Einführung in Theoretische Teilchenphysik

Lectures: Prof. Dr. M. M. Mühlleitner – Exercises: M.Sc. Martin Gabelmann, Dr. Sophie Williamson

Exercise Sheet 11

Hand-in Deadline: Friday 12.02.21, 14:00.
Discussion: Tuesday 16.02.21, Thursday 18.02.21.

1. **Higgs Masses in the Two-Higgs Doublet Model**:

One of the most minimal extensions of the Standard Model is the Two-Higgs Doublet Model (THDM), where a second Higgs doublet, with identical quantum numbers to the first, is added to the field content. The potential is then given by

$$
\begin{aligned}
V = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right) \\
& + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) \\
& + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \left(\Phi_2^\dagger \Phi_1\right)^2\right]
\end{aligned}
$$

where the two doublets , Φ_1 and Φ_2, and their respective vacuum expectation values (VEVs), are written

$$
\Phi_a = \begin{pmatrix} \phi_a^+ \\ \frac{v_a + \rho_a + i\eta_a}{\sqrt{2}} \end{pmatrix} , \qquad \langle \Phi_a \rangle = \begin{pmatrix} 0 \\ \frac{v_a}{\sqrt{2}} \end{pmatrix} , \qquad a = 1, 2 \, .
$$

with $v_{1,2}$ real. The parameters $m_{11,22}^2$, λ_{1-4} are necessarily real due to the hermiticity of the Lagrangian density. When m_{12}^2 and λ_5 are additionally real, then the potential is CP conserving. The component fields ρ_a and η_a are also real, while ϕ_a^+ are complex, with $(\phi_a^+)^* \equiv \phi_a^-$.

(a) **[3 points]** The doublet fields develop VEVs at the minimum of the potential, when

$$
\left. \frac{\partial V}{\partial \Phi_a^\dagger} \right|_{\Phi_i = \langle \Phi_i \rangle} = 0 \, , \qquad a = 1, 2 \, .
$$

Show that the above leads to the following two conditions that must be satisfied by the potential parameters:

$$
\begin{aligned}
m_{11}^2 + \frac{\lambda_1 v_1^2}{2} + \frac{\lambda_3 v_2^2}{2} &= m_{12}^2 \frac{v_2}{v_1} - (\lambda_4 + \lambda_5) \frac{v_2^2}{2} \, , \\
m_{22}^2 + \frac{\lambda_2 v_2^2}{2} + \frac{\lambda_3 v_1^2}{2} &= m_{12}^2 \frac{v_1}{v_2} - (\lambda_4 + \lambda_5) \frac{v_1^2}{2} \, .
\end{aligned}
$$

These equations are known as the *tadpole* equations.

(b) **[5 points]** To determine the mass terms of the fields, the potential needs to be developed in terms of each of its component fields ρ_a, η_a, $\phi_a^{\pm}$.

First consider the general term $\Phi_a^\dagger \Phi_b$ expressed in the component fields, to identify possible field combinations. Assume that you are working in a CP-conserving model, and make sure that there are no mixed terms of the form $\rho_a \eta_b$, $a, b = 1, 2$ which occur.

(c) **[5 points]** Determine the charged-mass matrix, $\mathcal{M}_C$, finding all terms of the form $\phi_a^- \phi_b^+$:

$$V|_{\phi^- \phi^+} = (\phi_1^-, \phi_2^-) \mathcal{M}_C \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} .$$

Then eliminate the potential parameters m_{11}^2 and m_{22}^2 using the minimum conditions. You should find something of the form

$$\mathcal{M}_C = X(\lambda_i, v_a, m_{12}^2) \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix} .$$

Calculate the eigenvalues of $\mathcal{M}_C$, finding the squared-masses of the two bosons.

(d) **[1 point]** Finally, calculate $\tan\beta$, where β denotes the angle of the rotation matrix U_C which diagonalises $\mathcal{M}_C$.

(e) **[4 points]** Extract all mass terms of the CP-odd components $\eta_a \eta_b$ and write the result in matrix form, $\mathcal{M}_P$, analogous to the previous task, and determine the two mass eigenvalues. You should again find the mass matrix of the form

$$\mathcal{M}_P = Y(\lambda_i, v_a, m_{12}^2) \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix} .$$

Show that the corresponding orthogonal transformation matrix is identical to the one we have already found for the charged case.

(f) **[2 points]** Now assume that the model is CP-violating, and that m_{12}^2 and λ_5 are complex. Show that in this case the $\rho_\alpha \eta_b$ -mixing terms do not vanish. Without explicit calculation, state what this means for the mass eigenstates.