Karlsruher Institut für Technologie

Einführung in Theoretische Teilchenphysik

Lectures: Prof. Dr. M. M. Mühlleitner – Exercises: M.Sc. Martin Gabelmann, Dr. Sophie Williamson

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Exercise Sheet 10

Hand-in Deadline: Friday 05.02.21, 14:00.
Discussion: Tuesday 09.02.21, Thursday 11.02.21.

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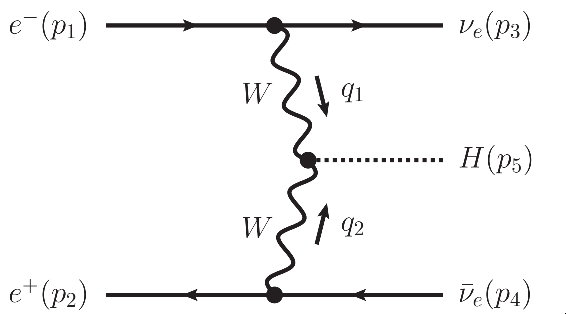
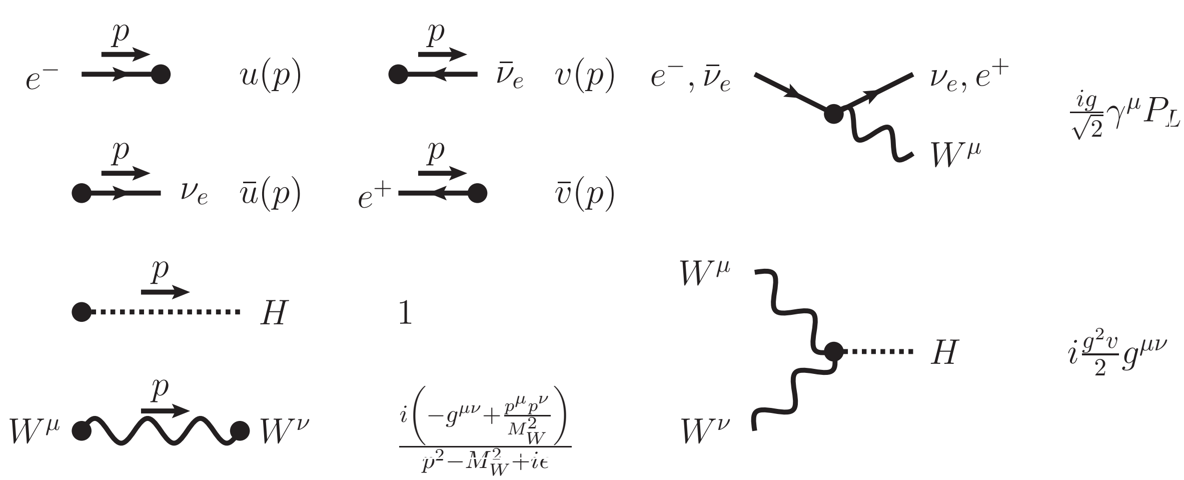
1. **[13 points] Higgs Production via W Fusion**: Higgs production via W fusion is one of the main Higgs production processes at electron-positron colliders. At leading order, this process is given by a single Feynman diagram:



The corresponding Feynman rules are

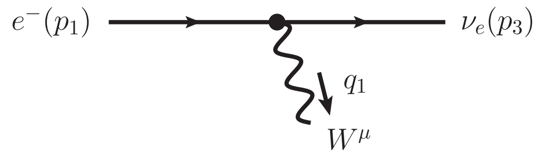


with the weak coupling constant $g \simeq 0.65$ and the vacuum expectation value of the Higgs field $v \simeq 246$ GeV.

(a) **[2 points]** Show the following properties of the chirality projection operators $P_{\substack{R\\L}} = \frac{1\pm\gamma^5}{2}$:

$$\left(P_{\substack{L\\R}}\right)^2 = P_{\substack{L\\R}}\,, \qquad P_R + P_L = 1\,, \qquad P_R - P_L = \gamma^5\,,$$
$$\left(P_{\substack{L\\R}}\right)^\dagger = P_{\substack{L\\R}}\,, \qquad P_{\substack{L\\R}}\gamma^\mu = \gamma^\mu P_{\substack{R\\L}}\,, \qquad \gamma^0\left(P_{\substack{L\\R}}\right)\gamma^0 = P_{\substack{R\\L}}\,.$$

(b) **[3 points]** Consider first the following sub-diagram:



where the W boson is "amputated" in that the subdiagram is not connected to the rest of the diagram, i.e. the contraction of the W boson with its polarisation vector is omitted and instead the matrix element contains an open Lorentz index μ.

i. **[1 point]** Write down the corresponding matrix element $\mathcal{M}_1^\mu$ and calculate $q_{1\mu}\mathcal{M}_1^\mu$, eliminating the explicit momentum dependence of the expression, assuming also that the neutrino has a mass m_ν.

ii. **[2 points]** What happens in the limit $m_e = m_\nu$? Show that for $m_e = m_\nu = 0$, $q_{1\mu}\mathcal{M}_1^\mu$ vanishes.

(c) **[3 points]** Considering only massless fermions, $m_e = m_\nu = 0$, show that one obtains, after spin summation, the following for the squared matrix element of the sub-diagram:

$$\sum_{s_1,s_3} |\mathcal{M}_1|^{2,\mu\nu} \equiv \sum_{s_1,s_3} \mathcal{M}_1^\mu \mathcal{M}_1^{\dagger,\nu} = g^2\left(p_1^\mu p_3^\nu + p_3^\mu p_1^\nu - p_1\cdot p_3 g^{\mu\nu} + i\epsilon^{\mu\nu\rho\sigma}p_{1,\rho}p_{3,\sigma}\right)\,.$$

(d) **[5 points]** Use this to calculate the spin-averaged squared matrix element of the whole Feynman diagram. You should obtain

$$\overline{\sum}|\mathcal{M}|^2 = \frac{g^8 v^2}{4}\frac{1}{(q_1^2 - M_W^2)^2(q_2^2 - M_W^2)^2}\; p_1\cdot p_4\; p_2\cdot p_3\,.$$

Hint: Consider first the result of the "middle part" (W propagators, HWW vertex) separately. The limit $\epsilon \to 0$ of the $i\epsilon$ terms in the propagators can be taken immediately. Take care to contract the correct Lorentz indices.

2. **[7 points] Weak Boson Decay:** Consider a massive vector field Z^μ (with mass M and momentum $k = p_1 + p_2$) and a Dirac fermion field Ψ, interacting via

$$\mathcal{L}_{\text{int}} = Z^\mu \bar{\Psi}(g_V - g_A\gamma_5)\Psi\,.$$

The amplitude for the decay of a massive vector boson to two fermions is given by

$$\mathcal{M} = \epsilon^{*\mu}\bar{v}_2\gamma_\mu(g_V - g_A\gamma_5)u_1\,,$$

and this result holds both if Ψ and $\bar{\Psi}$ are related, e.g. in the decay $Z^0 \to e^+e^-$, and if $\bar{\Psi}$ is a different Dirac field than Ψ, e.g. in the decay $W^+ \to e^+\bar{\nu}$. Compute the rates for the decay processes,

(a) $W^+ \to e^+\bar{\nu}_e$,

(b) $Z^0 \to e^+e^-$,

(c) $Z^0 \to \bar{\nu}_e \nu_e$,

neglecting the electron mass. Express your results in GeV.

Hint: start by computing the decay rate $|\mathcal{M}|^2$ without specifying the quantities $g_{V,A}$. Sum over the final spins and average over the three initial polarisations. Find the total decay rate, Γ, remembering to divide by the symmetry factor. Generally you should find, for distinguishable massless outgoing particles, that

$$\Gamma = \frac{1}{12\pi}(g_V^2 + g_A^2)M \, .$$

Useful data:

$W^+ \to e^+\bar{\nu}_e$	$g_V = g_A = g_2/2\sqrt{2}$
$Z^0 \to \bar{\nu}_e \nu_e$	$g_V = g_A = \frac{e}{4\sin\theta_W \cos\theta_W}$
$Z^0 \to e^+e^-$	$g_V = (-\frac{1}{4} + \sin^2\theta_W)\frac{e}{\sin\theta_W \cos\theta_W}$, $\quad gA = -\frac{1}{4}\frac{e}{\sin\theta_W \cos\theta_W}$

with $g_2 = \frac{e}{\sin\theta_W}$ and $e^2 = 4\pi\alpha$.