Übungen zu „Elektronische Eigenschaften von Festkörpern II: Supraleitung" (SS2023)

Exercise sheet 10 · Tutorial on 19.07.2023 · (A.Ustinov/G.Fischer)

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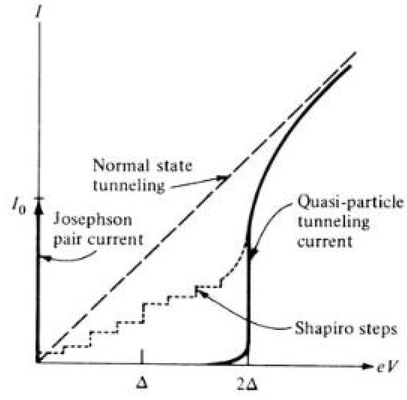
27) Shapiro steps

Overview: the IV characteristic of a Josephon junction without (solid) and with (dotted) an RF field imposed. The dashed line indicates the tunneling IV curve for the equivalent diode with normal (not superconducting) electrodes
(from: https://cas.ee.ic.ac.uk/people/dario/files/E302/1-Sensors.pdf)



The oscillations of a virtual particle can be driven by applying a small ac component to an ideal voltage bias: $V = V_0 + V_{ac}\cos(\omega_{ac}t)$ with $\omega_{ac} = 2\pi f_{ac}$.

a) Using the second Josephson relation and integrating, express the phase difference across the junction $\varphi(t)$.

b) With the first Josephson relation and the following expansion into Bessel functions of the first kind write down an expression for the super current.

$$\sin(z\sin(x)) = \sum_{n=-\infty}^{\infty} (-1)^n J_n(z)\ sin(-nx)$$

c) Show that there is only a dc component if $V_0 = V_m = m\hbar\omega_{ac}/(2e)$.

d) Show that the half-width of the mth step is $I_m = I_c J_m(2eV_{ac}/(\hbar\omega_{ac}))$.

28) Ambegoakar-Baratoff relation

For a Josephson Junction between two identical superconductors the temperature dependence of the product of the maximum supercurrent I_c and the normal state resistance R_n is given by

$$I_c R_n = \frac{\pi}{2e}\Delta_0(T)\tanh\left(\frac{\Delta_0(T)}{2k_B T}\right)$$

a) Show that the product is nearly constant as $T \to 0$.

b) Show that the product approaches 0 linearly as $T \to T_c$. Hint: remember the empirically relation $\frac{\Delta(T)}{\Delta(0)} \approx 1.74\left(1 - \frac{T}{T_c}\right)^{1/2}$ and the temperature dependence of the energy gap in BCS theory.

c) How can the normal state resistance R_n be measured?

29) Phase dynamics in a long junctions

Consider a long Josephson junction formed by an aluminum-oxide tunnel barrier between two Nb films. The specific capacitance of the junction is $C^* = 5\mu\mathrm{F/cm}^2$ and the London penetration depth for Nb thin films is $\lambda_L = 90$ nm.

a) How large is the Swihart velocity $\tilde{c}$ in this junction? (compare it to the speed of light in vacuum.)

b) How does the Swihart velocity change if the barrier thickness is doubled?

c) How does the critical current density change in this case?

d) How does the Josephson penetration depth λ_J change?

30) Lab tour

The lab tour starts on July 26 at 11:00 a.m.. Meeting point is the exercise room (3-01).