4\,\mathrm{GeV}\,, && m_Z = 91.2\,\mathrm{GeV}\,, \\ &\text{Quarks with } N_c = 3: && m_t = 173.2\,\mathrm{GeV}\,, && m_b = 4.8\,\mathrm{GeV}\,, && m_c = 1.3\,\mathrm{GeV}\,, \\ &\text{Leptons with } N_c = 1: && m_\tau = 1.8\,\mathrm{GeV}\,. \end{aligned}$$

Make a graphic that shows the branching ratios of the Higgs boson as a function of the Higgs boson mass m_H between 10 and 1000 GeV.

2. **[5 points] SU(5) Symmetry Breaking:**

In 1974, Howard Georgi and Sheldon Glashow proposed one of the first forms of grand unified theory (GUT) – based on the SU(5) symmetry group – into which the Standard model could be embedded. It was postulated that the SU(5) symmetry would break down to the Standard Model SU(3) × SU(2) × U(1) gauge group at some high energy scale below the so-called "grand-unification scale". This would happen when the GUT Higgs field, H, transforming in its adjoint representation (dimension $N^2 - 1 = 24$), acquired a vacuum expectation value. The potential for the GUT Higgs field is given by

$$V(H) = -m_1^2(\mathrm{tr}H^2) + \lambda_1(\mathrm{tr}H_2^2) + \lambda_2(\mathrm{tr}H^4)\,.$$

(a) **[2 points]** Owing to the SU(5) symmetry, $\langle H \rangle$ can be taken as diagonal. Use the parametrisation

$$H = \mathrm{diag}(a + b + 2d, -a + 2d, -b + 2d, c - 3d, -c - 3d),$$

where a, b, c and d are real scalar fields, to rewrite the potential.

(b) **[3 points]** Show that when $a = b = c = 0$, and d is non-vanishing, d is an extremum of the potential, such that

$$\langle H \rangle = v_1 \mathrm{diag}(2, 2, 2, -3, -3).$$

Compute v_1 in terms of the parameters in the potential.