Tutorial 7

1. The return of the Chromium

On exercise sheet 6 we learned that the experimentally determined value of the magnetic moment of the Cr^{3+} ion is μ=3.8 μ_B.

a) You learned now the reason why this deviates from the value you obtained via Hund's rules (0.77 μ_B). What was it again?
b) Show that the theory fits the experimental data better when taking into account the mechanism in (a). Why is this assumption legitimate for Cr^{3+}?
c) Discuss the term scheme considering the crystal field splitting of Cr^{3+} in tetrahedral and octahedral symmetry. How does this affect its spin state?

2. Spin waves

In the lecture, we discussed /will discuss spin waves as the mechanism that governs the magnetization of ferromagnets at low temperatures. We here want to derive the dispersion relaxation of 1D spin waves in a semiclassical approach.

a) Show that the equation of motion of a 1D spin wave can be written as

$$\frac{d\boldsymbol{S}_j}{dt} = -\frac{g\mu_B}{\hbar}\left(\boldsymbol{S}_j \times \boldsymbol{B}_{\text{ext}}\right) + \frac{J_A}{\hbar^2}\left[\boldsymbol{S}_j \times \left(\boldsymbol{S}_{j-1} + \boldsymbol{S}_{j+1}\right)\right]$$

(Hint: use a mean field approach with an effective field $B_{\text{eff}} = B_{\text{ext}} + B_{A,i}$ and an exchange field $B_{A,i} = -\frac{J_A}{g\mu_B\hbar}\sum_{i=1}^{N}(\boldsymbol{S}_{j-1} + \boldsymbol{S}_{j+1})$)

b) Now, linearize the equation by assuming $|S_{i,x}|, |S_{i,y}| \ll |S_{i,z}|$ and $\boldsymbol{B}_{\text{ext}} || \hat{\boldsymbol{z}}$ (→ neglect terms quadratic in $|S_{i,x}|, |S_{i,y}|$ and assume $|S_{i,z}| \sim -S$).
c) Solve the linearized equation by a plane wave ansatz of the form

$$S_{i,x} = S_x \exp\left(i[qia - \omega t]\right)$$
$$S_{i,y} = S_y \exp\left(i[qia - \omega t]\right)$$

Here, a is the lattice constant and ω the Larmor frequency. Solve for $\hbar\omega$

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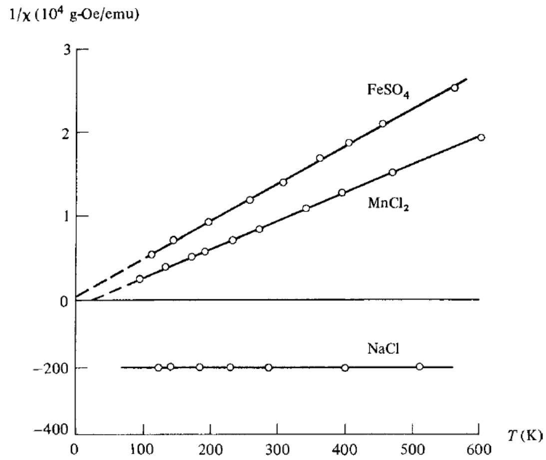
d) Plot $\hbar\omega$ as a function of q in the case of $B_{\text{ext}} = 0$.
e) What is the relation between S_x and S_y? What physical picture is connected with that?
f) The dispersion relation of an antiferromagnet is given by

$$\hbar\omega = \frac{2J_A S}{\hbar} |\sin qa|$$

Add this to the plotted result for the ferromagnet. What do they have in common? What is different?

3. Magnetic Susceptibility

a) The figure below shows the magnetic susceptibility of three compounds. Explain the behavior and mention the likely dominating source of magnetism.
b) For $FeSo_4$ and $MnCl_2$. Assuming equal density, estimate the ratio of effective magnetic moments per Fe/Mn atom (this assumption is not so bad: $FeSo_4$: 2,84 g/cm3 and $MnCl_2$ 2.977 g/cm3)
c) Both FeSo4 and MnCl2 show an offset from zero. Explain the reason behind it and the difference that a positive or a negative offset makes.



(*Taken from Cullity, Graham: Introduction to Magnetic Materials*)