Übungen zu „Elektronische Eigenschaften von Festkörpern II: Supraleitung“ (SS2023)

Exercise sheet 9 · Tutorial on 12.07.2023 · (A.Ustinov/G.Fischer)

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24) Josephson junction in magnetic field

Consider a Josephson junction, which is fabricated from two lead films, and placed in a magnetic field aligned with the junction plane. In this case, the Ferrell-Prange equation describes the dependence of the Josephson phase ϕ on the coordinate x, where $x = 0$ corresponds to the center of the junction:

$$\frac{d^2\phi}{dx^2} = \frac{1}{\lambda_J^2}\sin\phi. \tag{1}$$

The Josephson penetration depth is $\lambda_J = \sqrt{\Phi_0/(2\pi\mu_0 j_c \Lambda)}$ and the effective junction thickness is $\Lambda \approx 2\lambda_{\mathrm{Pb}} \approx 80$ nm. Assume the junction to have a critical current density of $j_c = 10\ \mathrm{A/cm^2}$.

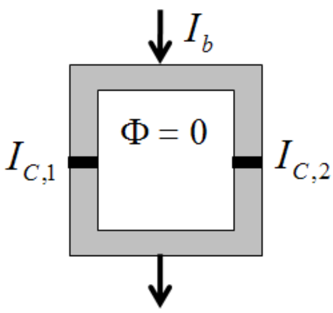
a) The Ferrel-Prange equation has a solution: $\phi(x) = 4\arctan[\exp(x/\lambda_J)]$.
   Create a sketch of $\phi(x)$, the corresponding magnetic field distribution in the junction $H(x)$, and the supercurrent density $j_s(x)$.

b) Find the field $H(0)$ at the center of the vortex.

Hint: $\frac{d}{dx}\arctan(x) = 1/(1+x^2)$.

25) The DC-SQUID without applied flux

Consider two Josephson junctions which are connected in parallel by superconducting wire. Assume critical currents of $I_{C,1} = 5\mu A$ and $I_{C,2} = 7\mu A$. For an applied bias current of $I_b = 10\mu$A, and the case that there is no applied magnetic flux Φ through the loop that is formed by the wire and junctions, find the current flowing through each junction.

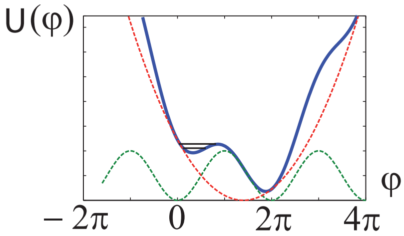


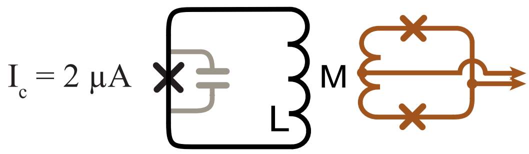
26) "Designing" a Phase Qubit (without SQUID or Qubit knowledge)

A phase qubit is formed from an RF-SQUID. Its potential energy $U(\varphi)$ is given by:

$$U(\varphi) = E_J\left[1 - \cos(\varphi) + \frac{\left(\varphi - \frac{2\pi}{\Phi_0}\Phi\right)^2}{2\beta_L}\right], \qquad \beta_L = 2\pi L I_c/\Phi_0 \tag{2}$$

The qubit states are phase eigenstates confined to the shallow potential well of $U(\varphi)$ (see sketch on back page) located at $\varphi_0 = 1.01$.





a) During qubit readout, the excited state tunnels to the neighboring potential well located at $\varphi_1 = 5.91$. Calculate the associated change of the magnetic flux in the qubit inductance $L = 500$ pH.
Hint: Treat the problem like a simple Josephson junction and calculate the difference in the phase and the Josephson current first.

b) To detect this flux change, a DC-SQUID shall be used. How large must one design the mutual inductance M between the qubit and the SQUID to obtain a flux signal of $\Delta\Phi_{\text{squid}} = 10\text{m}\Phi_0$?