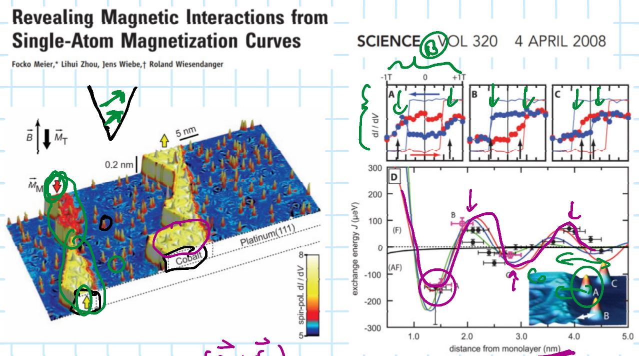
Lecture 23

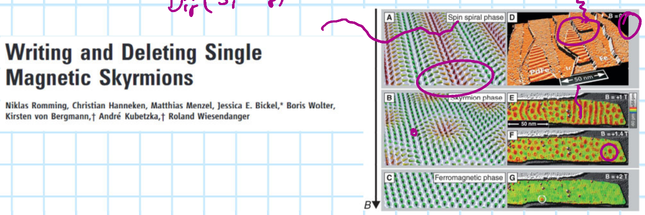
Friday, January 14, 2022 5:06 PM

Review

$$\mathcal{J}_{RKKY} \propto \frac{\cos(2k_F r)}{(2k_F r)^3}$$

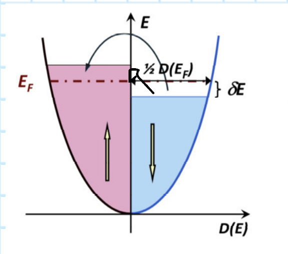


$D_{ij}(\vec{S}_i \times \vec{S}_j)$



$\Uparrow \ominus$

Step #2 : Band ferromagnetism in metals (Stoner Criterion)



Assume that e^- levels renormalize due to electron correlation of e^- with the same spin
↳ some ↓ will switch to ↑

↳ kinetic energy improved : $\delta N = \frac{1}{2} \cdot D(E_F) \cdot \delta E$

$$\boxed{\Delta E_{kin} = \frac{\delta N}{V} \cdot \delta E = \frac{1}{2V} \cdot D(E_F) \cdot \delta E^2}$$

- Potential energy:

$$N_{\uparrow\downarrow} = \frac{N}{2} \pm \delta N \quad ; \quad n_{\uparrow\downarrow} = \frac{n}{2} \pm \delta n \qquad n = \frac{N}{V}$$

- Magnetization

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- Magnetization

$$M_A = -\frac{1}{2}\cdot \overset{2}{g}\cdot \mu_B \left(n_\uparrow - n_\downarrow\right) \simeq -\mu_B \cdot \frac{D(E_F)}{V}\cdot \delta E$$

- imagine M_A creating a ficticious inner „molecular B-field"

$$B_A = \mu_0 \boxed{\lambda} \cdot M_A$$

↳ molecular field constant

$$\Delta E_{pot} = -\int_0^{B_A} M \cdot dB = -\mu_0 \cdot \lambda \int_0^{M_A} M\, dM = -\frac{1}{2}\cdot \mu_0 \cdot \lambda \cdot M_A^2$$

$$\boxed{\Delta E_{pot} = -\frac{1}{2}\mu_0 \cdot \lambda \cdot \mu_B^2 \left[\frac{D(E_F)}{V}\cdot \delta E\right]^2}$$

$$U = \frac{1}{V}\cdot 2 \cdot \mu_0 \cdot \mu_B^2 \cdot \lambda$$ ← only parameter here

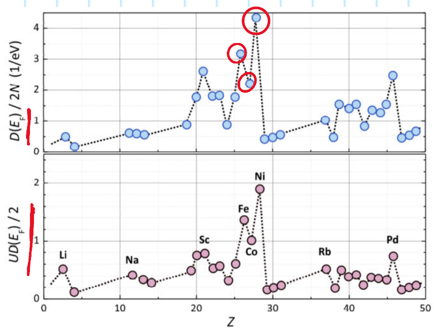
(Coulomb term / „Exchange energy term")

$$\Delta E_{pot} = -\frac{1}{4}\cdot U \cdot \left(2\,\delta N\right)^2$$ ⇒ Sign of ΔE_{pot} is negative
↳ good for FM

$$\Delta E = \Delta E_{kin} + \Delta E_{pot} = \frac{1}{2V}\cdot D(E_F)\cdot [\delta E]^2 \cdot \overbrace{\left[1 - \frac{1}{2}\cdot U \cdot D(E_F)\right]}^{1/[\cdot]\,:\ \text{Stoner factor}}$$

$$\boxed{\frac{1}{2} U \cdot D(E_F) > 1}$$

↳ FM requires high U and/or a high density of dates



Susceptibility

- add external field to the total energy

$$\Delta E = \frac{1}{2V} \cdot D(E_F)(\delta E)^2 \left[1 - \tfrac{1}{2} U \cdot D(E_F)\right] - M \cdot B_{ext}$$

$\longrightarrow$ minimize $\Delta E \Rightarrow \frac{\delta E}{\partial M} = 0 \longrightarrow$ solve for M, $\chi = \mu_0 \cdot \frac{\partial M}{\partial B_{ext}}$

$$\boxed{\chi = I \cdot \chi_p}$$

χ_p : Pauli – susceptibility

$$I = \frac{1}{1 - \frac{1}{2} U \cdot D(E_F)}$$

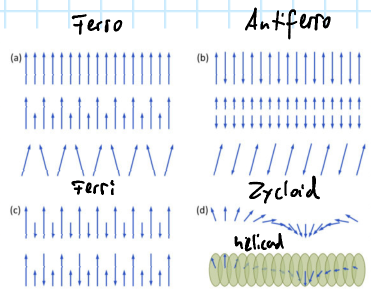
$\uparrow \lambda$

$I = 1 \Rightarrow \chi = \chi_p$ no correlations

$I > 1 + < \infty$: correlations, boost in χ

$I = \infty$: ferromagnet

4. Magnetic Order



Recap:
Paramagnet:

$$\chi = \frac{C}{T} \qquad C = \frac{\mu_0 \cdot n \cdot J(J+1) \cdot g_J^2 \cdot \mu_B^2}{3 k_B}$$

Goal: • Determine $\chi(T)$ for a magnetically ordered structures

- Use mean-field theory

1. Ferromagnetism

below T_C : Magnetic order $\rightarrow$ (later, spin waves)

above T_C : no order $\rightarrow$ mean-field theory

Mean-field theory (molecular field approximation)

- Phenomenological description by Pierre Weiss before QM (1907)
- Use (fictitious) exchange or molecular field

- Exchange interaction

$$E_i = -\frac{J_A}{\hbar^2} \sum_{j=1}^{z} \vec{J}_i \cdot \vec{J}_j \quad \xrightarrow{\text{temporal average of } \vec{J}_j} \quad E_i = -z \cdot \frac{J_A}{\hbar^2} \langle \vec{J}_j \rangle \cdot \vec{J}_i$$

z : # of neighbors

$\langle \vec{J}_i \rangle$

Z = # of neighbours

$$\vec{M} = -n \cdot g_J \cdot \mu_B \cdot \frac{\langle \vec{J}_j \rangle}{\hbar} \iff \langle \vec{J}_j \rangle = \frac{-\hbar}{n \cdot g_j \mu_B} \cdot \vec{M}$$

$$E = -\vec{\mu} \cdot \vec{B}_A$$

$$\vec{\mu} = -g_J \mu_B \cdot \frac{\vec{J}_i}{\hbar}$$

$$\vec{B}_A = \frac{Z \cdot J_A}{n \cdot g_J^2 \mu_B^2} \cdot \vec{M} \quad \text{or} \quad \mu_0 \cdot \gamma \vec{M}$$

$$\gamma = \frac{1}{\mu_0} \cdot \frac{Z \cdot J_A}{n \cdot g_J^2 \cdot \mu_B^2}$$

(molecular field constant)

$$B_{eff} = B_{ext} + B_A$$

contains $\vec{M}$ again

$$M = n \cdot g_J \mu_B \cdot J \underbrace{B_J(y)}_{\text{Brillouin functions}} \quad \text{with} \quad y = \frac{g_J \cdot \mu_B \cdot J \cdot B_{eff}}{k_B T}$$

$$\boxed{T_c = \gamma \cdot C} \qquad \boxed{\chi = \frac{C}{T - T_c}}$$

Curie-Weiss-law