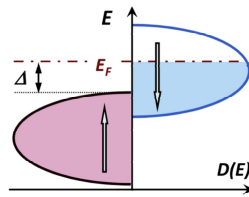


Lecture 28 - WiSe2022/2023

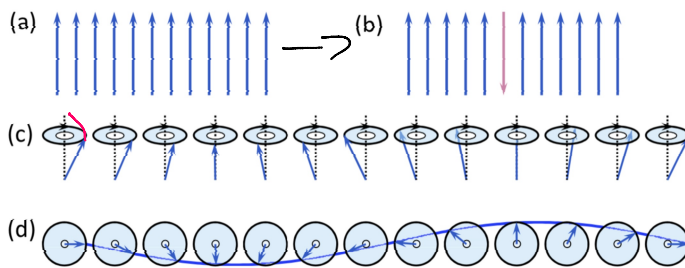
Wednesday, February 8, 2023 6:18 PM

Experiment: Magnetization shows a different dependence on temperature at low temperatures than what is expected from exchange or the Stoner model



- smallest unit of change in M : $\uparrow\uparrow\uparrow\uparrow\uparrow \rightarrow \uparrow\uparrow\downarrow\uparrow\uparrow$
- Other possibility: Change all spins by a fraction
 - ↳ collective excitation (similar phonon)
 - ↳ spin waves

- oscillations in the relative orientation of magnetic moments



- energy is quantized; quanta of excitation is called a magnon

↳ Quasi-particles that obey Bose-Einstein Statistics

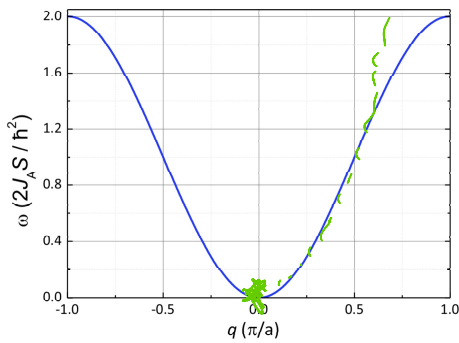
$$E_H = - \frac{J_A}{\hbar^2} \sum_{i=1}^N \vec{S}_i \cdot (\vec{S}_{i-1} + \vec{S}_{i+1}) \quad \text{with} \quad \vec{\mu}_i = -g \vec{S}_i$$

$$g = \frac{g_S \mu_B}{\hbar}$$

$$2J_A \cdot S \cdot \Gamma_1 \dots \Gamma_N$$

$$\hbar \omega = g \cdot \mu_B \cdot B_{\text{ext}} + \frac{2\gamma_A \cdot S}{\hbar} [1 - \cos(q \cdot a)] \quad \text{in 1D}$$

Wave vector Lattice constant

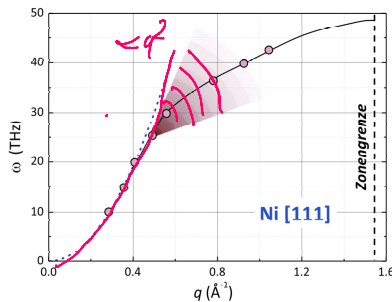
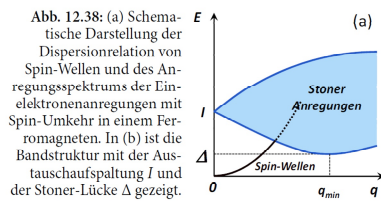


* no energy gap:
 ↳ gapless excitations
 ↳ long wavelength excitation with vanishing energy cost

for $qa \ll 1$ (assume $B_{\text{ext}} = 0$)

$$\omega \approx \underbrace{\frac{2J_A \cdot S \cdot a^2}{\hbar^2}}_{A/\hbar} \cdot q^2$$

↳ (Spin-wave stiffness)



- next-nearest neighbor excitations
- lifetime decay due to Stoner excitations

Temperature-dependence of ferromagnets @ low temperatures

- Spin wave excitation (magnons) lower magnetization

↳ @ $T=0$: total spin $N \cdot S / \hbar$

@ $T>0$: $N \cdot S / \hbar - \sum_q \langle n_q \rangle$

$$\langle n_q \rangle = \left[\exp\left(\frac{\hbar \omega}{k_B T}\right) - 1 \right]^{-1} \quad (\text{BE-statistics})$$

$$\sum_q \langle n_q \rangle = \int d\omega \cdot D(\omega) \cdot \langle n(\omega) \rangle$$

• same calculation

$$\frac{\Delta M}{M(T=0)} \propto \left(\frac{k_B T}{2A \cdot S / \hbar} \right)^{3/2} \quad \text{Bloch } T^{3/2} \text{ law}$$