Übungen zu „Elektronische Eigenschaften von Festkörpern II: Supraleitung“ (SS2023)

Exercise sheet 8 · Tutorial on 05.07.2023 · (A.Ustinov/G.Fischer)

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21) Josephson inductance

The inductance of an element is defined as the voltage drop that occurs over it divided by the rate of change of an applied current: $L = V/(dI/dt)$.

a) Use the two Josephson equations to derive the inductance of a Josephson junction L_J.

b) Sketch the inductance of a Josephson junction $L(\phi)$. Can you comment on the negative inductance if the junction's Josephson phase φ is between $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$?

22) Josephson energy

Consider the Josephson junction, $I_s = I_c \sin\phi$, biased with external current I.

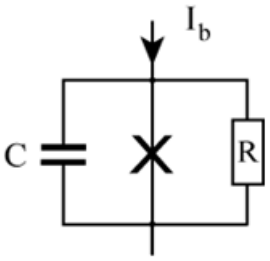
a) Calculate the associated energy, $E_{JJ} = \int_0^t I_s V dt$, accumulated by the Josephson element for constant current and Josephson voltage V.

b) Plot/sketch $I_s(\phi)$ and $E_{JJ}(\phi)$ and comment on the ground state.

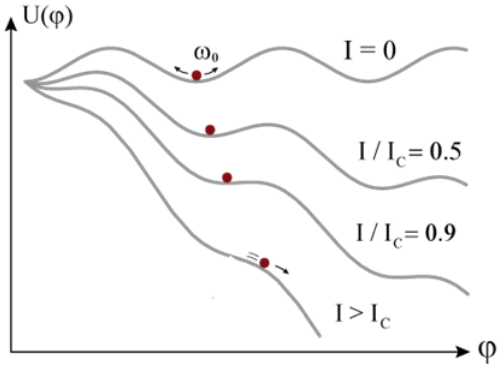
23) The RCSJ model

The resistively and capacitively shunted junction model includes the geometric capacitance C of an SIS tunnel junction and its normal (quasiparticle tunneling) resistance R. The dynamics of the Josephson phase φ are described by an equation of motion:

$$m\ddot{\varphi} + m\gamma\ \dot{\varphi} + \frac{\partial U}{\partial \varphi} = 0. \qquad (1)$$



a) Using Kirchhoff's law for the current that flows through each of the components, express the effective mass m and friction coefficient γ in terms of the system parameters: the critical current I_c, the effective junction shunting resistance R, and the junction capacitance C.



b) To resonantly drive the virtual particle, a small ac component can be added to a static bias current I_b. What is the resonance frequency $\omega_0(I_b)$ of small amplitude oscillations for the particle inside a well of the washboard potential? *Hint:* you already know the Josephson inductance.

c) Equation (1) describes the motion of a virtual particle moving in the washboard potential $U(I_b)$, whose 'tilt' is given by the applied bias current. Can you explain the current-voltage characteristics of the Josephson junction from the motion of this particle when the bias current is changed? See four different cases in the sketch of $U(\varphi)$.