Electron Microscopy I
Lecture 05

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Contents

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Radiation damage

"Direct"
Radiation damage under the microscope

- **Nuclear displacement damage** ("knock-on damage")
- **Electron excitation damage** (radiolysis)

"Indirect"
Radiation damage under the microscope

- **Contamination** *(polymerization of hydrocarbon molecules* ($C\,H_{nm}$) on the sample)
- **Heating** (radius r_0 of the illuminated sample area proportional to the temperature increase)

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"Direct" radiation damage under the microscope

Nuclear displacement damage ("knock-on damage")

Electron - atomic nucleus interaction

Nuclear relocation
→ leads directly to a bond dissociation

Electron excitation damage (radiolysis)

Electron - valence electron interaction

Nuclear relocation
→ Leads to ionization
→ Leads to local sample heating

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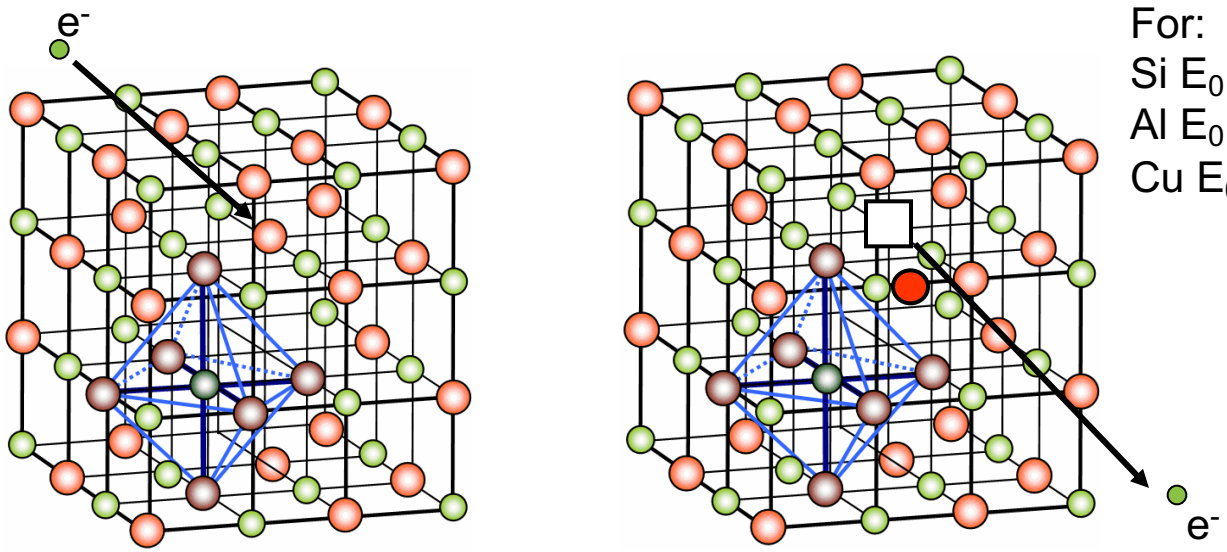
"Direct" radiation damage under the microscope

- Nuclear displacement damage ("knock-on damage")

Nuclear displacement damage



Minimum electron energy
"Threshold energy"
For:
Si E_0 > 145 keV
Al E_0 > 170 keV
Cu E_0 > 300 keV

- Formation of vacancies and interstitial atoms when
  Binding energy of the atom < energy transferred during impact

- Formation of extensive defects (stacking faults, cavities) with high void and
  Interstitial atom concentrations

→ Limitation of the irradiation time of an object, reduction of electron energy

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Nuclear displacement damage ("knock-on damage")

Grain boundary in Si, 300 keV electron energy

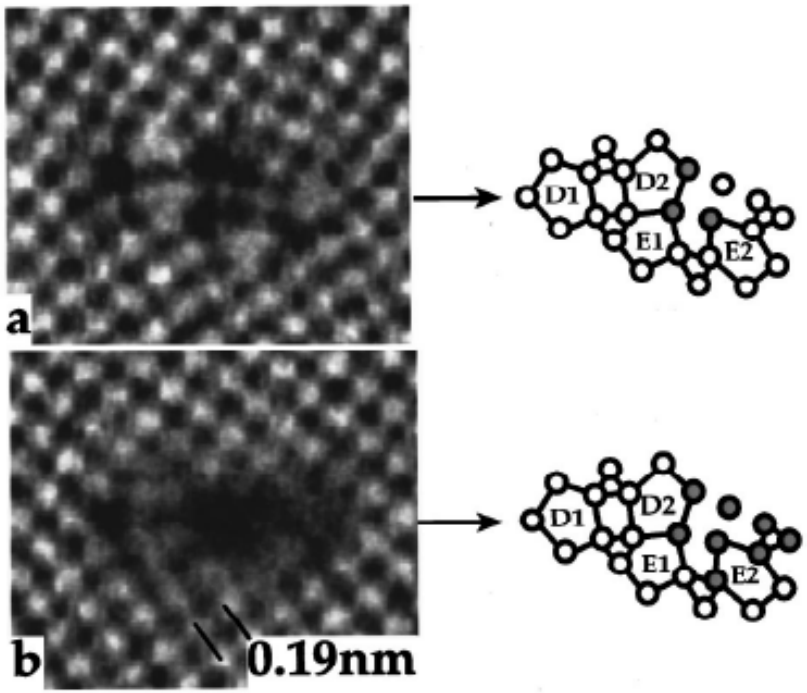
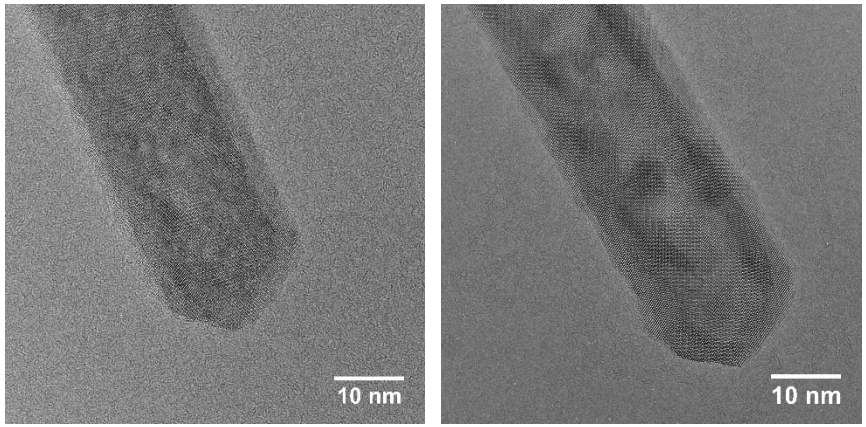
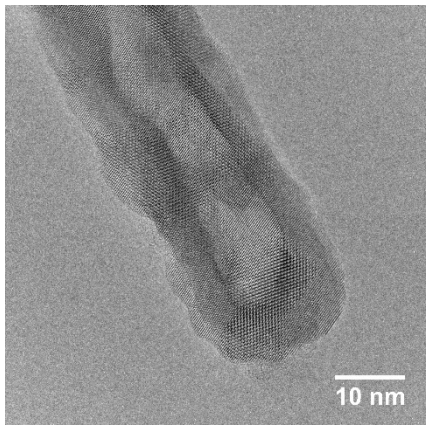


FIG. 1. Z-contrast images and derived structures of the $\Sigma=25$ {710} ⟨001⟩ symmetric tilt boundary at two stages of exposure to electron irradiation: (a) a nearly unaffected core with all columns visible but those shaded showing reduced intensity; (b) a partially affected core with several columns appearing darker.

A. Maiti et al, Appl. Phys. Lett. 75, 2380 (1999)

Electron beam-induced transformation of a nanopillar into a nanotube

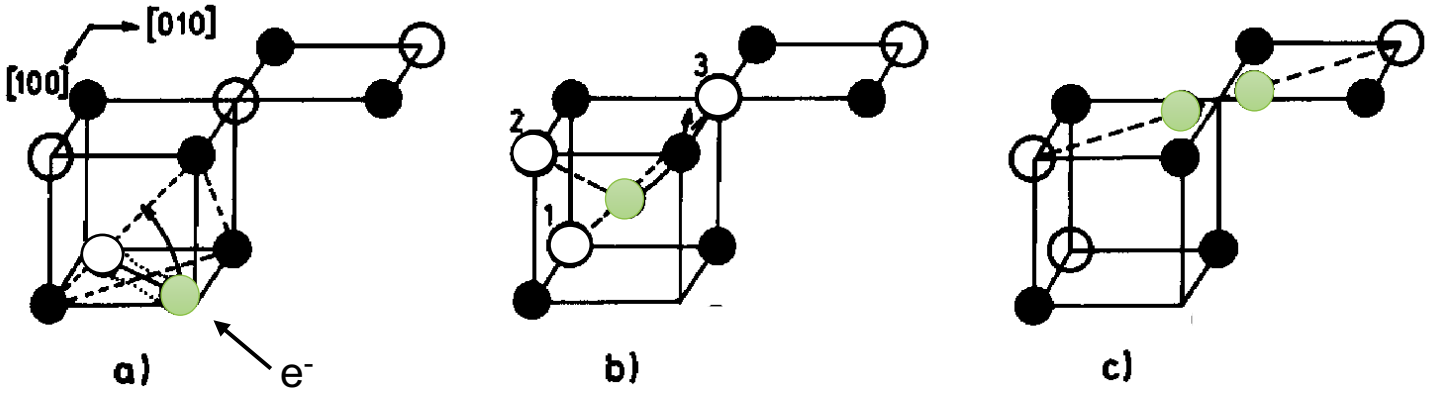




R. Popescu (LEM)

Radiolysis in electrically insulating materials

Example NaCl structure:



L. Reimer, Transmission Electron Microscopy, Fig. 11.8

Cl Na

- Structural rearrangement by excitation of electrons in the sample material in higher Energetic or non-binding states
  → *Reduction by cooling the sample in a sample holder cooled with liquid nitrogen*
  → *Increase* in electron energy (probability of an electronic excitation process decreases)
- Radiolysis in electrically conductive samples insignificant
- Radiolysis dominant in ionic crystals, (non-conductive) ceramic oxides, polymers and samples from the life sciences

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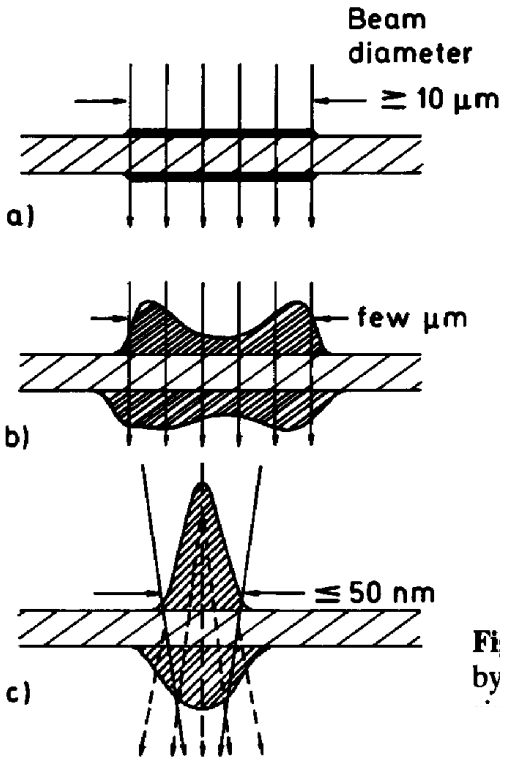
"Indirect" radiation damage

- Contamination
- Heating

Contamination



Fi
by

L. Reimer, Transmission Electron Microscopy, Fig. 10.10

Polymerization of hydrocarbon molecules (C_nH_m):

- Residual gas atmosphere (e.g. pump oils, vacuum grease from rubber seals)
- Contamination from sample preparation residues

on the sample surface due to the influence of the electron beam

Remedy:

- Extremely clean work (gloves)
- Cleaning in Ar/O plasma
- Improvement of the vacuum by cold trap to liquid nitrogen temperature (condensation of C_nH_m)
- Liquid nitrogen cooling holder (reduction of surface diffusion)

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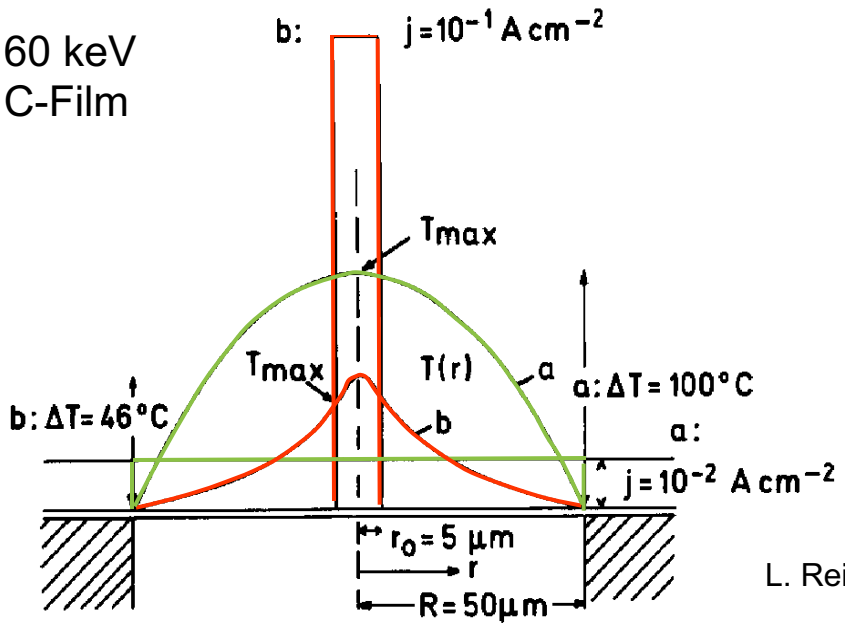
Sample heating



- Hardly any experimental data available
- Thermal conductivity of the sample and thermal coupling to the sample holder crucial
- Melting of small particles and small excretions in the microscope observed without heating
- Calculation:
  Dissipated heat by electron beam =
  Dissipated heat through heat conduction

L. Reimer, Transmission Electron Microscopy, Fig. 11.2 and Tab. 11.3

Table 11.3. Rise of temperature ΔT in the center of a circular diaphragm ($R = 50$ μm) covered with a supporting film and irradiated with 100 keV electrons.

	Uniform illumination	Small-area illumination
Substance	$R = 50$ μm, $j = 100$ A m^{-2}	$r_0 = 0.5$ μm, $j = 10^4$ A m^{-2}
Formvar	62°C	6°C
Glass (SiO)	27°C	2.5°C
Metal (Cu)	0.3°C	0.03°C

Radius r_0 of the illuminated sample area is decisive for temperature rise

Summary chapter 2

- The structure of a transmission electron microscope (TEM) is comparable to that of a light microscope: "light source", condenser, large-area illumination of the sample, imaging lens system, image plane with camera
- In scanning transmission mode (STEM), illumination is achieved with a focused electron beam that is scanned across the sample. The image is generated by the locally detected electron intensity, which controls the brightness of the corresponding image pixel. An imaging lens system is not required for image generation.
- However, images in TEM and STEM mode are comparable (see Chapter 4), as the interaction of electrons with the sample determines the image.
- Images (real space) and diffraction images (reciprocal space) of the same sample area can be generated by extending the focal length of the intermediate lens.
- Diffraction images provide local crystal structure information and allow the sample to be specifically oriented in relation to the electron beam, e.g. in a 2-beam condition or zone axis orientation.
- In the rear focal plane of the objective lens there are diaphragms with different diameters that determine the imaging mode.
- "Conventional" TEM bright-field or dark-field imaging (imaging with one reflex) is created by selecting a small aperture. A large aperture must be used for high-resolution TEM images so that electrons from at least 2 reflections interfere in the image.

Summary chapter 2

- A homogeneous magnetic field focuses electrons with the exception of electrons that move parallel to the optical axis
- Real electron lenses therefore generate inhomogeneous magnetic fields
- Magnetic and electrostatic round lenses for electrons are subject to lens aberrations that depend on the object-side aperture angle a_0 of the beam bundle
- The resolving power of electron-optical systems is limited by lens aberrations and not by the electron wavelength
- The spherical aberration is dominant for large a_0 . The error disk diameter is given by $\frac{1}{2} c_s a_0^3$ (c_s : spherical aberration constant). Modern devices contain a corrector lens system for the spherical aberration.
- Other lens defects include chromatic aberration and astigmatism, although astigmatism can be routinely corrected under the microscope.
- Specimens for TEM must be very thin, as electrons and solids interact strongly. Maximum sample thicknesses depend on the electron energy and material density. Typical maximum thicknesses are 1 mm for "conventional" TEM and a few 10 nm for high-resolution TEM.
- There are different techniques for sample preparation - depending on the material, purely mechanical, chemical, electrochemical or with a focused ion beam for "target preparation". The aim is to prepare the sample material free of artifacts.
- The interaction of the sample with electrons can damage the sample in the microscope. A distinction is made between direct damage (displacement damage and radiolysis) and indirect damage (contamination and heating of the sample)

3. Electron diffraction in the solid state

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Contrast through interaction between electrons and sample

→ Contrast interpretation based on a detailed understanding of the Interaction between electrons and sample

Approach

Interaction of electrons with a single atom

Interaction with crystalline solids

Kinematic diffraction theory (single scattering)

Dynamic diffraction theory (multiple scattering)
Ch. 5

General validity of the kinematic diffraction theory for
Electron, X-ray, neutron diffraction, ...

Elastic and inelastic scattering processes

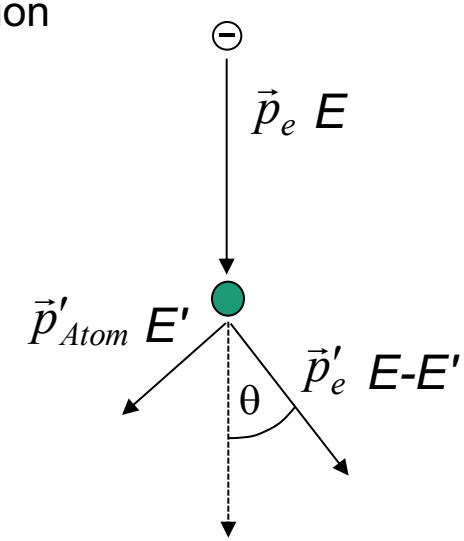
- *Elastic scattering process:*
  Prerequisite: conservation of momentum and energy

- *Inelastic scattering process:*
  Conservation of momentum and energy **does not apply**!
  Generation of excited states or ionization of the target atom (radiolysis)
  in the solid state additionally: excitation of phonons, plasmons

Elastic collision of an electron with a single atom: "billiards" physics
Collision of two particles without detailed knowledge of the interaction

Energy transferred to the target atom during elastic impact

$$E' = \frac{2E(E+2E_R)}{Mc^2}\sin^2\theta/2 = \frac{E(E+1.02)}{496A}\sin^2\theta/2$$



E: kinetic energy of the electron (in 10^6 eV) before the collision
E_R : Rest energy of the electron 0.511*10^6 eV
M: Mass of the atom in A m_p (m_p : atomic mass unit)
c: Speed of light
θ : Scattering angle
p: Pulse (e: Electron, At

Elastic scattering

Energy transferred during impact = energy loss of the primary electron

E	100 keV			1 MeV		
	C	Cu	Au	C	Cu	Au
θ	$(A = 12)$	$(A = 63.5)$	$(A = 197)$			
0.5°	0.5 meV	0.1 meV	0.03 meV	9 meV	1.7 meV	0.54 meV
10°	0.15 eV	29 meV	9 meV	2.7 eV	0.5 eV	0.17 eV
90°	10 eV	1.9 eV	0.6 eV	179 eV	34 eV	11 eV
180°	20 eV	3.8 eV	1.2 eV	359 eV	68 eV	22 eV

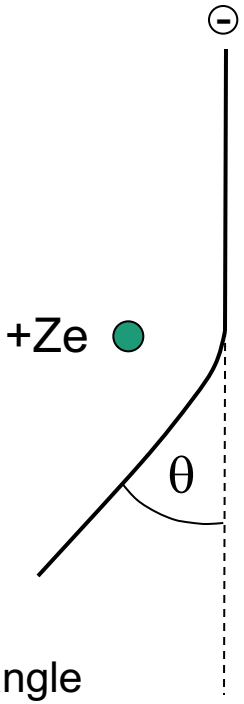
L. Reimer, Transmission Electron Microscopy, Table 5.1

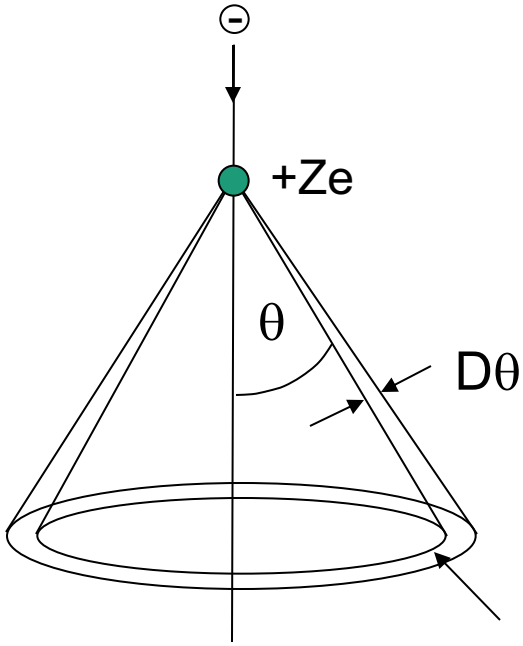
For the small scattering angles that dominate in the TEM, θ applies as a good approximation:
The energy and wavelength of the electron do not change with elastic scattering!

Displacement damage from approx. 100 keV for scattering processes with large scattering angle for typical binding energies of atoms in solids between 5 and 30 eV

Description of elastic scattering processes: Particle image

Classical by Rutherford (1911): Coulomb interaction between charged atomic nucleus and electron without taking into account the shielding effect of the electron shell





dΩ: Circular segment

θ: Scattering angle
Z: Atomic number of the scattering atom

Differential cross section:
Probability for scattering process with scattering angle θ

$$\frac{d\sigma(\theta)}{d\Omega}$$

Differential Rutherford scattering cross section (not relativistic)

Scattering probability:

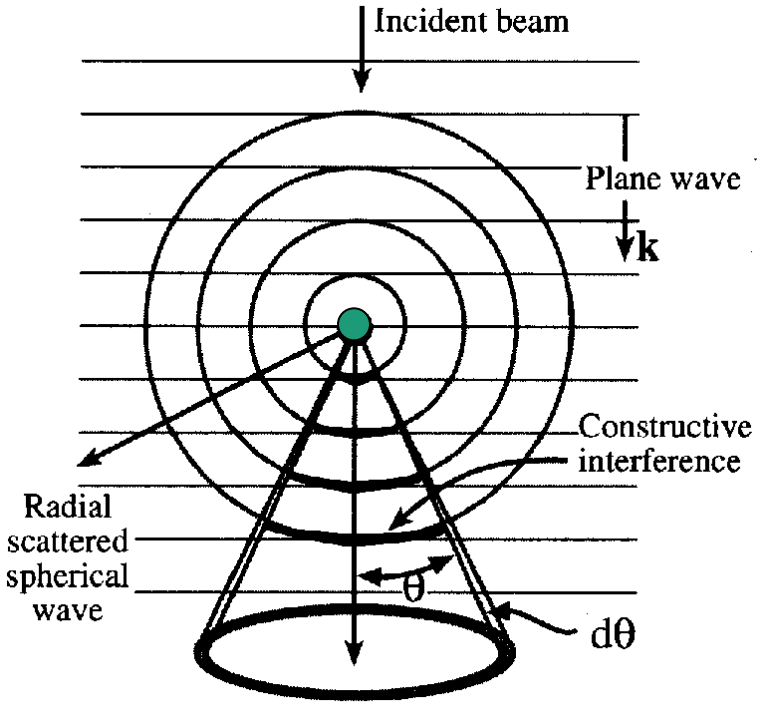
$$\frac{d\sigma(\theta)}{d\Omega} = \frac{e^4 Z^2}{4\left(4\pi\varepsilon_0\right)^2 m^2 v^4 \sin^4\frac{\theta}{2}}$$

Z: Atomic number of the scattering atom
v: electron velocity
m: electron mass
e: Elementary charge

Problem: Differential scattering cross-section diverges for $\theta \to 0$, as shielding of the atomic nucleus by the electron shell is not taken into account. (*To be solved quantum mechanically, time-independent and relativistic Schrödinger equation*)

Solution of the scattering problem taking into account the shielding by the Electron shell by quantum mechanical calculation: Solution of the time-independent relativistic Schrödinger equation

Description of elastic scattering processes (*stationary*): Wave pattern



D.B. Williams, C.B. Carter, Transmission Electron Microscopy, Fig.3.6, p.42

Incident plane electron wave

$$\psi = \psi_0 e^{\left(2\pi i \vec{k}\vec{r}\right)}$$

Wave scattered on a point charge

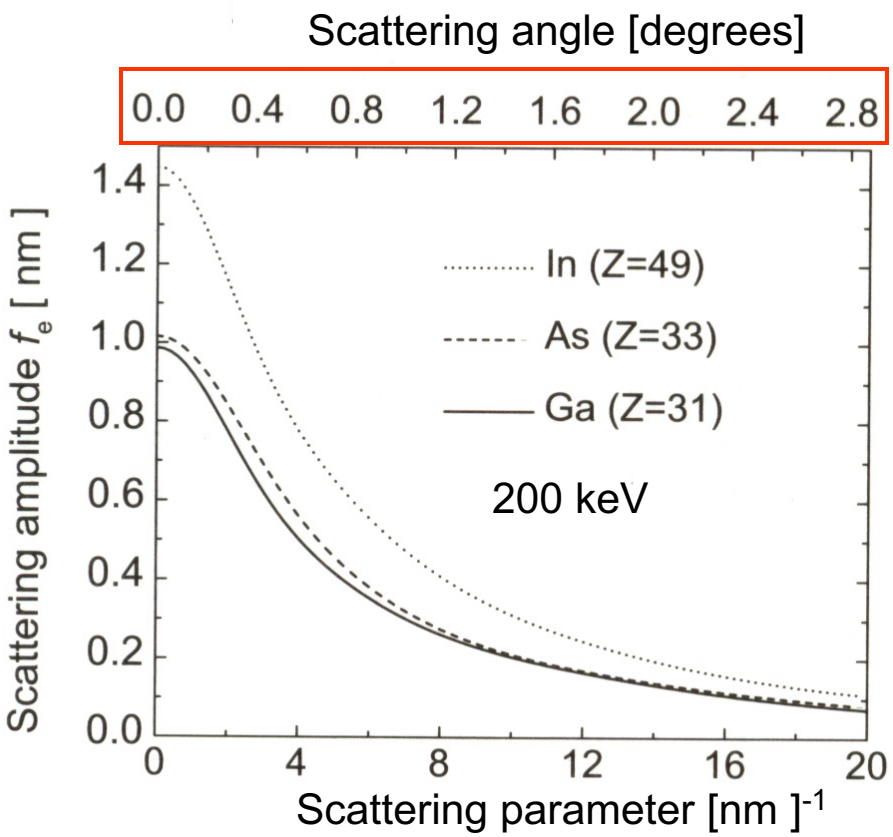
$$\psi_S = \psi_0 f(\theta) \frac{e^{(2\pi i k r)}}{r}$$

f(θ): atomic form factor, angular dependence of the Amplitude of the scattered wave

Resulting wave

$$\psi = \psi_0 \left(e^{\left(2\pi i \vec{k}\vec{r}\right)} + i f(\theta) \frac{e^{(2\pi i k r)}}{r} \right)$$

Atomic form factors for Ga, In, As

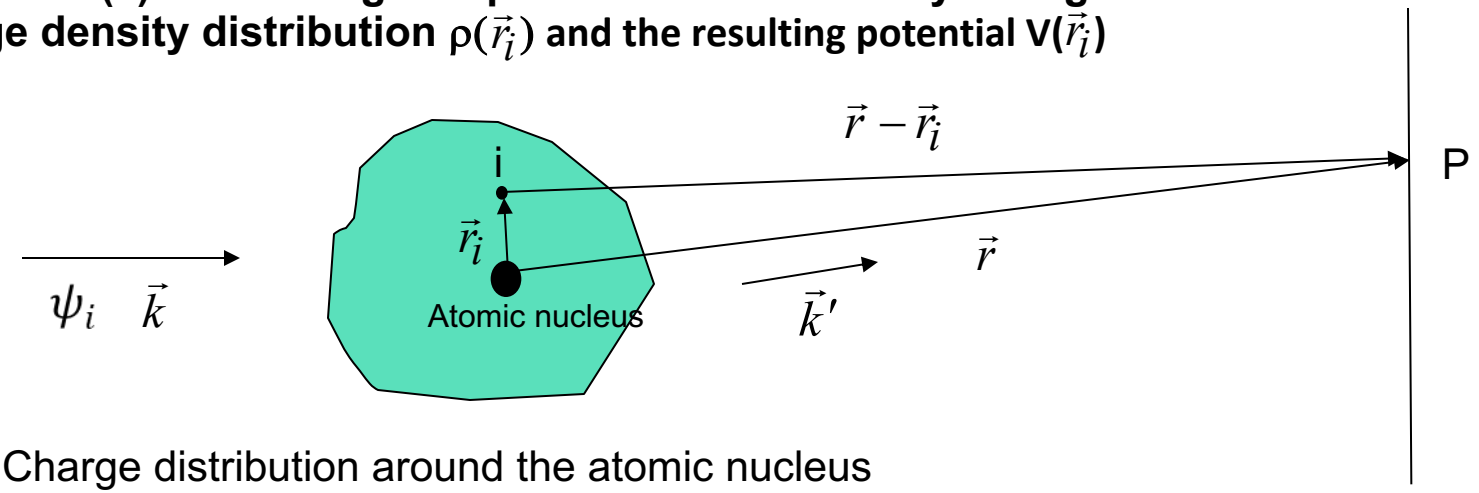


A. Rosenauer, Transmission Electron Microscopy of Semiconductor Nanostructures", Fig. 2.2

The following applies to f(θ):

- The amplitude of the scattered spherical wave decreases very quickly as the scattering angle increases
  → Strong tendency towards forward scattering
- The amplitude increases with the atomic number Z
- Not shown here, as f(θ) is only shown for 200 keV: the amplitude decreases with the Electron energy

Calculation of f(θ): Scattering of a plane electron wave by a single atom with charge density distribution $\rho(\vec{r}_i)$ and the resulting potential $V(\vec{r}_i)$



Blue area: Charge distribution around the atomic nucleus

Conversion of the Schrödinger equation into an integral equation using Green's function

$$\psi(\vec{r}) = \psi_i + \psi_s = \psi_i + \frac{2\pi me}{h^2} \int_{Atom} V(\vec{r}_i) \frac{\exp\left(2\pi i k |\vec{r} - \vec{r}_i|\right)}{|\vec{r} - \vec{r}_i|} \psi(\vec{r}_i)\, d^3\vec{r}_i$$

Incident plane wave

Scattered wave

$V(\vec{r}_i)$ Potential on site $\vec{r}_i$

Green's function

Total wave function on Location $\vec{r}_i$

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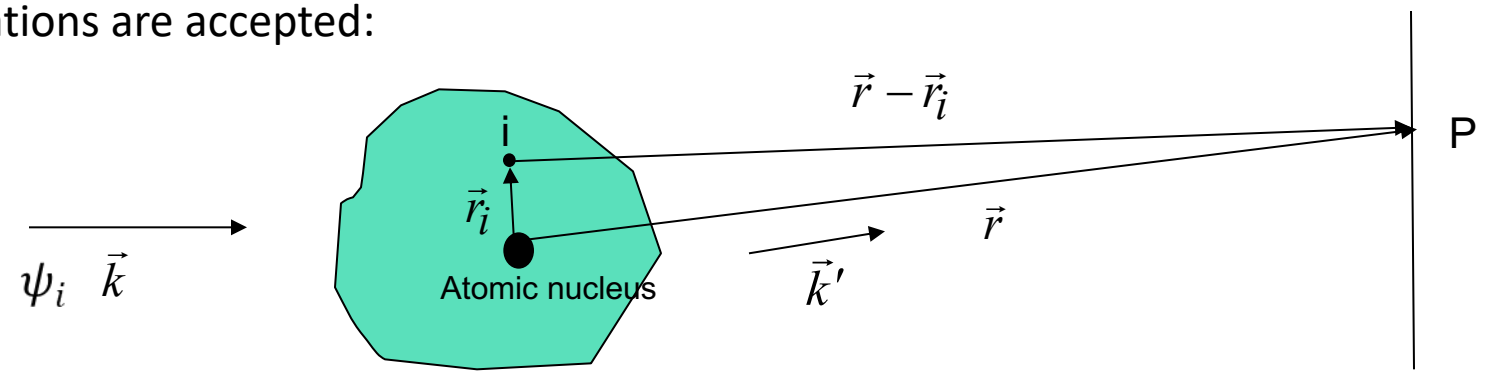
Simplifications are accepted:



1st Born's approximation $eV(\vec{r}_i)$ << Electron energy

→ incident wave ψ_i is not significantly weakened when passing through V

→ $\psi(\vec{r}_i) = \psi_i$ and $|\psi_s| << |\psi_i|$

→ Single scattering, i.e. ψ_s is no longer scattered

Approximation $|\vec{r}|$ and $|\vec{r} - \vec{r}_i| >> |\vec{r}_i|$

$$\psi_s = \frac{\exp(2\pi i k r)}{r} \frac{8\pi^2 me}{h^2} \int_{Atom} V(\vec{r}_i) \exp\left(2\pi i \left(\vec{k} - \vec{k}'\right)\vec{r}_i\right) d^3\vec{r}_i$$

with $\vec{k} - \vec{k}' = \vec{g}$

$$\psi_s = \frac{\exp(2\pi i k r)}{r} \frac{8\pi^2 me}{h^2} \int_{Atom} V(\vec{r}_i) \exp(2\pi i \vec{g}\vec{r}_i) d^3\vec{r}_i$$

f(θ)

- Scattering potential from integration of the Poisson equation for the charge distribution ρ
- Atomic form factors are dependent on the charge distribution in the atom
- Amplitude of the scattered wave is proportional to the Fourier transform of the scattering potential

Atomic form factor under

- Consideration of scattering at the atomic nucleus and shielding by the electron shell
- Use of Born's 1st approximation (does not apply to heavy atoms!)
- Consideration of relativistic effects

$$|f(\theta)|^2 = \frac{d\sigma(\theta)}{d\Omega} = \frac{4R^4Z^2(1+E/E_0)^2}{a_H{}^2} \frac{1}{\left[1+(\theta/\theta_o)^2\right]^2}$$

$$\theta_o = \frac{\lambda}{2\pi R}$$

$$R = a_H Z^{-1/3}$$

Connection between
Particle and wave image

Z: Atomic number of the scattering atom
E_0 : Rest energy of the electron
E: kinetic energy of the electron
a_H : Bohr radius
R: Shielding radius

- There are numerous different approaches for calculating differential scattering cross sections
- Calculated atomic form factors for electron scattering on individual atoms
  e.g. by Doyle and Turner, Acta Cryst. A24, 390 (1968), which, however, do not optimally describe the scattering behavior of atoms in ionic and covalent crystals (error a few %)!
- Charge distributions of atoms in crystals must be known as well as possible.

Inelastic scattering processes: Generation of characteristic X-rays and bremsstrahlung

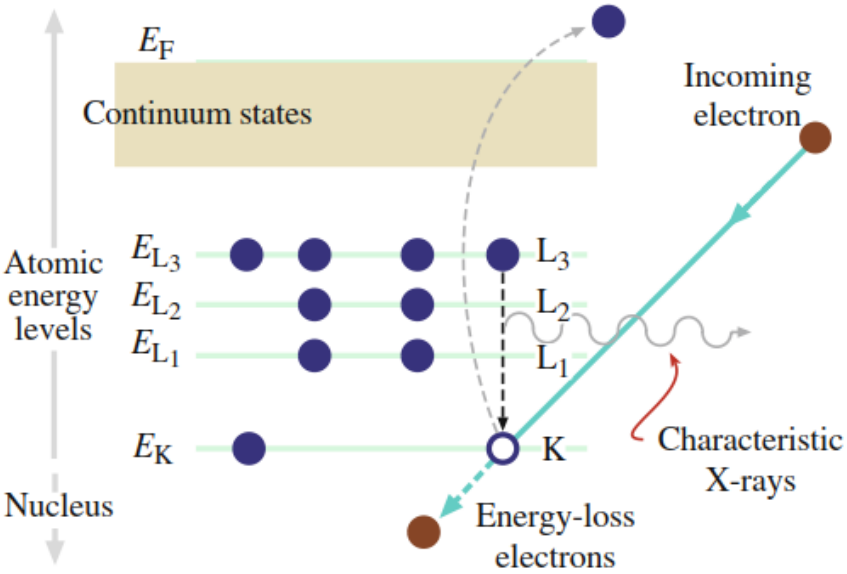


FIGURE 4.2. The ionization process. An inner (K) shell electron is ejected from the atom by a high-energy electron. When the hole in the K shell is filled by an electron from the L shell, characteristic (K_α) X-ray emission occurs. The beam electron loses energy but continues on through the specimen.

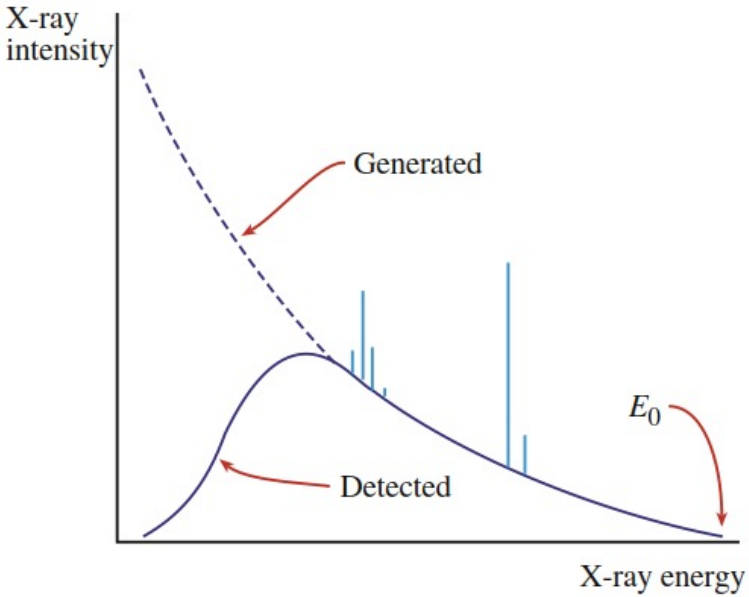


FIGURE 4.6. The bremsstrahlung X-ray intensity as a function of energy. The generated intensity increases rapidly with decreasing X-ray energy but at energies $<\sim 2$ keV the bremsstrahlung is absorbed in the specimen and in any detector being used so the observed intensity in the detected spectrum drops rapidly to zero. E_0 is the energy of the electrons that cause the X-ray emission. Two families of characteristic lines at specific energies are also shown superimposed on the bremsstrahlung.

D.B. Williams, C.B. Carter, Transmission Electron Microscopy, Fig.4.2 and Fig.4.6

Scattering on collective excitations of the solid: Phonon scattering

Generation of a phonon

- small energy change of the electron in the magnitude of $k_B T \approx 25$ meV at 20°C
  → Heating of the sample
- Large scattering angles

(order of magnitude 10 degrees)

→ Darkening of the image through suppressing electrons by the lens aperture

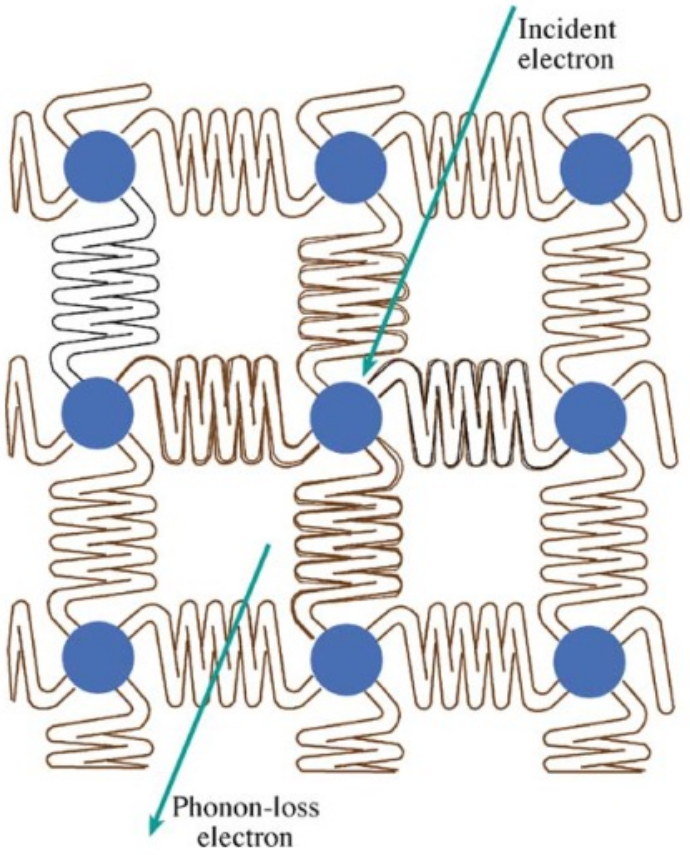


FIGURE 4.10. An illustration of the crystal lattice as a group of atoms linked elastically by springs. The bonds vibrate when struck by a high-energy electron creating lattice oscillations or phonons and these vibrations are equivalent to heating the specimen.

D.B. Williams, C.B. Carter, Transmission Electron Microscopy, Fig. 4.10

Scattering on collective excitations of the solid: Plasmon scattering

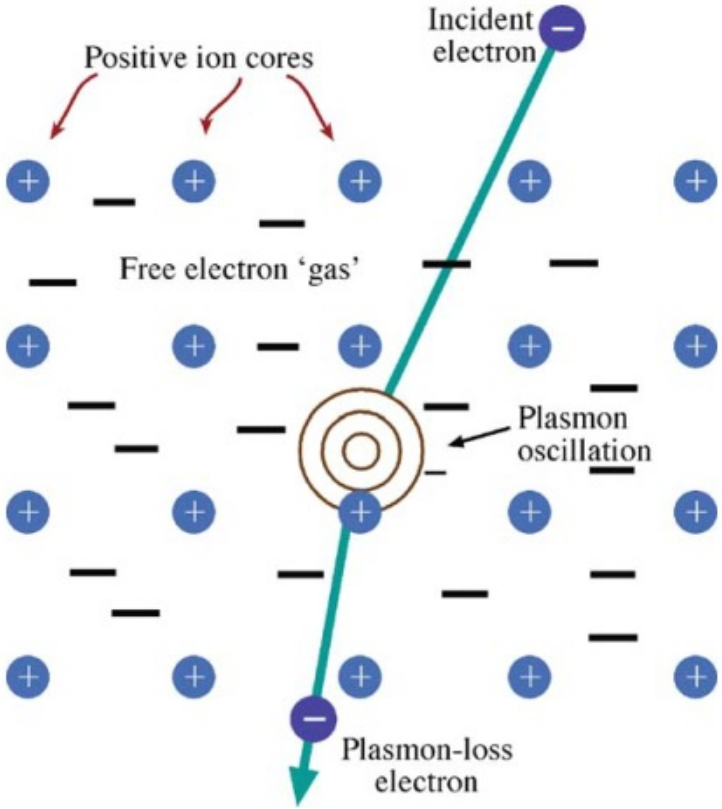


FIGURE 4.9. Schematic diagram of a high-energy beam electron exciting a plasmon oscillation in a free-electron gas that permeates the ion cores in a metal.

- Oscillations of the free electron gas in metals and semiconductors, vibrations of valence electrons in insulators
- Energy losses of the primary electrons between a few eV and 40 eV

D.B. Williams, C.B. Carter, Transmission Electron Microscopy, Fig.4.9

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Elastic scattering of electrons on a group of atoms

Prerequisite for kinematic description: single scattering

Scattering by two atoms B, C
Scattered wave at a distance R >> r
Phase shift between the two scattered waves Dφ

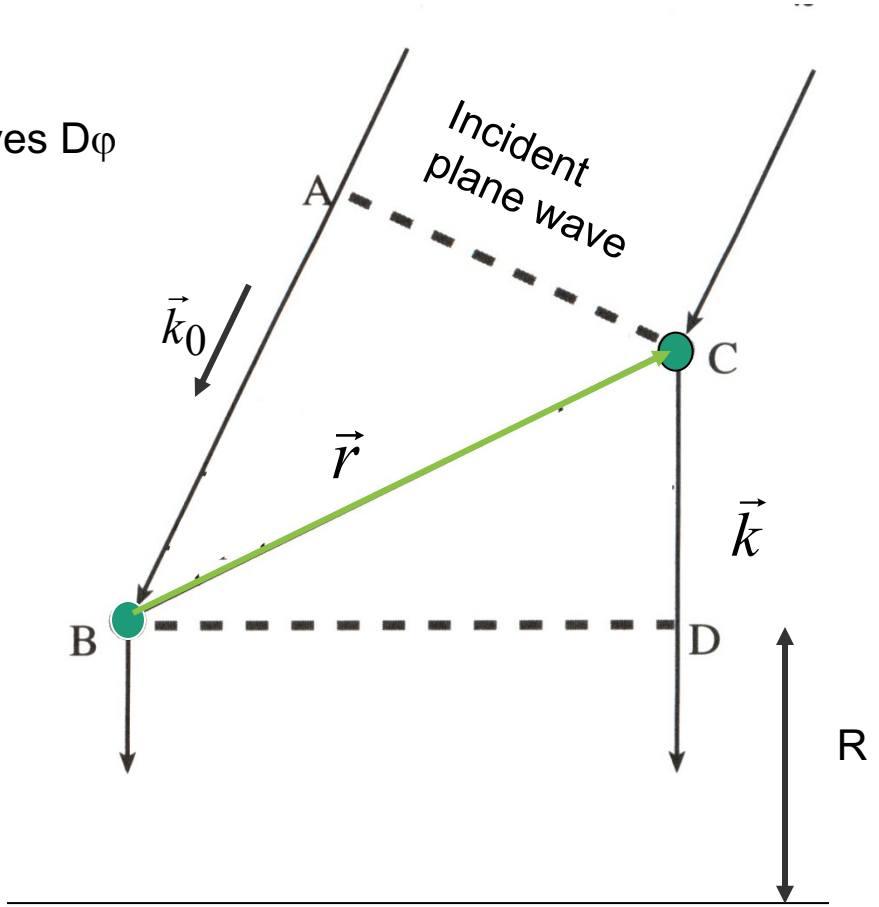
with $\left|\vec{k}\right| = \left|\vec{k}_0\right| = 1/\lambda$ for elastic scattering

$$\Delta\varphi = 2\pi\frac{\Delta l}{\lambda}$$

$$\Delta l = AB - CD$$

$$\Delta\varphi = 2\pi\vec{r}\left(\vec{k} - \vec{k}_0\right)$$

Δl: *path difference*



Scattered plane wave

Observation screen at distance R >> r

Scattered wave at a distance R (R>>r_i) and in the direction of $\vec{k}$

$$\psi_S = \psi_0 \frac{\exp(2\pi i\, kR)}{R}\left\{f_B(\theta) + f_C(\theta)\exp\left(2\pi i\left[\vec{k}-\vec{k}_o\right]\vec{r}\right)\right\}$$

Scattering on M arbitrarily arranged atoms

$$\psi_S = \underbrace{\psi_0 \frac{\exp(2\pi i\, kR)}{R}} \sum_{i=1}^{M} f_i(\theta)\exp\left(2\pi i\left[\vec{k}-\vec{k}_o\right]\vec{r}_i\right)$$

Is described below neglected

Scattering amplitude $F_{(\theta)}$

$\vec{r}_i$: Atom positions in relation to the atom at the origin

Intensity of the scattered wave on the screen: $I \propto |F(\theta)|^2$

Scattering at a crystal lattice (ordered arrangement of atoms)

Scattering amplitude (general)

$$F(\theta) = \sum_i f_{i(\theta)} \exp\left(2\pi i \left[\vec{k} - \vec{k}_o\right] \vec{r}_i\right)$$

For crystals: $\vec{g} = \vec{k} - \vec{k}_0$

$$F(\theta) = \sum_i f_{i(\theta)} \exp\left(2\pi i \, \vec{g} \, \vec{r}_i\right)$$

Condition for constructive interference (maximum scattered intensity) :

$$\vec{g}\vec{r}_i = integer$$

Suitable definition of $\vec{g}$ for maximum scattering intensity

Relationship between real and reciprocal lattice

lattice point in local space characterized by

$$\vec{r} = u\vec{a}_1 + v\vec{a}_2 + w\vec{a}_3$$

u,v,w $\in$ integers

lattice point in the reciprocal lattice characterized by

h,k,l

$$\vec{g} = h\vec{a}_1^{\,*} + k\vec{a}_2^{\,*} + l\vec{a}_3^{\,*}$$

$\vec{a}_i$: basis vectors of the lattice in the spatial domain

$\vec{a}_i^{\,*}$: basis vectors of the lattice in reciprocal space

Construction rule for base vectors of the reciprocal lattice

$$\vec{a}_i \cdot \vec{a}_j^{\,*} = \delta_{ij}$$ with $\delta_{ij} = 0$ for $i \neq j$, $\delta_{ij} = 1$ for $i = j$

So that $$\vec{g}\vec{r} = uh\,\vec{a}_1\vec{a}_1^{\,*} + uk\vec{a}_1\vec{a}_2^{\,*} + \ldots\ldots wl\vec{a}_3\vec{a}_3^{\,*} = \text{integer}$$

Relationship between real and reciprocal lattice

Base vectors of the reciprocal lattice

$$\vec{a}_1^* = \frac{\vec{a}_2 \times \vec{a}_3}{V_e} \qquad \vec{a}_2^* = \frac{\vec{a}_1 \times \vec{a}_3}{V_e} \qquad \vec{a}_3^* = \frac{\vec{a}_1 \times \vec{a}_2}{V_e}$$

Volume of the unit cell: $V_e = \vec{a}_i \left(\vec{a}_j \times \vec{a}_k \right)$

Definition for reciprocal lattice vectors also applies to lattice with non-orthogonal basis vectors in real space

$$\vec{g} = h\vec{a}_1^* + k\vec{a}_2^* + l\vec{a}_3^*$$

h, k, l: Miller's indices for the characterization of planes (sets) in the reciprocal lattice

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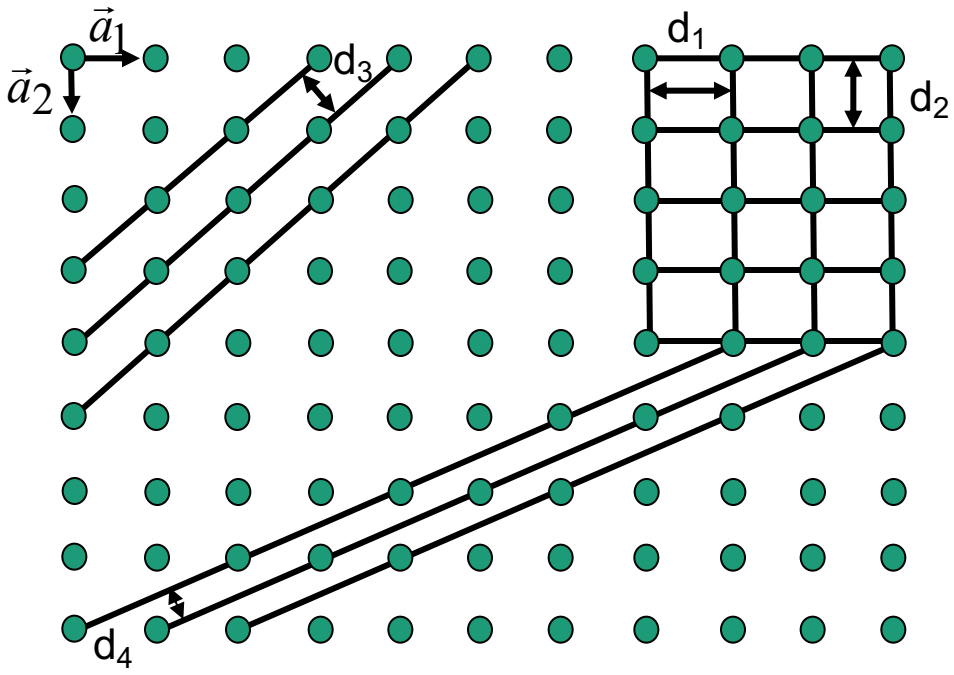
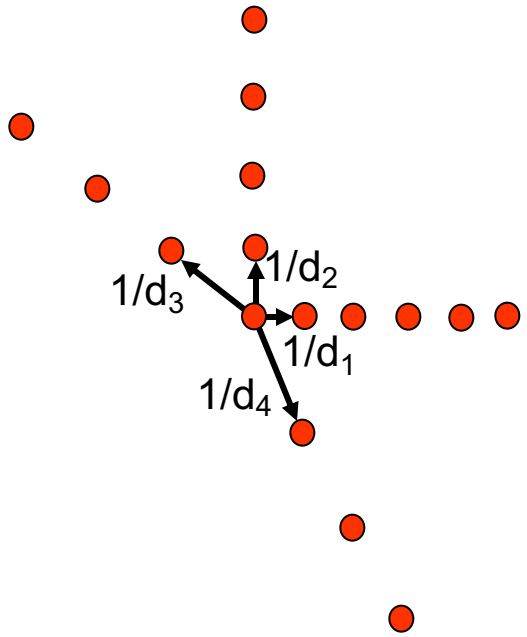
Relationship between lattice in real space and reciprocal lattice (2-dimensional)

lattice in real space

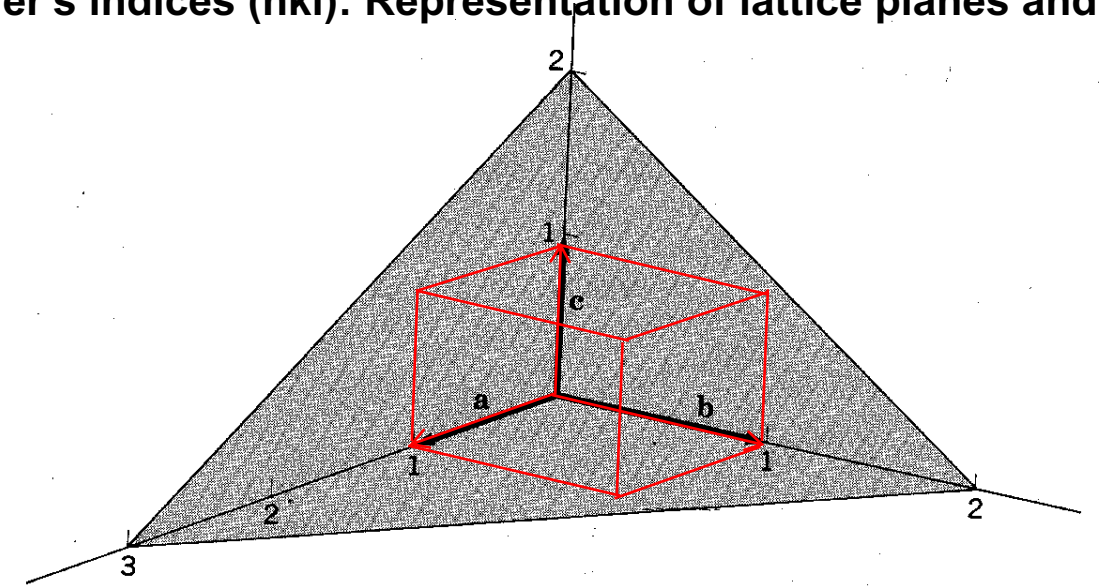


lattice in reciprocal space



Each lattice plane (set) in real space has a point in the reciprocal lattice

Miller's indices (hkl): Representation of lattice planes and their distances



Points of intersection of the plane with the axes: 3a, 2b, 2c
reciprocals 1/3, 1/2 , 1/2
Miller indices: smallest integers that are in the same ratio to each other (2 3 3)

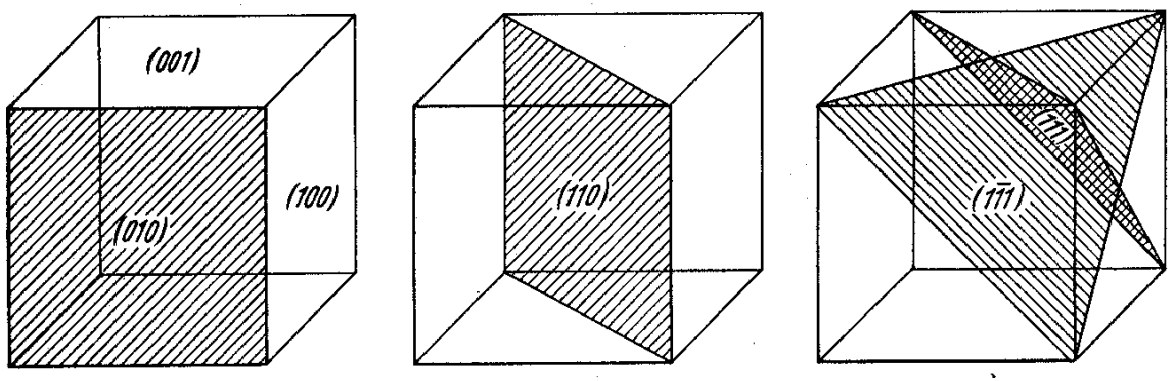
lattice plane(s) characterized by reciprocal lattice vector

$$\vec{g} = 2\vec{a}_1^* + 3\vec{a}_2^* + 3\vec{a}_3^*$$

With the length

$$|\vec{g}_{hkl}| = \sqrt{\left(ha_1^*\right)^2 + \left(ka_2^*\right)^2 + \left(la_3^*\right)^2} = \frac{1}{d_{hkl}}$$

Miller indices: Representation of lattice planes



Presentation conventions:
(hkl): certain lattice plane(s)
{hkl}: Types of plane sets with the same symmetry properties
e.g. in cubic lattices (100), (010), (001), (100), ...
[hkl]: certain direction
<hkl>: Types of directions with the same properties

In cubic crystals the following applies: [hkl] $\perp$ (hkl)

Miller indices: Representation of planes and plane distances

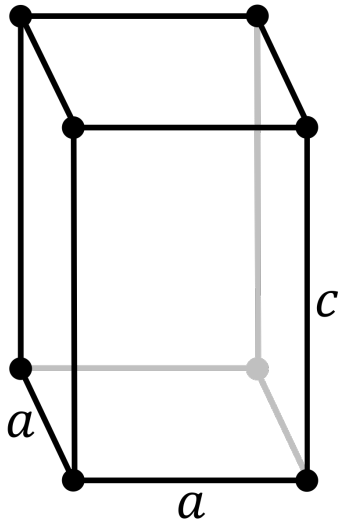
The following realation holds for **cubic** lattices: $d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

with lattice parameter a

because of $\left|\vec{a}_1^*\right| = \left|\vec{a}_2^*\right| = \left|\vec{a}_3^*\right| = \frac{1}{a}$

In tetragonal lattices with valid: $\left|\vec{a}_1\right| = \left|\vec{a}_2\right| = a \quad und \quad \left|\vec{a}_3\right| = c$

$$d_{hkl} = \sqrt{\frac{a^2}{\left(h^2 + k^2\right)} + \frac{c^2}{l^2}}$$



3.2 Kinematic Diffraction Theory

Cubic:

$$\frac{1}{d^2} = \frac{h^2 + k^2 + l^2}{a^2}$$

Tetragonal:

$$\frac{1}{d^2} = \frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2}$$

Hexagonal:

$$\frac{1}{d^2} = \frac{4}{3}\left(\frac{h^2 + hk + k^2}{a^2}\right) + \frac{l^2}{c^2}$$

Rhombohedral:

$$\frac{1}{d^2} = \frac{(h^2 + k^2 + l^2)\sin^2\alpha + 2(hk + kl + hl)(\cos^2\alpha - \cos\alpha)}{a^2(1 - 3\cos^2\alpha + 2\cos^3\alpha)}$$

Orthorhombic:

$$\frac{1}{d^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}$$

Monoclinic:

$$\frac{1}{d^2} = \frac{1}{\sin^2\beta}\left(\frac{h^2}{a^2} + \frac{k^2\sin^2\beta}{b^2} + \frac{l^2}{c^2} - \frac{2hl\cos\beta}{ac}\right)$$

Triclinic:

$$\frac{1}{d^2} = \frac{1}{V^2}\left(S_{11}h^2 + S_{22}k^2 + S_{33}l^2 + 2S_{12}hk + 2S_{23}kl + 2S_{13}hl\right)$$

where

$$S_{11} = b^2c^2\sin^2\alpha$$
$$S_{22} = a^2c^2\sin^2\beta$$
$$S_{33} = a^2b^2\sin^2\gamma$$
$$S_{12} = abc^2(\cos\alpha\cos\beta - \cos\gamma)$$
$$S_{23} = a^2bc(\cos\beta\cos\gamma - \cos\alpha)$$
$$S_{13} = ab^2c(\cos\gamma\cos\alpha - \cos\beta)$$

Lattice plane spacing can be calculated for all 7 crystal systems on the basis of Miller's indices (hkl)

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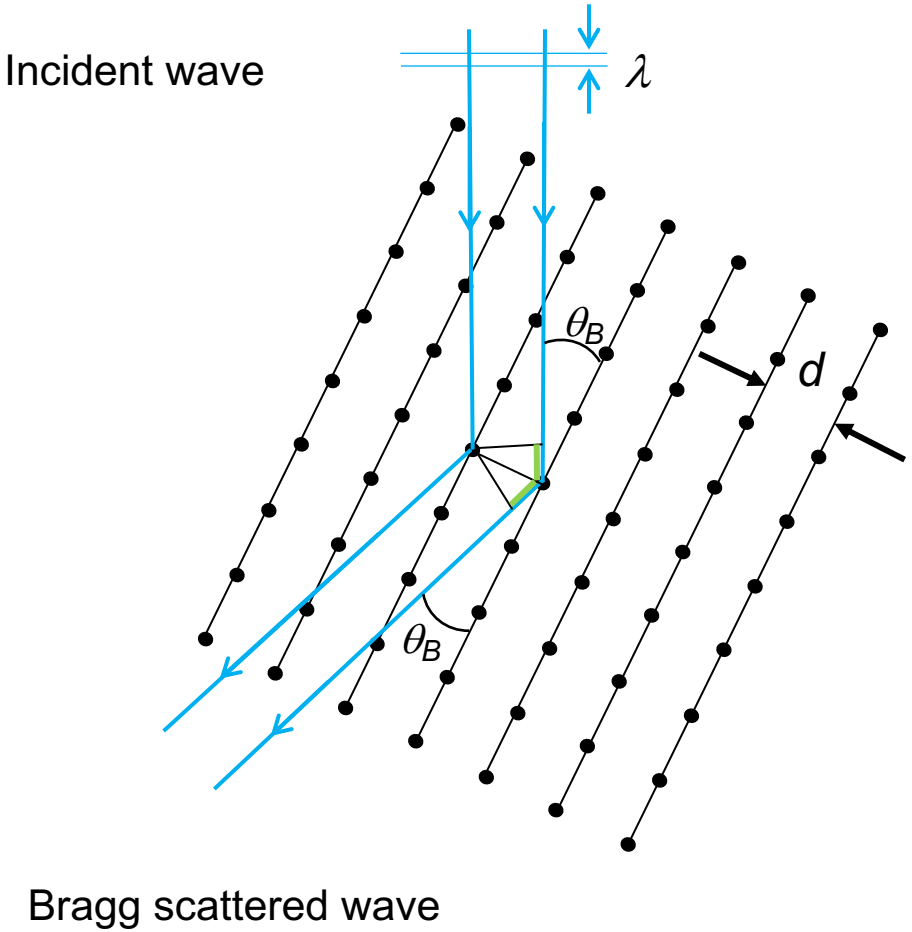
Bragg condition: alternative condition for maximum scattering intensity



Bragg condition for constructive interference (maximum scatter intensity)

$$2d \sin\theta_B = n\lambda$$

Bragg reflex in the diffraction image

d: lattice spacing
λ: Electron wavelength
θ_B : Bragg angle

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Ewald construction

Bragg condition

$$2d \sin\theta_B = n\lambda$$

and Laue condition

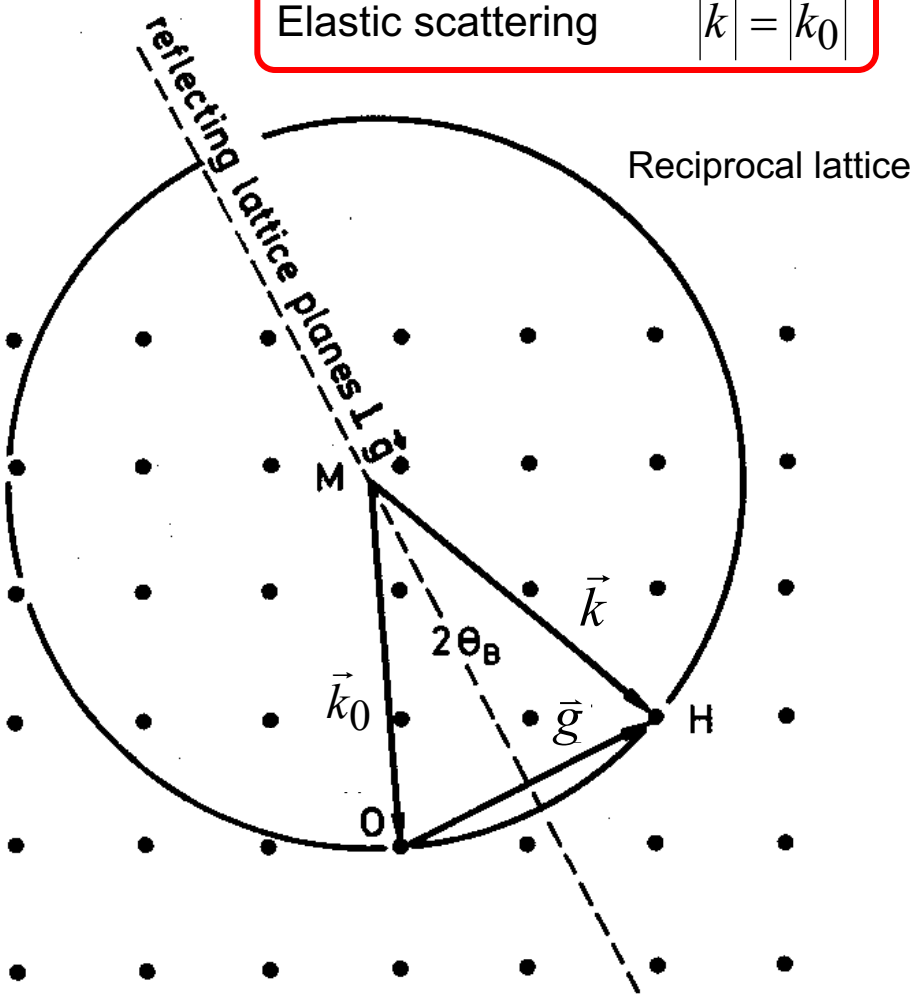
$$\vec{g} = \vec{k} - \vec{k}_0$$

are equivalent

For reciprocal lattice points of planes with (hkl), which are cut by the Ewald Kugel, the Bragg condition is fulfilled

Determination of all directions with maximum scattering intensity, which belong to a specific angle of incidence

Elastic scattering $|\vec{k}| = |\vec{k}_0|$



L. Reimer; Transmission Electron Microscopy, Fig.7.8

Summary of the properties of the reciprocal lattice:

Definition of reciprocal lattice vectors $\vec{g}_{hkl}$ with the condition of constructive interference

$\vec{g}_{hkl} = \vec{k} - \vec{k}_0$ Laue equation with $\vec{g}_{hkl}$ perpendicular to "reflective" lattice plane set (hkl)

$$|\vec{g}_{hkl}| = \frac{n}{d_{hkl}}$$

$2d \sin\theta_B = n\lambda$ Bragg condition equivalent to Laue equation

Constructive interference is independent of the arrangement of the atoms on the lattice plane