# Lecture 19 - WiSe2022/2023

Tuesday, December 27, 2022 2:06 PM

## 2.5. Larmor Diamagnetism

$$\Delta E = \frac{e^2 \cdot B^2}{8m} \cdot \sum_i \langle n | x_i^2 + y_i^2 | n \rangle$$

↳ relevant for atoms and ions with filled electron shells

↳ A filled shell has: $\langle 0 | x_i^2 | 0 \rangle = \langle 0 | y_i^2 | 0 \rangle = \frac{1}{3} \langle 0 | r_i^2 | 0 \rangle$

$$\Delta E = \frac{e^2 \cdot B^2}{8m} \cdot \frac{2}{3} \sum_i \langle n | r_i^2 | n \rangle$$

$$\Rightarrow \chi_{Dia} = -\frac{\mu_0 \cdot N}{V} \cdot \left( \frac{\partial^2 F}{\partial B^2} \right) = -\mu_0 \cdot \frac{e^2}{6m} \cdot \frac{N}{V} \cdot \sum_i \langle n | r_i^2 | n \rangle$$

← Density of atoms

↑ # $e^-$ in the atom

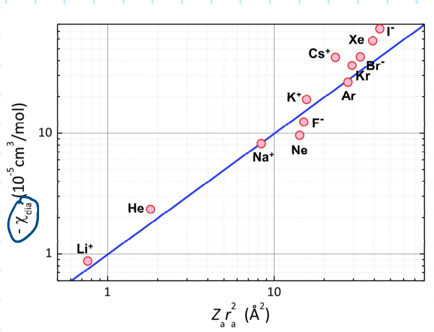
### Assumptions

- contribution of $e^-$ in the outermost shell dominates

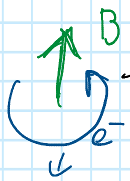
↳ $\sum_i \langle n | r_i^2 | n \rangle \approx r_a^2 \cdot Z_a$ → # of $e^-$ in the outermost shell

atom/ion radius

$$\chi_{Dia} \approx -\mu_0 \cdot \frac{e^2}{6m} \cdot \rho \cdot Z_a \cdot r_a^2 \Rightarrow \text{always negative } \chi_{Dia} < 0$$




⇒ independent of T




→ Lenz law

## 2.6. Magnetism of conduction electrons

conduction electrons

- are spatially delocalized
- still obey the Pauli principle

$$E = \underbrace{\left(n+\tfrac{1}{2}\right)\hbar\omega_c}_{\substack{\text{Landau cylinders} \\ \hookrightarrow \text{Landau diamagnetism}}} + \frac{\hbar^2}{2m}\cdot k_z^2 \;\underbrace{\pm\, \mu_B\cdot B}_{\text{Pauli - Paramagnetism}} \qquad \omega_c = \frac{eB}{m}$$

Solution $\tilde{E} = \left(n+\frac{1}{2}\right)\hbar\omega_c = E - \frac{\hbar^2 k_z^2}{2m}$

Eigen energies of the particle in the field are:

$$\boxed{E_n = \underbrace{\left(n+\tfrac{1}{2}\right)\hbar\omega_c}_{\text{quantized orbital motions } \perp \vec{B}} + \frac{\hbar^2 k_z^2}{2m}}$$

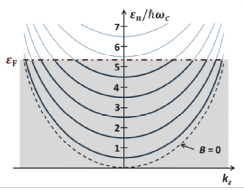


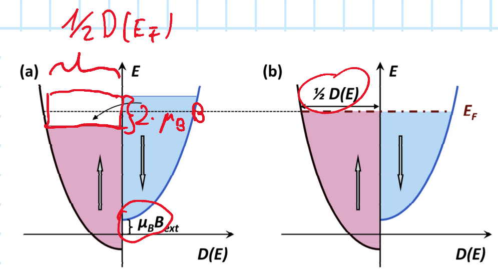

**Abb. 9.37:** Ladungsträgerenergie im Magnetfeld als Funktion der Wellenzahl $k_z$ parallel zur Feldrichtung. Die gestrichelte Kurve zeigt die für $B = 0$ erwartete Abhängigkeit. Der Abstand der Subbänder (Landau-Niveaus) beträgt $\hbar\omega_c$. Die Subbänder sind bis zur Fermi-Energie $\varepsilon_F$ besetzt.

Orders of magnitude: $B \approx 1T$

$\Delta E = \hbar\omega_c = \frac{eB}{m} = 0.1\,meV$

$\frac{\hbar\omega_c}{k} = 1K$

# Pauli - paramagnetism




$$M = (n_+ - n_-)\cdot\mu_B = \frac{N_+ - N_-}{V}\cdot\mu_B$$

$$\approx \frac{1}{V}\frac{1}{2}D(E_F)\cdot 2\mu_B B\,\mu_B = \frac{\mu_B^2\cdot B}{V}\cdot D(E_F)$$

proper way: better solve the integrals

- Use the DOS of a free $e^-$ gas:

$$D(E_F) = \frac{3}{2}\cdot\frac{n\cdot V}{k_B\cdot T_F} \qquad E_F = k_B\cdot T_F$$

$$D(E_F) = \frac{3}{2} \cdot \frac{n \cdot V}{k_B \cdot T_F} \qquad E_F = k_B \cdot T_F$$

$$\Rightarrow \quad M = \frac{3}{2} \cdot \frac{\mu_B^2 \cdot B}{k_B \cdot T_F} \cdot n$$

$$\boxed{\chi_P = \mu_0 \cdot \left(\frac{\partial M}{\partial B}\right) = \mu_0 \cdot \mu_B^2 \cdot \frac{D(E_F)}{V} = n \cdot \frac{3}{2} \cdot \overbrace{\frac{\mu_0 \cdot \mu_B^2}{k_B \cdot T}}^{\text{Langevin}} \cdot \frac{T}{T_F}}$$

Pauli Susceptibility

- $T_F \gg T \Rightarrow$ Only a small fraction of $e^-$ contributes

$\Rightarrow$ temperature-independent (up until very high temperatures)

$\Rightarrow$ Depends on the DOS, for example high for transition metals

## Landau Diamagnetism

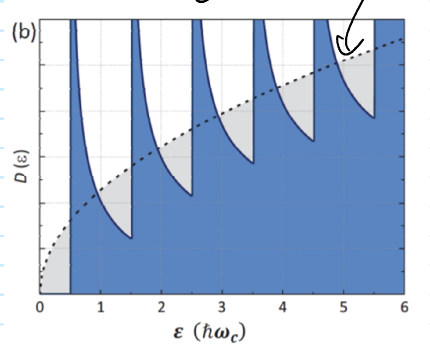
Turn on B-field $\rightarrow$ quantized Landau Levels

↳ change of energy of the system

↳ redistribution of $e^-$

high B field

$E^{1/2}$




$\Rightarrow$ Magnetization is a change in energy of a system due to a change in B

(p. 151 Blundell)

$$\Delta U = \frac{V \cdot k_F \cdot e^2 \cdot B^2}{24 \cdot \pi^2 \cdot m}$$

$$\rightarrow -\frac{\mu_0}{V}\left(\frac{\partial^2 \Delta U}{\partial B^2}\right)$$

for a free $e^-$ $\quad \chi_L = -\frac{1}{3}\chi_P$