9) A thin film in the Ginzburg-Landau theory

For a thin film of thickness $d \ll \lambda, \xi$ of a type I superconductor and choosing a gauge for A so that ψ is real, the first and second Ginzburg-Landau equations can be written as

$$-\left[1-\left(\frac{2\pi\xi A}{\Phi_0}\right)^2\right]\psi+\psi^3=0 \tag{1}$$

$$\frac{d^2A}{dx^2}=\frac{\psi^2}{\lambda^2}A \tag{2}$$

a) Using formulas from lecture 4, show that the critical field H_{cm} of a bulk superconductor is related to the penetration depth λ and the coherence length ξ by

$$H_{\mathrm{cm}}=\frac{\Phi_0}{2\sqrt{2}\pi\lambda\xi},$$

where $\Phi_0 = hc/2e$ is the magnetic flux quantum.

b) Calculate the critical field for the film placed in a parallel external field H_0 for arbitrary d and apply the results to the thin film limit.
Take the vector potential A along the $y-$axis. The surfaces of the film conincide with the planes $x = \pm d/2$, which gives you the boundary condition for the field $H(\pm d/2)$.

c) Calculate the critical field of a thin superconducting film of $d = \lambda/10$ with (bulk) $H_{cm} =$ 1000 Oe.

d) Find the critical current density in absence of an external magnetic field for arbitrary d and apply the results to the thin film limit.

Hint: Maybe a look into to book of V. V. Schmidt will give you some ideas for b) and d).

10) Resistivity and Superconductivity

A ring is made of lead wire (diameter d_{lw} = 1 mm) formed into a circle (d_{ring} = 10 cm). The ring is in the superconducting state and has a 10 A current flowing in it. It is observed that there is no detectable change in current for the period of 1 year. If the detector is sensitive to a change in current as little as 1 μA, calculate the experimental upper limit for the resistivity of lead in the superconducting state.
Note: The ring may be considered as an RL-circuit. Assuming a uniform magnetic field distribution for a rough estimate, the self-inductance of the ring is approximately: $L = \frac{1}{2}\pi\mu_0 r$.