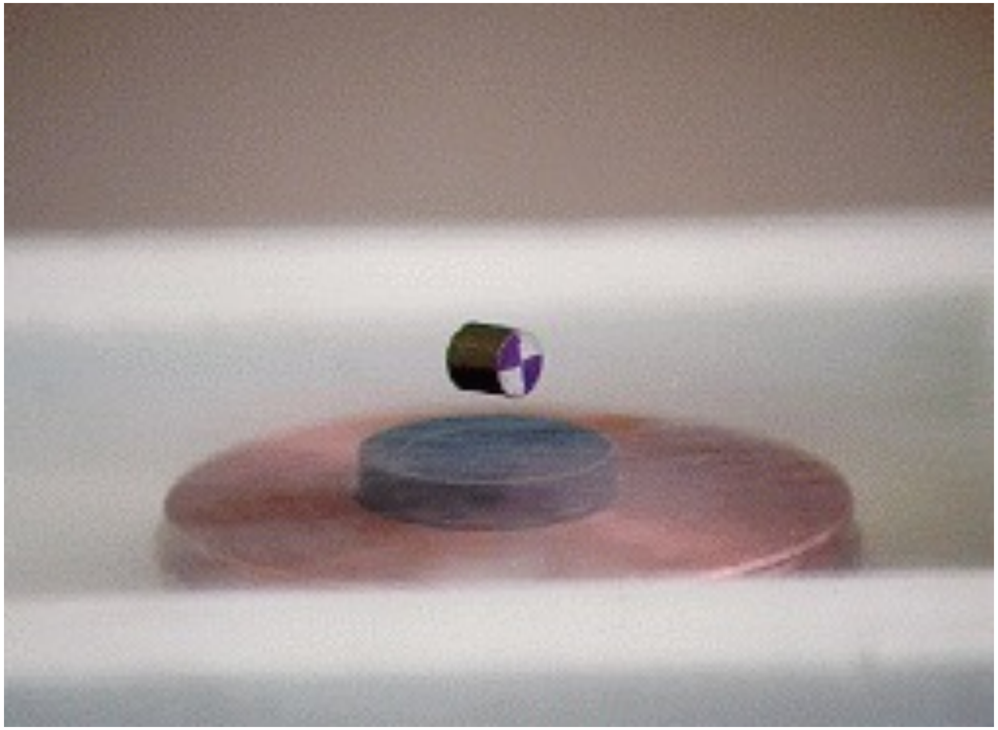
Superconductivity
Lecture 1

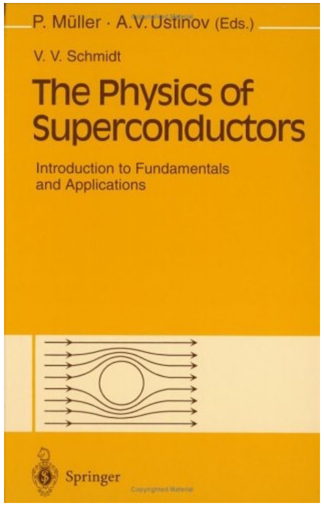


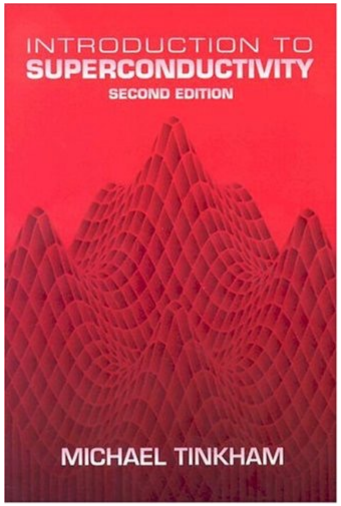
Literature

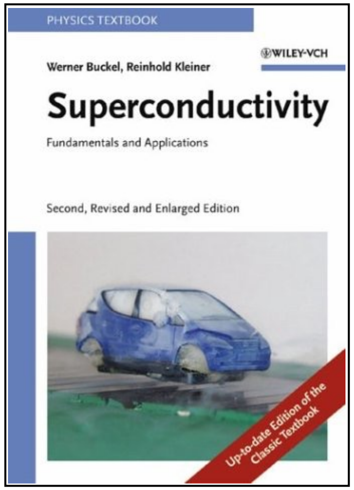
1. V.V. Schmidt "The physics of superconductors", Eds. P.Müller and A.V.Ustinov, Springer, 1997.

2. M. Tinkham "Introduction to superconductivity", McGraw-Hill, 2nd ed., 1996.

3. W. Buckel and R. Kleiner, "Superconductivity: Fundamentals and Applications", Wiley-VCH, 2004







Phenomenon of superconductivity

- Introduction
- Historic milestones
- Superconducting materials
- Meissner effect
- Levitation experiments
- Magnetic flux quantization
- BCS theory – main idea
- Energy gap
- Josephson effect

Historic milestones

 1908 Making liquid helium (4.2 K)

1911 Discovery of zero resistance

1933 Meissner effect

1935 Londons' theory

 1950 Ginzburg-Landau theory

 1957 Bardeen-Cooper-Schrieffer (BCS) theory

1960 Magnetic flux quantization

 1962 Josephson effect

 1986 High-temperature superconductors

1999 Superconducting qubit

 Nobel Prizes

The Nobel Prize in Physics 1913

"for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium"

Heike Kamerlingh Onnes

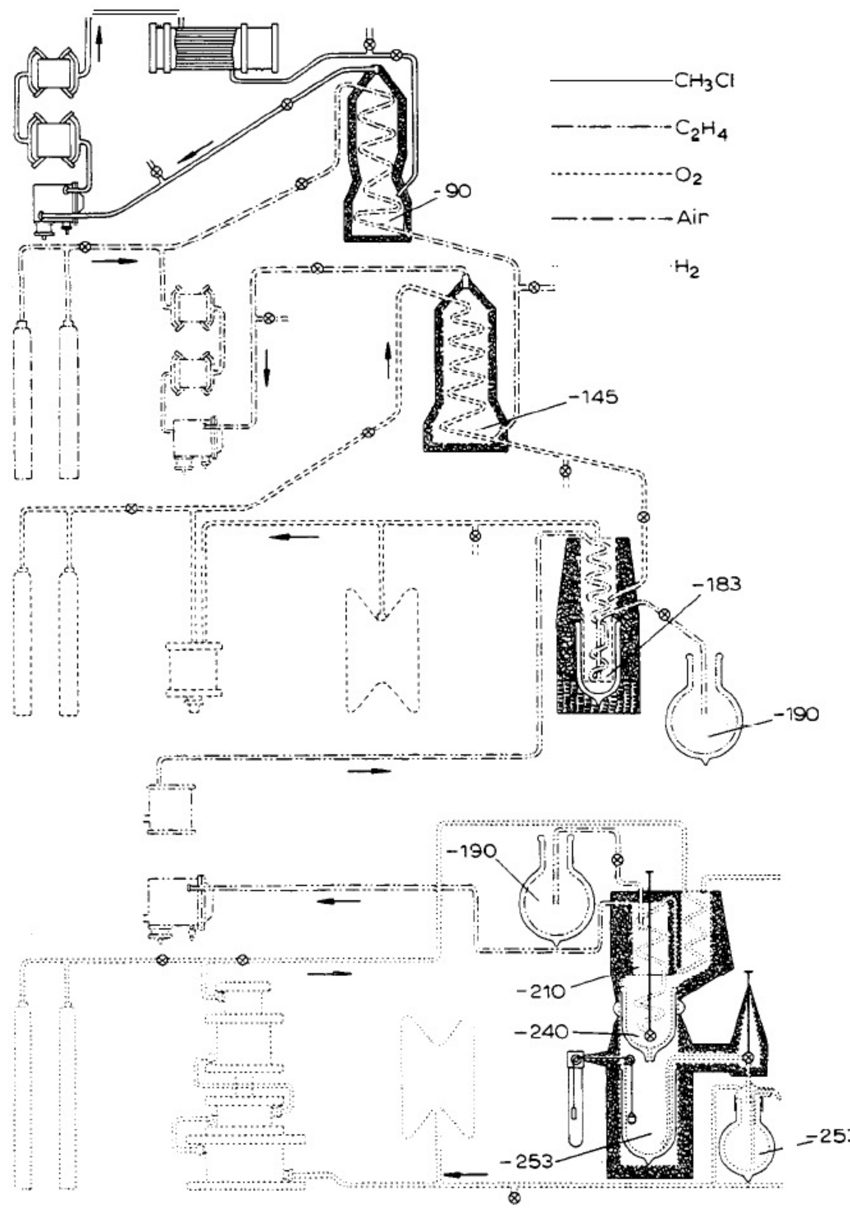
the Netherlands

Leiden University
Leiden, the Netherlands

b. 1853
d. 1926

methyl chloride
ethylene
oxygen
air
hydrogen

“The cycles for the various gases are joined in cascades, so that the cold bath, which can be maintained by a previous cycle, also serves to liquefy at a lower temperature the gas which circulates in the following cycle”.



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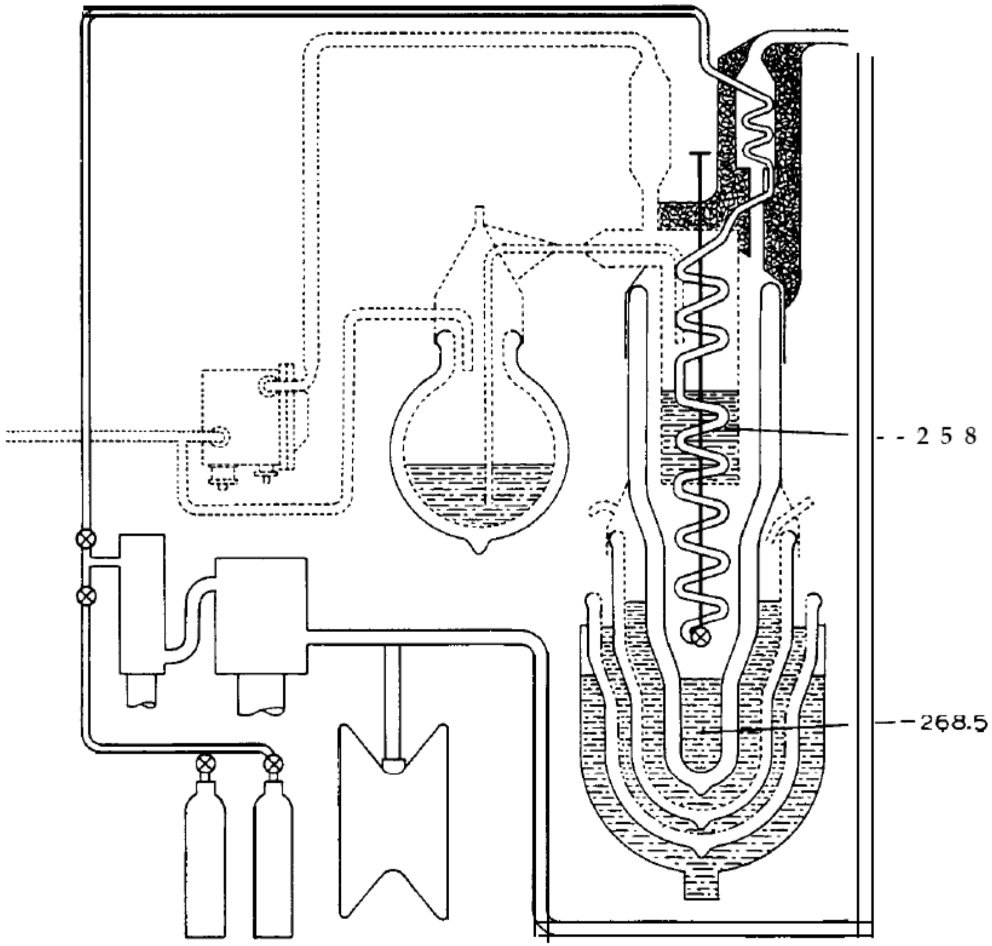
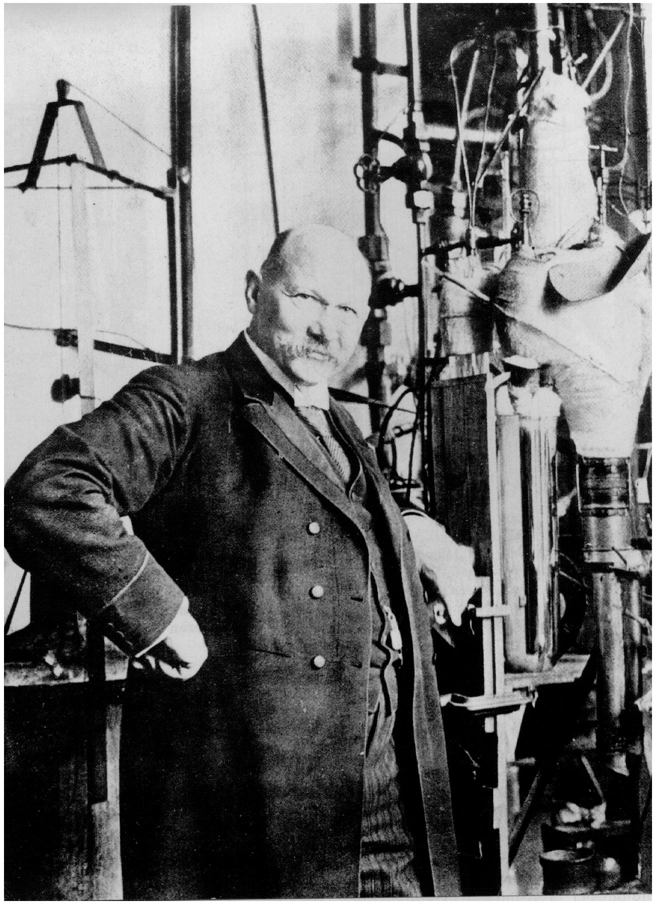


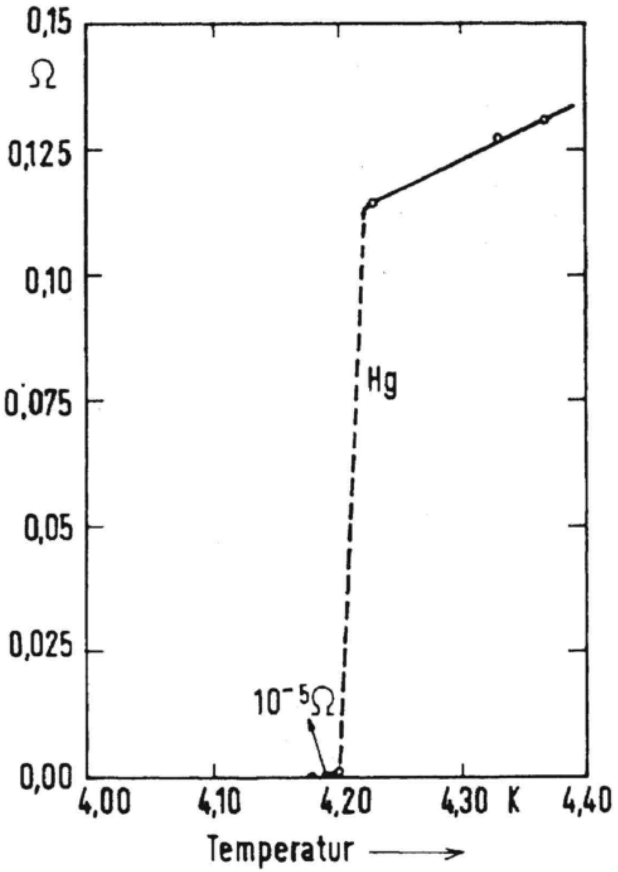
"The compressed helium comes, after suitable pre-cooling, into the refrigerator, where hydrogen is evaporating at the air pump, runs through the regenerator spiral and expands at the throttle valve. The liquid formed according to the Linde process gathers in the lower portion of the vacuum glass vessel. In order to be able to observe the liquid helium this portion is transparent and is surrounded by a vacuum glass vessel which is kept filled with liquid hydrogen, which is again protected by a transparent vacuum glass vessel containing liquid air".

Introduction

Superconductivity was discovered by Kamerlingh-Onnes in 1911 in mercury (Hg), having $T_c \approx 4$ K

".. Mercury has passed into a new state, which on account of its extraordinary electrical properties may be called the superconductive state"





Dependence of the resistance on temperature

Superconducting materials

Cold liquids required for reaching low temperatures:

- helium ^4He (4.2 K)
- hydrogen H_2 (20 K)
- neon Ne (27 K)
- nitrogen N_2 (77 K)

modern 100 liter liquid helium dewar

material	T_c,K	H_c,Oe	year	
Al	1.2	105	1933	pure metals
In	3.4	280		
Sn	3.7	305		
Pb	7.2	803	1913	
Nb	9.2	2060	1930	
NbN	15	$1.4\ 10^5$	1940	alloys
Nb_3Ge	23	$3.7\ 10^5$	1971	

Superconducting elements

30 elements superconduct at ambient pressure, 23 more superconduct at high pressure.

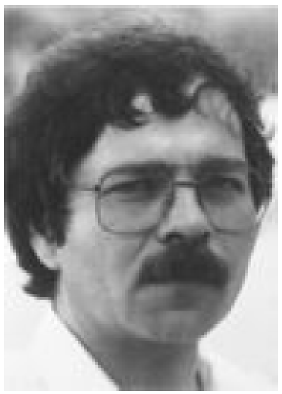
ambient pressure superconductor: T_c(K) / T_c^{max}(K) / P(GPa)

high pressure superconductor: T_c^{max}(K) / P(GPa)

H																	He
Li 0.0004 14 30	Be 0.026											B 11 250	C	N	O 0.6 100	F	Ne
Na	Mg											Al 1.14	Si 8.2 15.2	P 13 30	S 17.3 190	Cl	Ar
K	Ca 25 161	Sc 19.6 106	Ti 0.39 3.35 56.0	V 5.38 16.5 120	Cr	Mn	Fe 2.1 21	Co	Ni	Cu	Zn 0.875	Ga 1.091 7 1.4	Ge 5.35 11.5	As 2.4 32	Se 8 150	Br 1.4 100	Kr
Rb	Sr 7 50	Y 19.5 115	Zr 0.546 11 30	Nb 9.50 9.9 10	Mo 0.92	Tc 7.77	Ru 0.51	Rh .00033	Pd	Ag	Cd 0.56	In 3.404	Sn 3.722 5.3 11.3	Sb 3.9 25	Te 7.5 35	I 1.2 25	Xe
Cs 1.3 12	Ba 5 18	insert La-Lu	Hf 0.12 8.6 62	Ta 4.483 4.5 43	W 0.012	Re 1.4	Os 0.655	Ir 0.14	Pt	Au	Hg-α 4.153	Tl 2.39	Pb 7.193	Bi 8.5 9.1	Po	At	Rn
Fr	Ra	insert Ac-Lr	Rf	Ha													
		La-fcc 6.00 13 15	Ce 1.7 5	Pr	Nd	Pm	Sm	Eu 2.75 142	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu 12.4 174	
		Ac	Th 1.368	Pa 1.4	U 0.8(β) 2.4(α) 1.2	Np	Pu	Am 0.79 2.2 6	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

The Nobel Prize in Physics 1987

"for their important break-through in the discovery of superconductivity in ceramic materials"



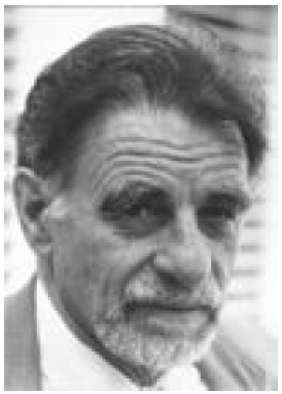
J. Georg Bednorz

1/2 of the prize

Federal Republic of Germany

IBM Zurich Research Laboratory
Rüschlikon, Switzerland

b. 1950



K. Alexander Müller

1/2 of the prize

Switzerland

IBM Zurich Research Laboratory
Rüschlikon, Switzerland

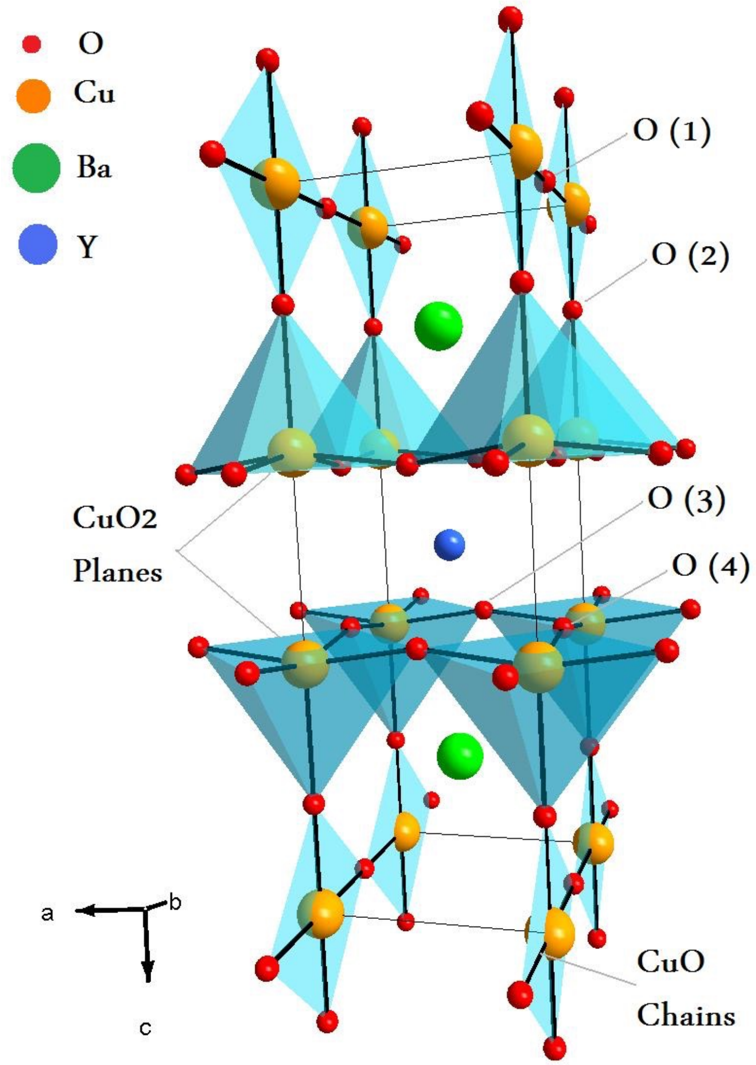
b. 1927

ceramic materials

material	T_c,K	year
$La_{1.85}Ba_{0.15}CuO_4$	35	1986
$YBa_2Cu_3O_7$	93	1987
$Bi_2Sr_2CaCu_2O_{8+x}$	94	1988
$Ta_2Ba_2Ca_2Cu_3O_{10+x}$	125	1988
$HgBa_2Ca_2Cu_3O_{8+x}$	150^*	1993

* under pressure

High-Tc superconductors



ceramic materials

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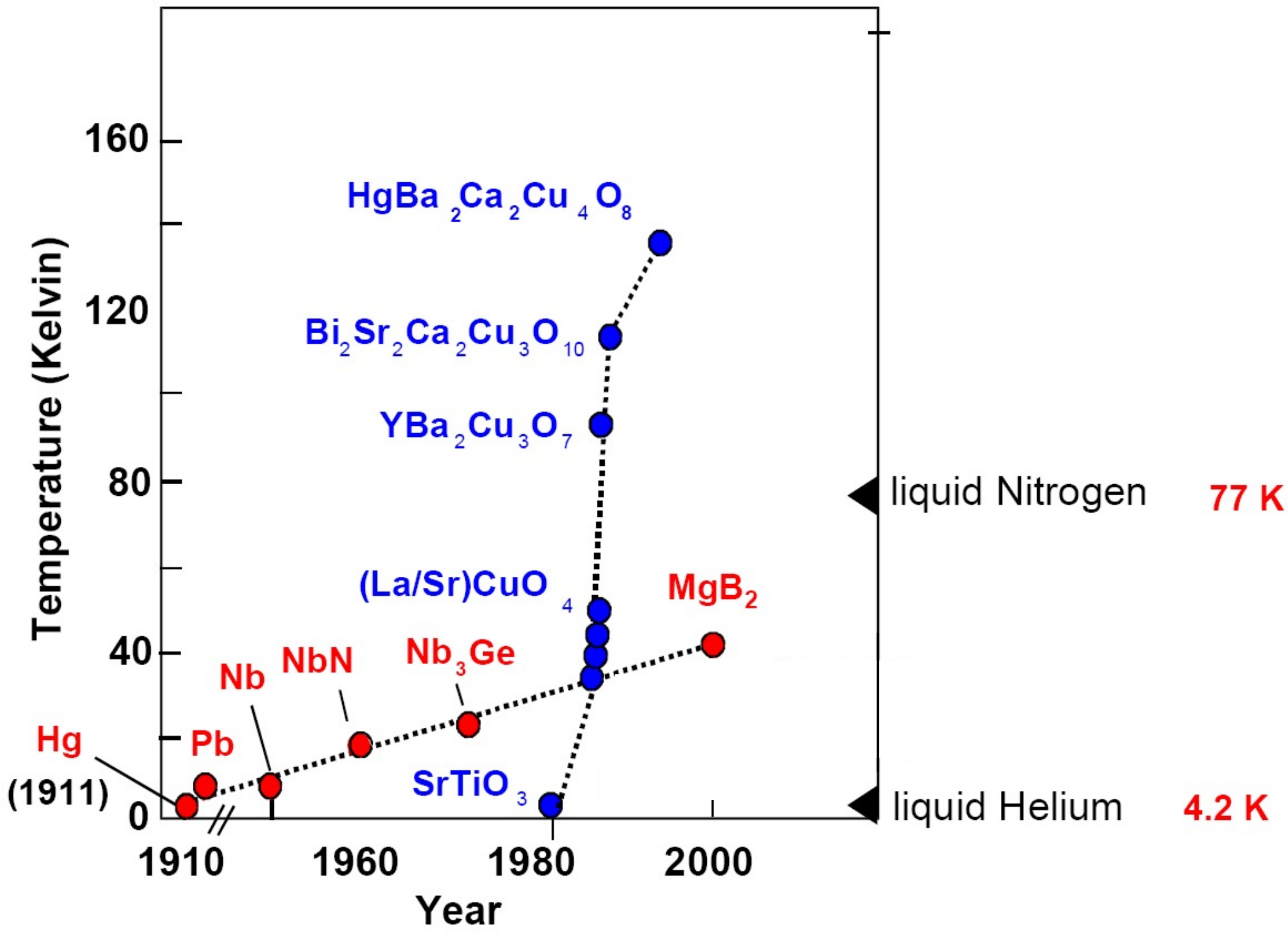
tetragonal
crystal structure

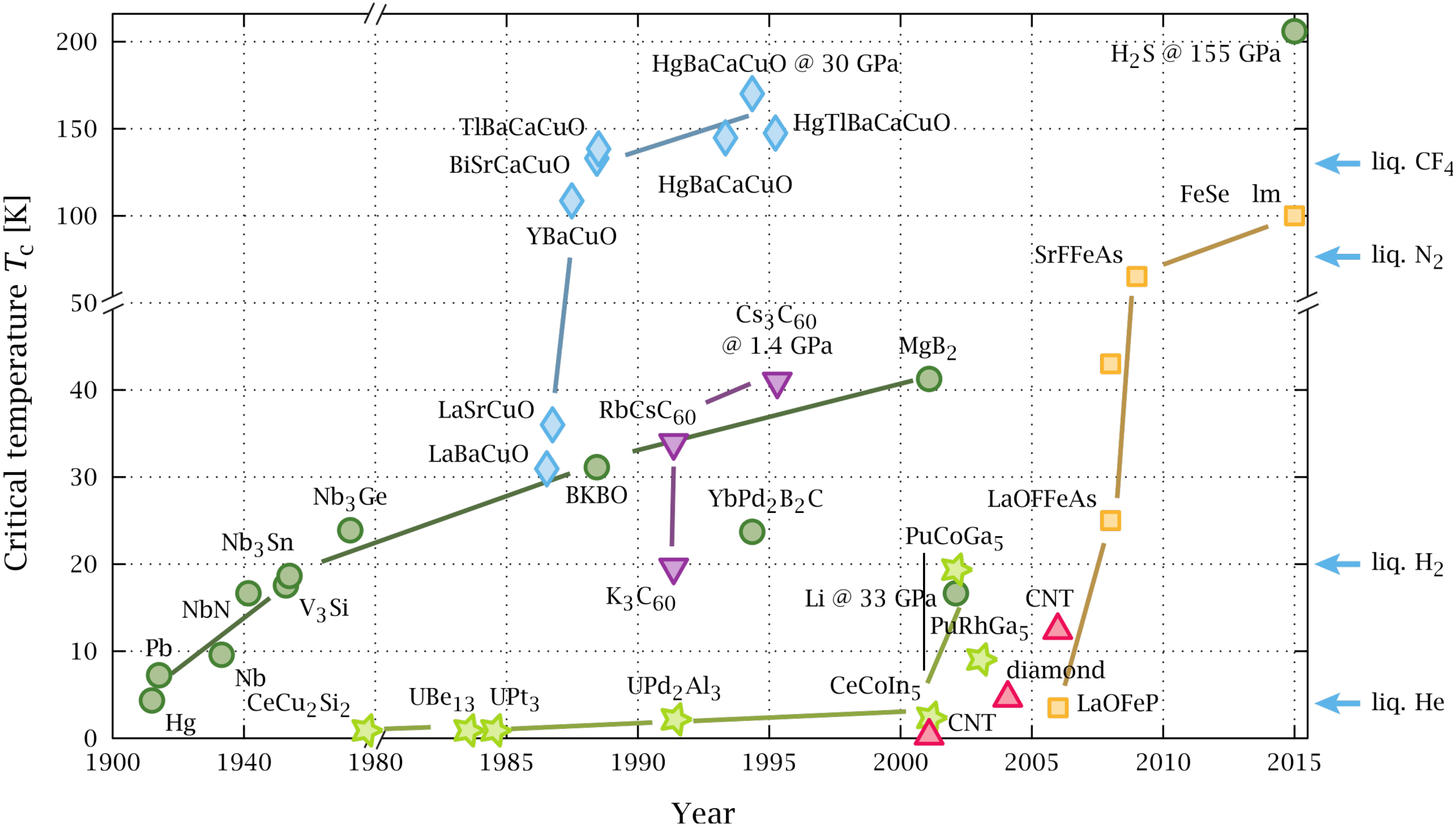
* under pressure

orthorhombic
crystal structure

Superconducting materials



Most recently discovered superconducting materials



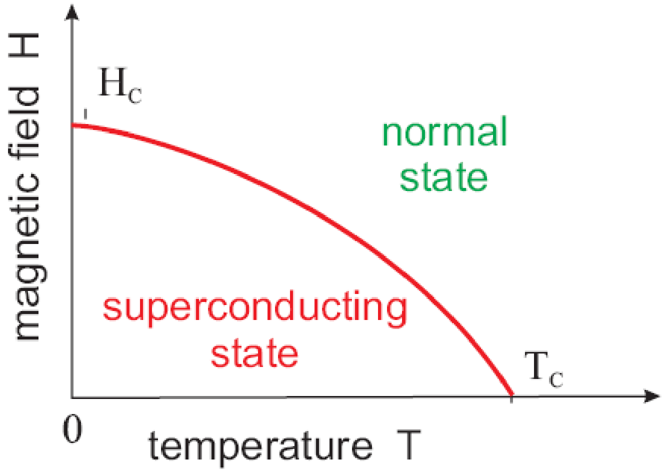
Introduction

Superconductivity is destroyed:

- by increasing temperature at $T > T_c$
- by large magnetic field $H > H_c$

Phase diagram of a superconductor in the $H - T$ plane is described by an empirically found formula:

$$\frac{H_c(T)}{H_c(0)} = \left[1 - \left(\frac{T}{T_c}\right)^2\right]$$



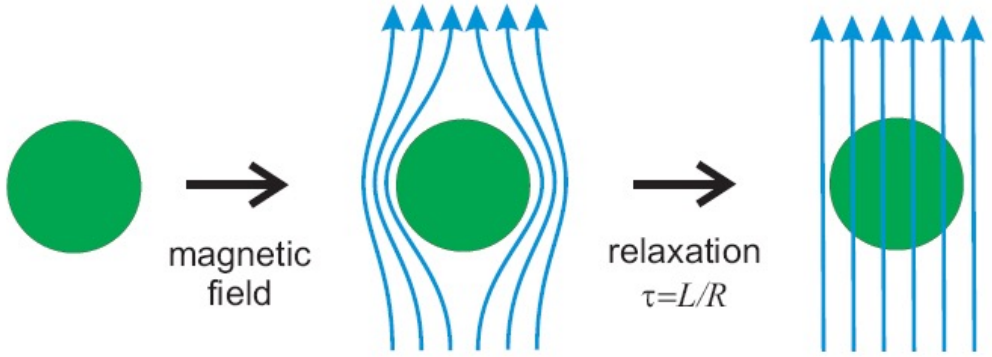
$H - T$ diagram for the superconducting state

Meissner effect

Discovered by Walther Meissner and Robert Ochsenfeld in 1933.

Example 1: a conductor

Magnetic field induces a screening current (Lentz' rule) which generates the opposite field.



A real conductor in magnetic field.

In an *ideal* conductor: $\vec{E} = \vec{j}\rho = 0$

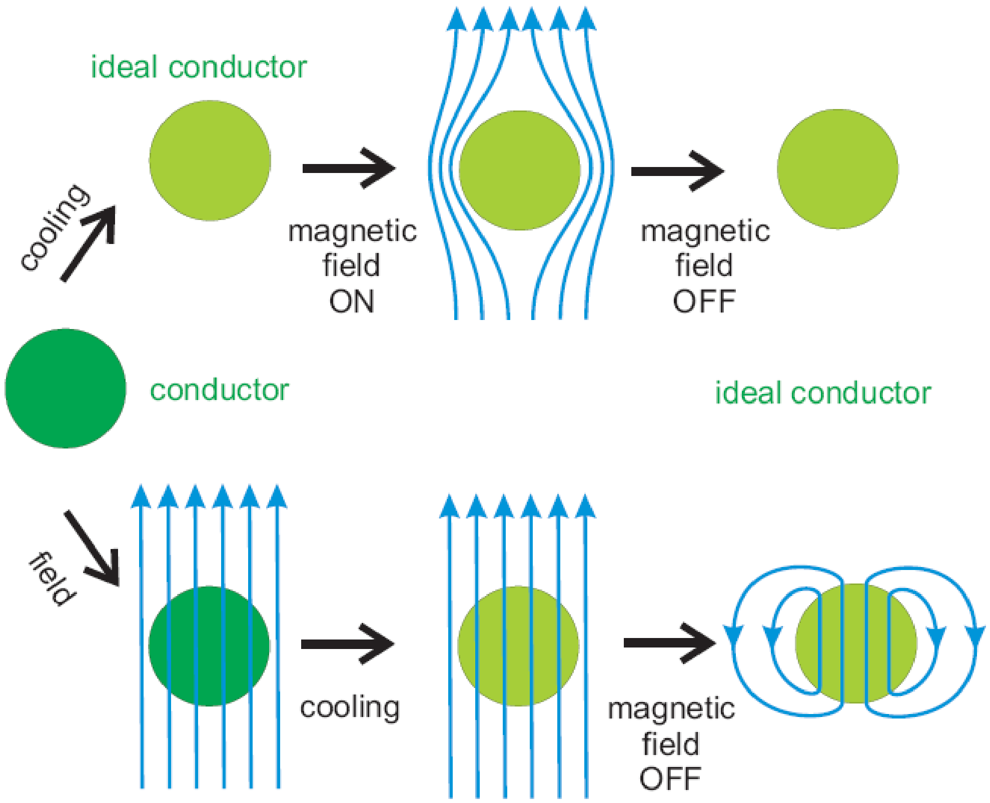
According to Maxwell's equation $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$

inside the conductor we have $\vec{B} = \text{const}$

Meissner effect

Example 2: an ideal conductor

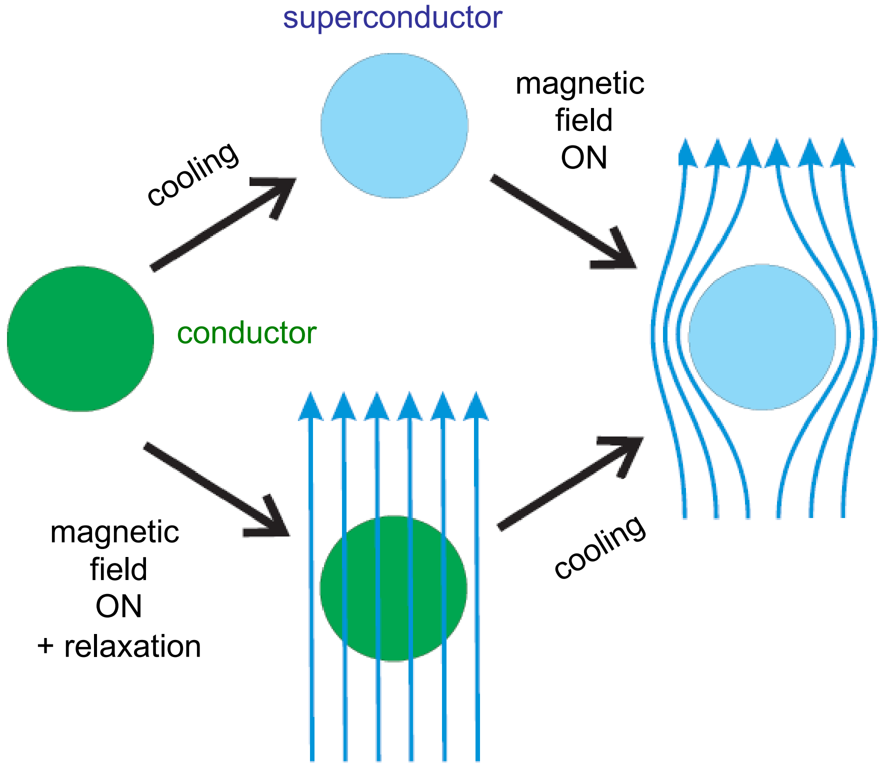
Magnetic field induces a screening current (Lentz' rule) which generates an opposite magnetic field inside the sample



An ideal conductor in magnetic field

Meissner effect

Example 3: finally, a superconductor



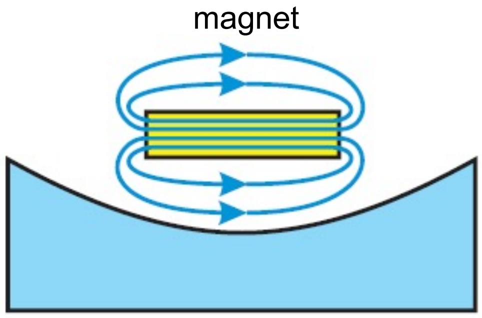
<u>Meissner effect:</u>

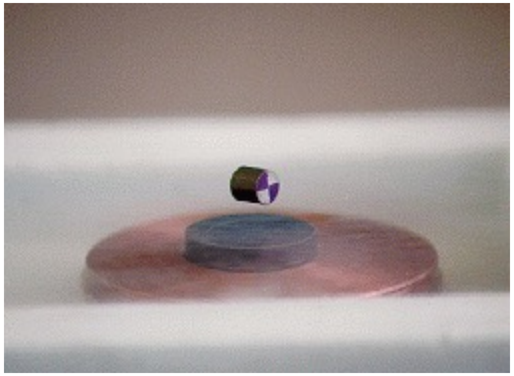
Superconductor always
expels the magnetic flux

A superconductor in magnetic field

Levitation experiments

Levitation of a magnet:





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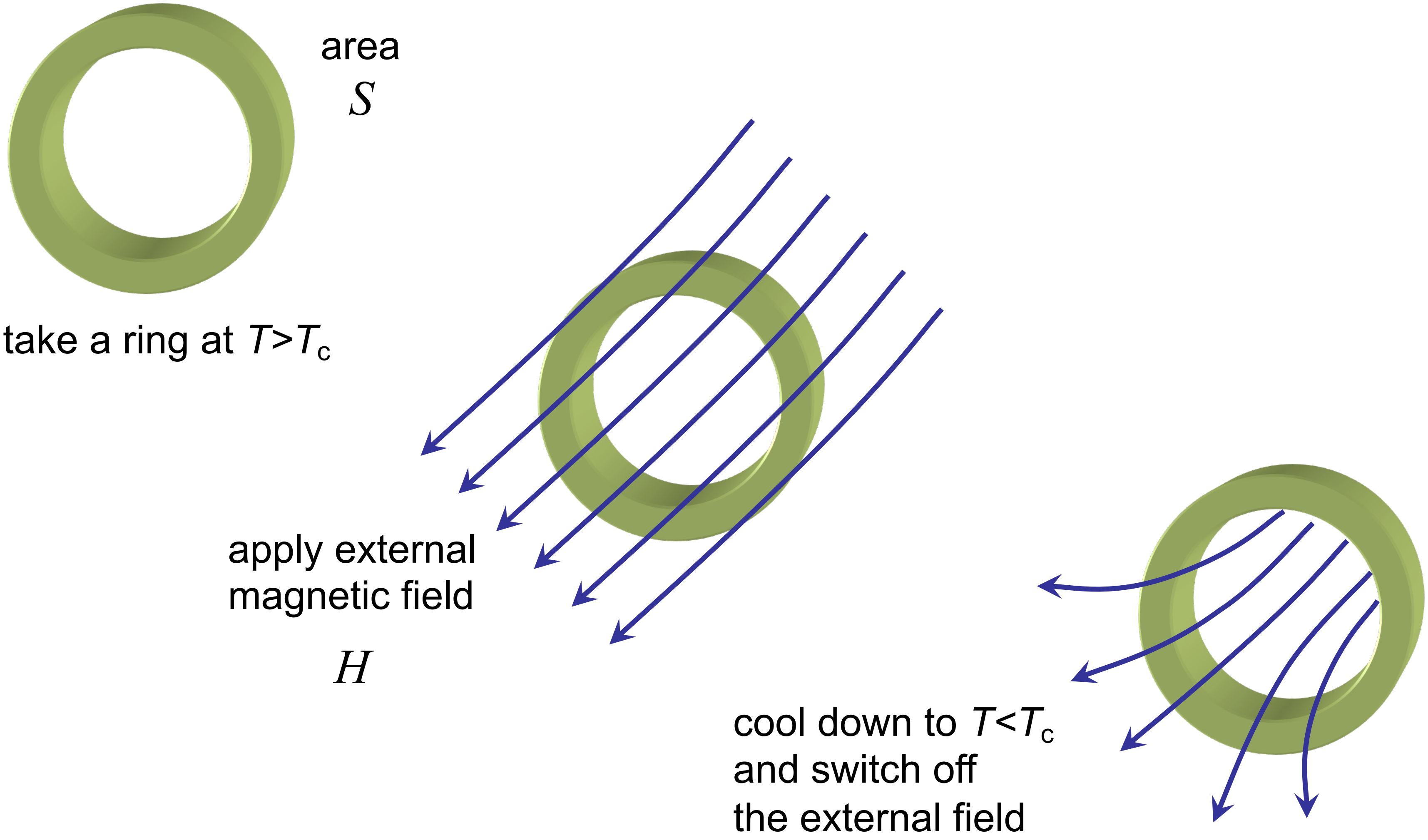
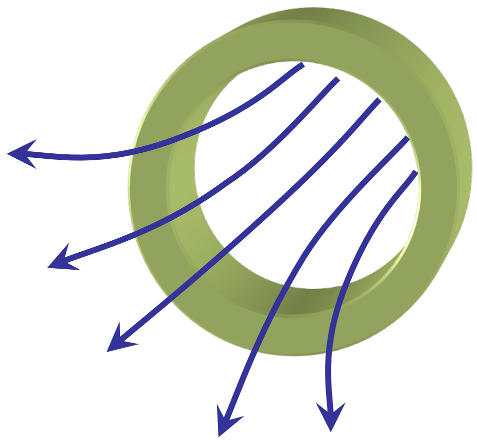
Trapping magnetic flux in a ring



Magnetic flux quantization



how large is
the magnetic
flux in the ring ?

$$\Phi = \oint \vec{A} \cdot \mathrm{d}\vec{\ell} \approx H \cdot S$$

Experimental result:

$$\Phi = n\Phi_0$$

where $\Phi_0 = 2.07 \times 10^{-15}\,\mathrm{V \cdot s}$

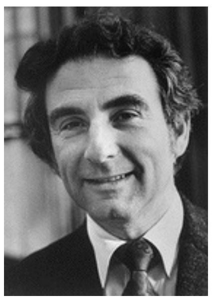
$$n = 0, \pm 1, \pm 2, \ldots$$

$$\Psi = |\Psi| \exp(\mathrm{i}\theta)$$

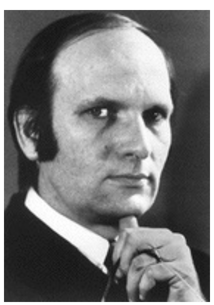
S. Deaver and W.M. Fairbank, *Phys. Rev. Lett.* **7**, 43 (1961)
R. Doll and M. Näbauer, *Phys. Rev. Lett.* **7**, 51 (1961)

BCS theory (1957): Cooper pairs

John Bardeen

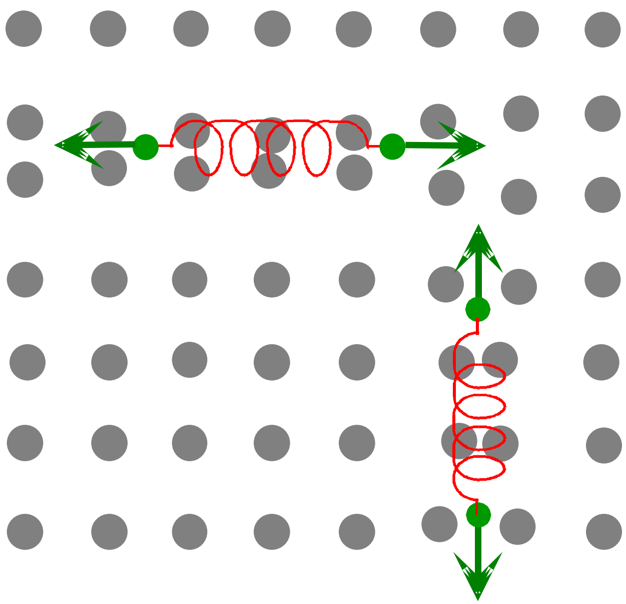


Leon Cooper



John Schrieffer

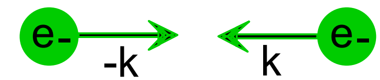
Nobel Prize in Physics 1972



ions

electrons

Cooper pair



spin singlet

angular momentum	0 (s-state)
attractive interaction	electron-phonon
coherence length	~ 100 nm >> atom spacing
order parameter	$\Delta = \Delta_0 e^{i\varphi}$

The physical mechanism of superconductivity became clear only 46 years after this phenomenon was discovered.

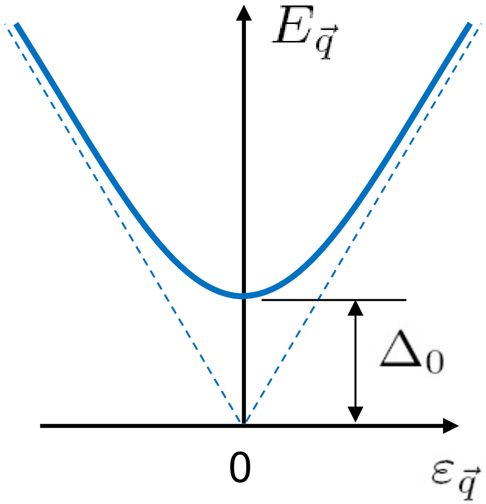
Superconducting energy gap

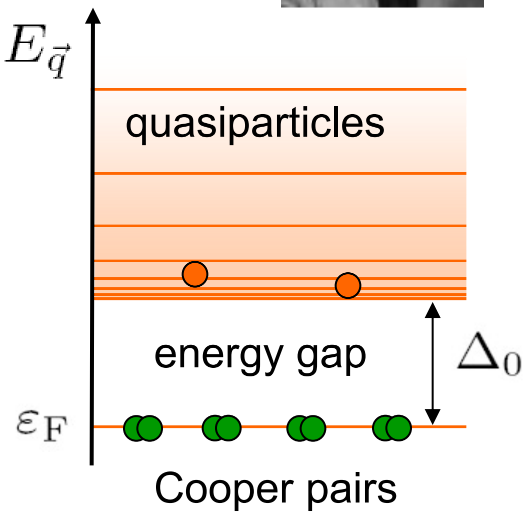
The energy of an elementary excitation

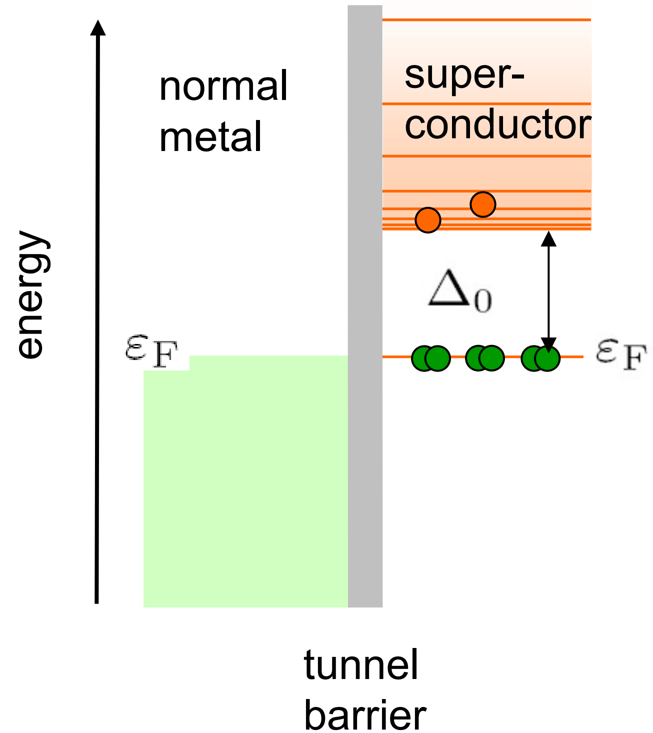
$$E_{\vec{q}} = \sqrt{\varepsilon_{\vec{q}}^2 + \Delta_0^2}$$

Giaever's experiments in 1960

Nobel Prize 1973



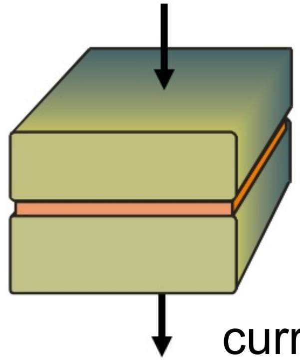




$$\varepsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m} - \frac{\hbar^2 k_F^2}{2m}$$

the „price“ of breaking one Cooper pair is $2\Delta_0$

Josephson effect



$$\Psi_1 = |\Psi_1| \exp(i\theta_1)$$

$$\Psi_2 = |\Psi_2| \exp(i\theta_2)$$

$$|\Psi_1| = |\Psi_2|$$

superconducting phase difference: $\varphi = \theta_1 - \theta_2$

Josephson relations

$$\begin{cases} I_s = I_c \sin\varphi \\ V = \dfrac{\hbar}{2e}\dfrac{d\varphi}{dt} \end{cases}$$

Electromagnetic radiation at the frequency f

$$f = \frac{V}{\Phi_0}$$