9. Cyclotron Frequency

If the magnetic field is applied in the z direction, the cyclotron effective mass is defined as:

$$m^* = \left(\frac{det|\overleftrightarrow{m}|}{m_{ZZ}}\right)^{1/2}$$

Where $\overleftrightarrow{m}$ is the effective mass tensor defined as $\overleftrightarrow{m}_{ij} = \left(\frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}\right)^{-1}$.

Calculate the cyclotron frequency for electrons at the Fermi surface in a nearly empty tight-binding band, described by:

$$E = -\left(E_1 cos k_x a + E_2 cos k_y b + E_3 cos k_z c\right),$$

And show that the result indeed corresponds to the cyclotron frequency of free electrons of mass m^*.

10. Charge Screening and Thomas Fermi Approximation

a) Use the set of equations we established in the lecture in order to derive the general expression for the dielectric function

$$\epsilon(\boldsymbol{q}) = 1 - \frac{1}{\epsilon_0 q^2}\chi(\boldsymbol{q}) = 1 - \frac{1}{\epsilon_0 q^2}\frac{\rho^{ind}(\boldsymbol{q})}{\phi^{tot}(\boldsymbol{q})}$$

b) What are the assumptions made for the Thomas Fermi approximation?

c) Using these assumptions, one can write $\rho^{ind}(r) = -e\{n_0(\boldsymbol{r}) - n_0\}$, where n_0 is the homogeneous charge carrier concentration of the unperturbed system. Moreover, $n_0(\boldsymbol{r}) = n_0(\mu + e\phi^{tot}\{\boldsymbol{r}\})$, leading to

$$\rho^{ind}(r) = -e\{n_0(\mu + e\phi^{tot}\{\boldsymbol{r}\}) - n_0(\mu)\}$$

Show that

$\epsilon(\boldsymbol{q}) = 1 + \frac{k_s^2}{q^2}$ with $k_s = \sqrt{\frac{e^2}{\varepsilon_0}\frac{D(E_F)}{V}}$

What happens for $q \to 0$?

[Hints: you can assume $e\phi^{tot} \ll \mu$ and use $\frac{D(E_F)}{V} = (\partial n_0 / \partial \mu)_{\mu = E_F}$]

d) How large is the screening length $\lambda = 1/k_s$ roughly for copper? How much for silicon? Compare them to the interatomic spacing.

11. de Haas-van Alphen effect

Note: we will discuss quantum oscillations and Landau levels in only on 01/12 but the problem has been reformulated in a way that should enable the boldest students to give it a try beforehand.

We have seen in the lecture that the measurement of the magnetic susceptibility $\chi = \mu_0 \partial M/\partial B$ shows, of clean metals and at sufficiently low temperatures, an oscillatory behavior as function of the applied magnetic field. These oscillations are periodic in 1/B, and this effect is called the de Haas-van Alphen-Effect. The origin of the effect is rooted in the formation of Landan tubes and the associated changes of quantization conditions under field.

a) Consider nearly free electron carriers of effective mass m^* in a cubic crystal of volume V in an homogeneous magnetic field B along the z direction. The energy dispersion now depends on two quantum numbers, the landau tube index ν and the crystal momentum along the field direction k_z so that it can be written: $E_\nu(k_z) = \frac{\hbar^2}{2\,m^*} k_z + \hbar\omega_c^* \left(\nu + \frac{1}{2}\right)$ where $\omega_c^* = \frac{eB}{m^*}$ is the cyclotron frequency.
Use this to and show that the density of states D(E) in a magnetic field B can be written as:

$$\mathrm{D(E)} = \frac{V}{(2\pi)^2}\sqrt{\left(\frac{2m^*}{\hbar^2}\right)^3}\sum_{\nu=0}^{n_{max}} \frac{\hbar\omega_c^*}{\sqrt{E - \left(\nu + \frac{1}{2}\right)\hbar\omega_c^*}}$$

where n_{max} the number of Landau cylinders with states of energies less than E.
[Hint: this is amounts to estimate the DOS for a collection of 1D systems]

b) For $E_F \gg \hbar\omega_c^*$ show that $D(E_F)$ has periodic discontinuities as function of 1/B.

c) Calculate the period $\Delta(1/B)$ which can be expected for the de Haas-van Alphen effect in Potassium (treat the electrons as free in this case, that is $m^* = m$). Potassium (electronic structure: $[Ar]4s^1$) crystallizes in a body centered cubic unit cell with lattice constant a $=$ 5.225Å.