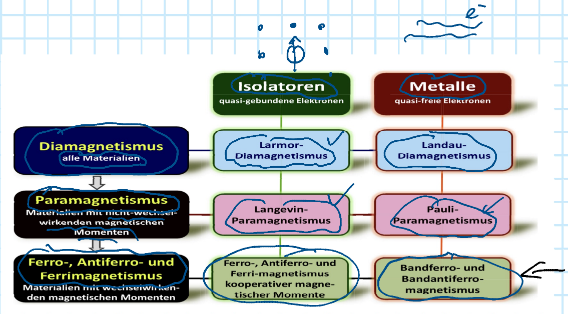
Lecture 17 – Magnetism

Tuesday, December 20, 2022 3:00 PM

1. Macroscopic description of magnetism



important questions

- Do I have conduction electrons?
- Do I have partially or fully filled states?
- Do I have interaction? (Exchange)

Blundell

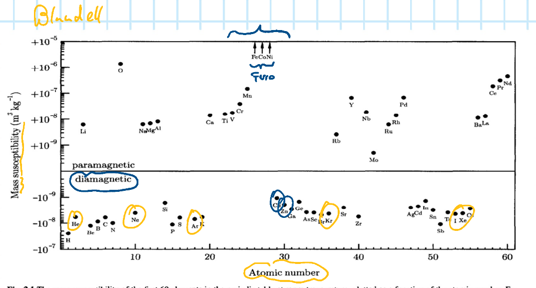


Fig. 2.1 The mass susceptibility of the first 60 elements in the periodic table at room temperature, plotted as a function of the atomic number. Fe, Co and Ni are ferromagnetic so that they have a spontaneous magnetization with no applied magnetic field.

2. Microscopic Origin of Magnetism

2.1. Microscopic theory

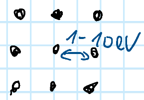
Atomic units of magnetism

- Bohr magneton $\mu_B = \frac{e\hbar}{2m_e}$
- Nuclear magneton $\mu_N = \frac{e\hbar}{2m_p}$

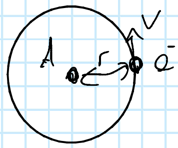
$\cdot\frac{1}{1500}$

$$\mu_B = 9.27 \cdot 10^{-24}\,\frac{J}{T} \approx 58\,\frac{\mu eV}{T} = 14\,\frac{GHz}{T}$$

$hf = E$



Bohr magneton → semiclassical solution of an e^- with an orbital momentum of $1\hbar$



1. atomic current $I = \frac{-e}{T} = \frac{-e}{(2\pi r/v)} = \frac{-e\cdot v}{2\pi r}$
2. atomic magnetic moment $m = I\cdot A = I\cdot \pi r^2 = -\frac{e v\cdot r}{2}$
3. angular momentum $L = m_e \cdot v\cdot r := \hbar$

$\hookrightarrow\ v\cdot r = \frac{\hbar}{m_e}$

$$\Rightarrow m = -\frac{e\hbar}{2\cdot m_e} := \mu_B$$

- Quantum mechanical description:

depends on the occupancy of e^- states

orbital angular momentum: $\hat{\mu}_\ell = -g_\ell \cdot \mu_B \cdot \frac{\hat{\ell}}{\hbar}$

spin angular momentum: $\hat{\mu}_s = -g_s \cdot \mu_B \cdot \frac{\hat{s}}{\hbar}$

$g_s \simeq 2$ (QED 2.0023)

2.2. Atoms in a homogenous magnetic field

- Consider a system of non-interacting atoms

$$H = \underbrace{\frac{1}{2m}\left[\vec{p} + e\vec{A}\right]^2}_{=T \text{ kinetic part}} + V(\vec{r})$$

$V(\vec{r})$: electrostatic potential of cores and other e^-

$\vec{p} = -i\hbar\vec{\nabla}$; Coulomb-gauge $\vec{\nabla}\times\vec{A} = 0$; $\phi = 0$

↳ choose $\vec{A} = -\frac{1}{2}\vec{r}\times\vec{B}_{ext}$

choose $\vec{B}_{ext} \parallel \hat{z} \rightarrow \vec{B}_{ext} = \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix}$

$$T = \frac{1}{2m}\sum_i \left[\overset{\text{"a"}}{\vec{p}_i} - \frac{e}{2}\overset{\text{"b"}}{\vec{r}_i\times\vec{B}_{ext}}\right]^2 = \underbrace{\frac{1}{2m}\sum_i \vec{p}_i^{\,2}}_{T_0} + \underbrace{\frac{e}{2m}}_{\frac{e\hbar}{2m} \to \mu_B}\underbrace{\sum_i (\vec{r}_i\times\vec{p}_i)_z}_{\frac{L_z}{\hbar}}\cdot B_z + \frac{e^2 B_z^2}{8m}\underbrace{\sum_i (x_i^2 + y_i^2)}$$

("a^2", "$2ab$", "b^2"; $\sum_i$ ↳ for all e^-)

$$T = T_0 + \underbrace{\mu_B \cdot \frac{L_z}{\hbar}\cdot B_z}_{} + \underbrace{\frac{e^2 B_z^2}{8m}\sum_i (x_i^2 + y_i^2)}_{} := T_0 + \Delta H_\ell$$

ΔH_ℓ: Orbital contribution

- Equivalently, spin contribution: $\rightarrow \Delta H_S = g_s \cdot \mu_B \sum_i \frac{\vec{s}_i}{\hbar}\cdot\vec{B}_{ext} = g_s\cdot\mu_B \frac{S_z}{\hbar} B_z$ $\quad S_z = \sum_i s_{i,z}$

$$\Delta H = \Delta H_\ell + \Delta H_S = \underbrace{\frac{\mu_B}{\hbar}\left(L_z + g_s\cdot S_z\right)\cdot B_z}_{\text{Langevin-paramagnetism}} + \underbrace{\frac{e^2 B_z^2}{8m}\sum_i (x_i^2 + y_i^2)}_{\text{Larmor diamagnetism}}$$

↳ 2nd order perturbation theory (because magnetic Zeeman interaction is much smaller than the electronic energies of the atomic levels E_n)

$$\Delta E_n = \langle n|\Delta H|n\rangle + \sum_{n\neq n'}\frac{|\langle n|\Delta H|n'\rangle|^2}{E_n - E_{n'}}$$

$= \frac{\mu_B\cdot B_z}{\hbar}\langle n|L_z + g_s\cdot S_z|n\rangle$ (1) atomic paramagnetism (Langevin) (usually dominating)

$+ \frac{e^2 B_z^2}{8m}\langle n|\sum_i (x_i^2 + y_i^2)|n\rangle$ (2) Larmor Diamagnetism; no spin!

$+ \frac{\mu_B^2 B_z^2}{\hbar^2}\sum_{n\neq n'}\frac{|\langle n|L_z + g_s S_z|n'\rangle|^2}{E_n - E_{n'}}$ (3) Van-Vleck paramagnetism

$+ \ldots$

(1) $\langle 0|L_z + g_s S_z|0\rangle \approx \hbar \Rightarrow \mu_B\cdot B_z \rightarrow 10^{-4}\,\text{eV}$

(2) $\langle n|\sum_i (x_i^2 + y_i^2)|n\rangle \approx a_B^2 \Rightarrow \frac{e^2 B_z^2}{8m}\cdot a_B^2 \hat{=} \frac{(\mu_B\cdot B_z)^2}{E_R}$ $\quad E_R = \frac{1}{2}\frac{\hbar^2}{m_e\cdot a_B^2} \approx 13\,\text{eV}$

(3) $\frac{(\mu_B\cdot B_z)^2}{\cancel{\hbar^2}}\frac{\cancel{\hbar^2}}{E_n - E_{n'}} \Rightarrow (E_n - E_{n'}) \sim 1\,\text{eV}$

((3) $\frac{\cdots}{\cancel{\hbar^2}}$ $E_n - E_{n'}$ $\rightarrow$ $(E_n - E_{n'}) \sim 1\,eV$

Both are much smaller than (1), but still comparable in energy