Problem set 8

Problem 21: Small field inflation *(double credit)*
Consider the inflationary potential

$$V(\phi) = V_0 - \frac{\lambda}{4}\phi^4 \,, \tag{1}$$

where the condition $\lambda\phi^4 \ll V_0$ is assumed to hold throughout inflation. Note that, in contrast to large-field inflation, the inflaton field evolves away from the origin, i.e. $0 < \phi_{\text{start}} < \phi_{\text{end}}$.

a) Calculate the slow-roll parameters ϵ_V and η_V. Show that $\eta_V < 0$ and $|\eta_V| \gg \epsilon_V$.

b) Inflation ends when $|\eta_V| \approx 1$. Find the corresponding field value ϕ_{end}.

c) Show that the initial value of the inflation field is approximately given by

$$\phi^2_{\text{start}} = \frac{4\pi V_0}{\lambda M_{\text{Pl}} \Delta N} \,, \tag{2}$$

where ΔN denotes the number of e-folds.

d) Assuming that most perturbations are generated for $\phi \approx \phi_{\text{start}}$, use the observed amplitude of curvature perturbations, $\Delta^2_{\mathcal{R}} \sim 10^{-9}$, as well as $\Delta N \sim 60$, to estimate the required value of λ.

e) Under the same assumptions, obtain an upper bound on the tensor-to-scalar ratio r

Problem 22: Phase transitions
Consider the effective potential

$$V(\phi, T) = V_0 + D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4 \tag{3}$$

with $V_0, D, E, \lambda > 0$.

a) Find expressions for T_1, T_2, T_3 and $\phi_m(T)$, such that the following description is correct: For $T > T_1$ the potential features a single minimum at $\phi = 0$. For $T < T_1$ a second local minimum appears at $\phi = \phi_m(T)$. At $T = T_2 < T_1$ the second minimum becomes the global minimum, i.e. the minimum at $\phi = 0$ becomes metastable for $T < T_2$. Finally, at $T = T_3 < T_2$ the minimum at $\phi = 0$ disappears entirely.

b) Draw the potential for different values of T and discuss why one expects a first-order phase transition. Give an upper and a lower bound on the temperature when the phase transition will occur.

c) Assuming that $V(\phi_m(T), T) = 0$ in the limit $T \to 0$, calculate the energy density of the effective potential before the phase transition. Assuming that this energy density is larger than the energy density of matter and radiation, calculate the corresponding Hubble expansion rate of the universe.

Problem 23: Relic anti-baryons

For temperatures $T \ll m_p$ baryons obtain a large chemical potential, $\mu_b \sim m_p$, such that $n_b^{\rm eq} = \eta_b n_\gamma$.

a) Use this relation to show that

$$n_{\bar{b}}^{\rm eq} = \left(\frac{m_p T}{2\pi}\right)^3 \frac{1}{\eta_b n_\gamma} \exp^{-2m_p/T} \ . \tag{4}$$

b) The annihilation rate of anti-baryons is given by $\Gamma_{\rm ann} = \langle \sigma_{\rm ann} v \rangle n_b^{\rm eq}$, where $\langle \sigma_{\rm ann} v \rangle \approx m_\pi^{-2}$ with $m_\pi = 140\,\mathrm{MeV}$. Use the standard condition for decoupling from thermal equilibrium to estimate the freeze-out temperature of anti-baryons.

c) Calculate the number density of anti-baryons at freeze-out and in the present universe.

d) Calculate the total number of anti-baryons remaining in the entire observable universe.