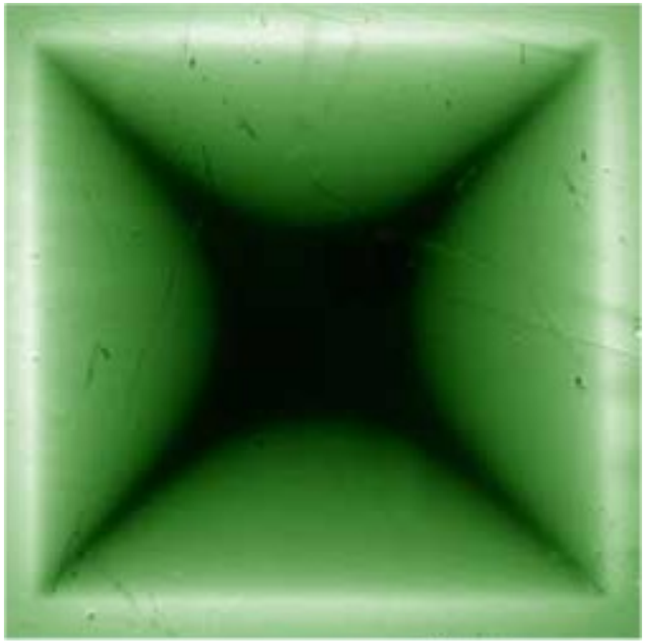
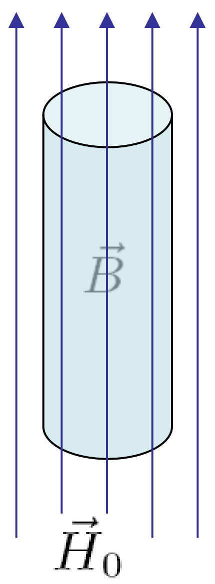
Superconductivity
Lecture 2



Magnetic properties of superconductors

- Magnetic properties of type I superconductors
- Demagnetizing factor
- Intermediate state
- Magnetic properties of type II superconductors
- Critical fields H_{c1}, H_{c2}, and H_{c3}
- Anisotropy of magnetic properties

Magnetic properties of type I superconductors



Magnetization curve

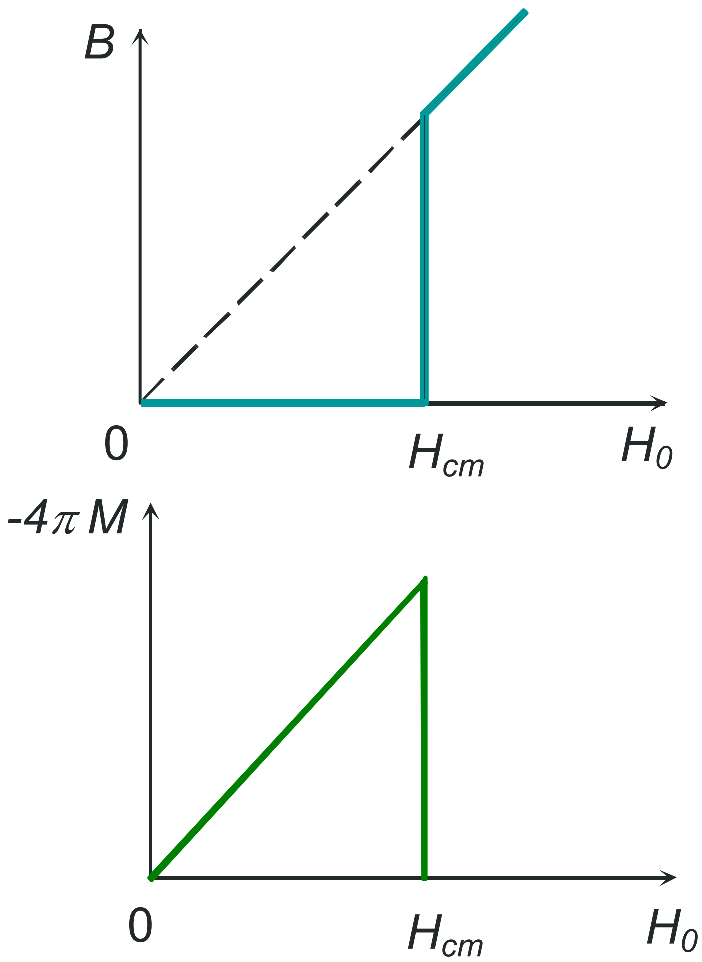
$$\vec{B} = \vec{H}_0 + 4\pi\vec{M} \quad \text{[cgs]}$$

$$\vec{B} = \mu_0(\vec{H}_0 + \vec{M}) \quad \text{[SI]}$$

$\vec{B}$ magnetic induction

$\vec{H}_0$ magnetic field intensity

$\vec{M}$ magnetic moment per unit volume



Magnetic properties can be derived from

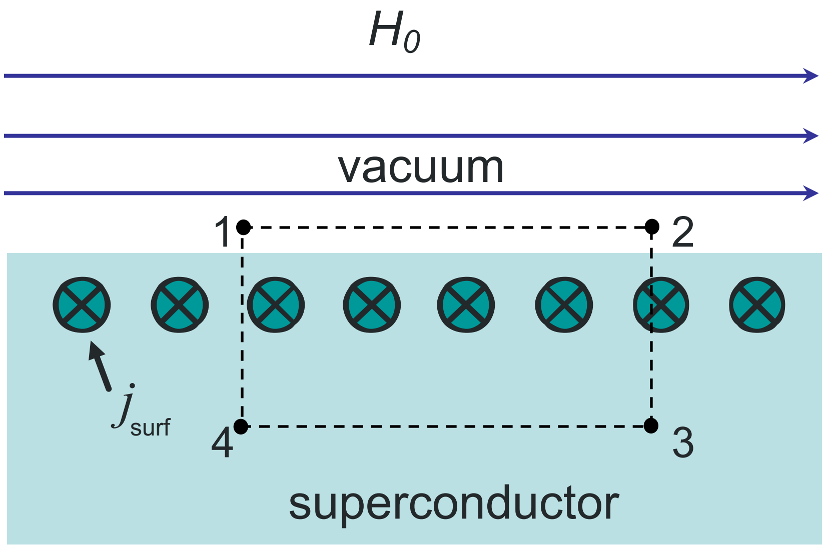
main equations for the superconducting state:

$$\begin{cases} \rho = 0 \\ \vec{B} = 0 \end{cases}$$

Type-I are all elements-superconductors except Nb

Magnetic field near the surface

Magnetic lines of force outside a superconductor are always tangential to its surface



$$\mathrm{div}\vec{B} = 0$$

since $\vec{B}_{\mathrm{n}}^{(\mathrm{i})} = 0 \Rightarrow \vec{B}_{\mathrm{n}}^{(\mathrm{e})} = 0$

A superconductor in an external magnetic field always carries an electric current near its surface

$\vec{\nabla}\times\vec{B} = (4\pi/c)\,\vec{j} \Rightarrow$ in the interior of the superconductor $\vec{j} = 0$

The circulation of vector $\vec{B}$ about contour 1-2-3-4 $\oint \vec{B}\,\mathrm{d}\vec{l} = H_0 l_{12}$

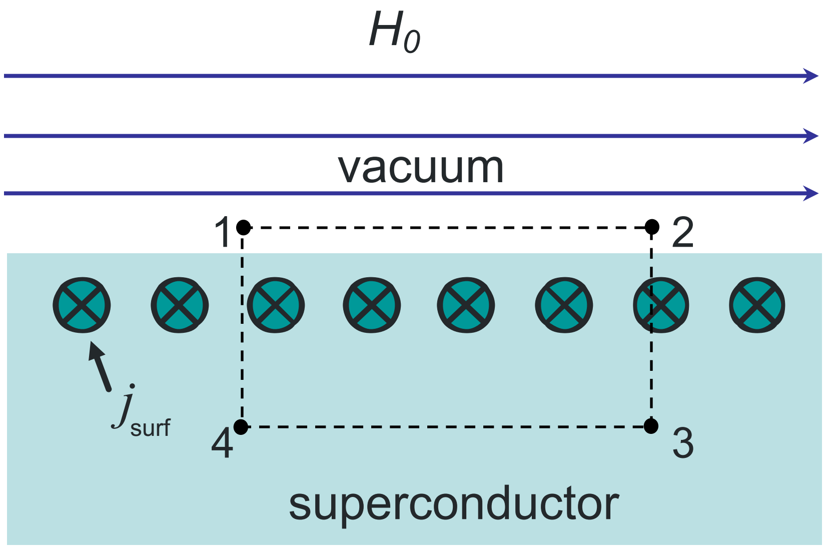
From the Maxwell equation $\oint \vec{B}\,\mathrm{d}\vec{l} = \frac{4\pi}{c} I$

Current at the surface of a superconductor

From above two equations

$$H_0 l_{12} = \frac{4\pi}{c} j_{\rm surf}\, l_{12}$$

$$\vec{j}_{\rm surf} = \frac{c}{4\pi} \vec{n} \times \vec{H}_0$$



Thus, the surface current $\vec{j}_{\rm surf}$ is completely defined by the magnetic field $\vec{H}_0$ at the surface of a superconductor.

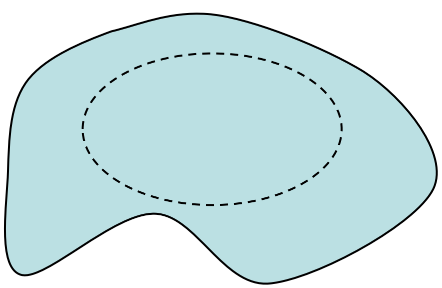
Type I superconductor at zero field

$$\vec{B} = 0 \quad \Rightarrow \quad \vec{j} = 0$$

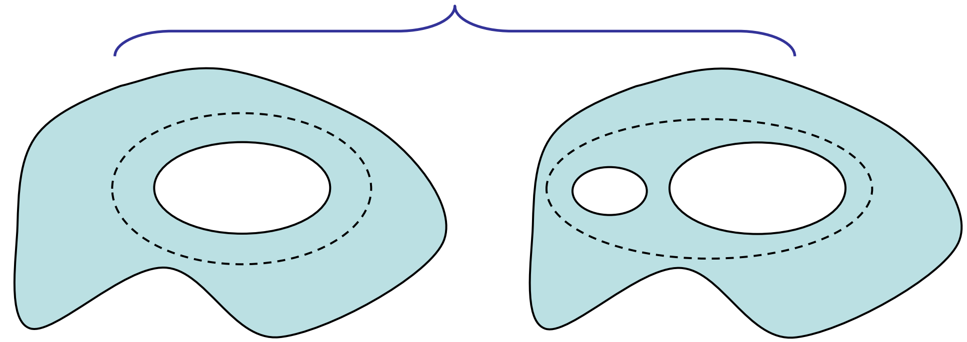
In a *single-connected* superconductor the surface currents vanish at zero magnetic field.

A single-connected body refers to a body inside which an arbitrary closed path can be reduced to a point without having to cross the boundaries of the body.

not true for these



single-connected



multiple-connected

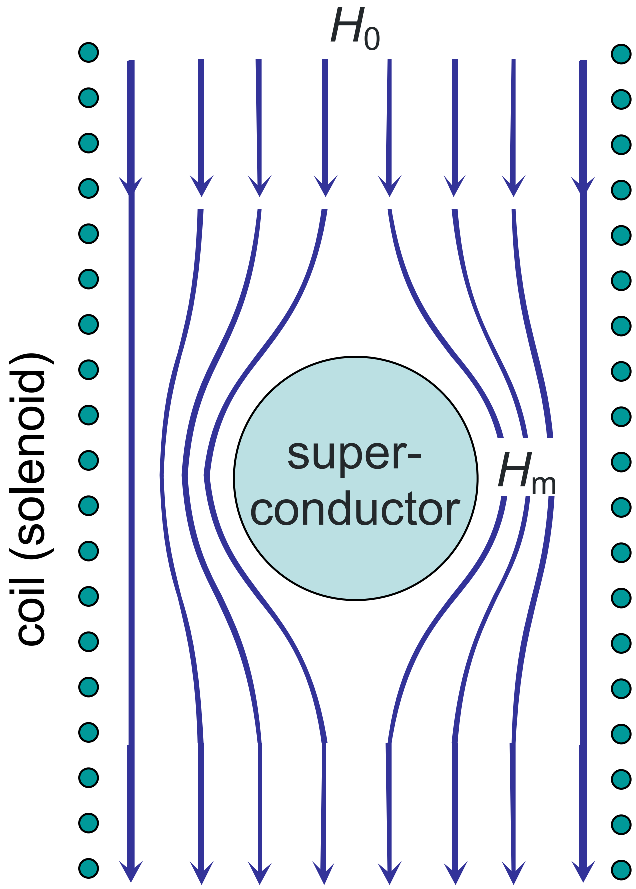
Dependence on sample shape

The superconductivity is destroyed when the field reaches the critical value H_{cm}.

The field lines will have a higher density at the 'equator' and there will be a local increase in magnetic field.

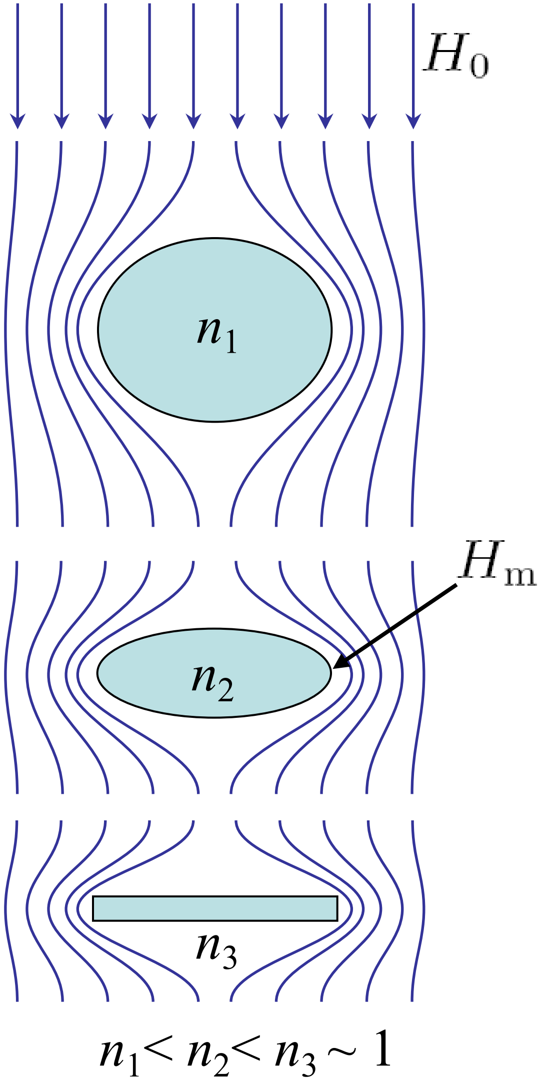
$H_m > H_0$

So what happens when the equatorial field H_m reaches the critical value H_{cm} ?



Superconducting sphere in a homogeneous field of a solenoid

Demagnetizing factor



H_m the maximum field at the surface

H_0 the external field far away from the body

$$H_\mathrm{m} = \frac{H_0}{1-n}$$

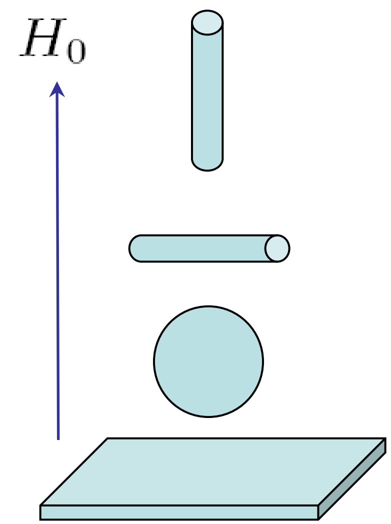
the factor n depends on geometry and is called demagnetizing factor

cylinder in parallel field $n = 0$

cylinder in transverse field $n = 1/2$

sphere $n = 1/3$

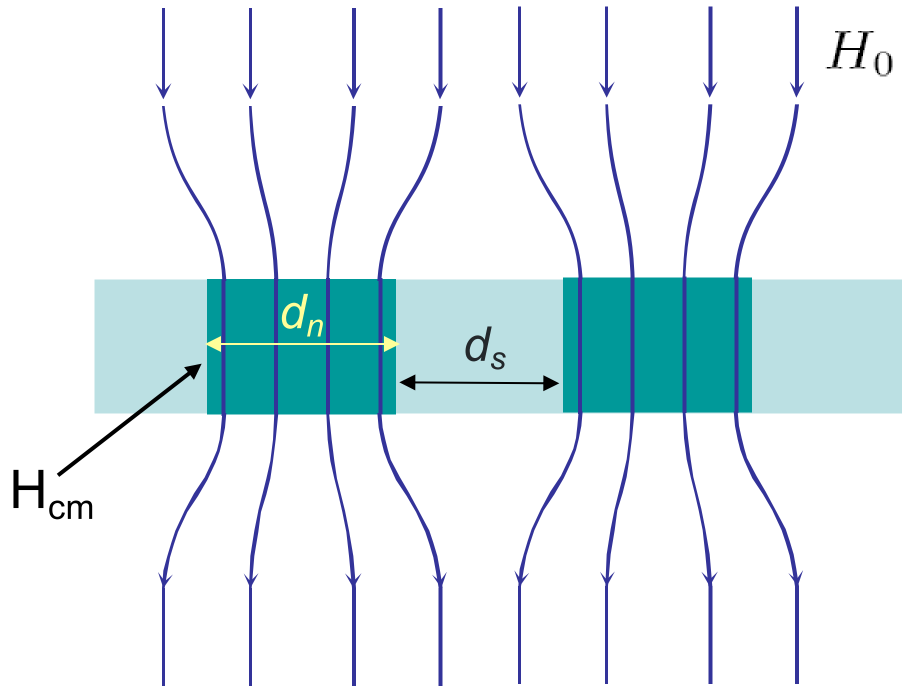
thin plate in perpendicular field $n = 1$

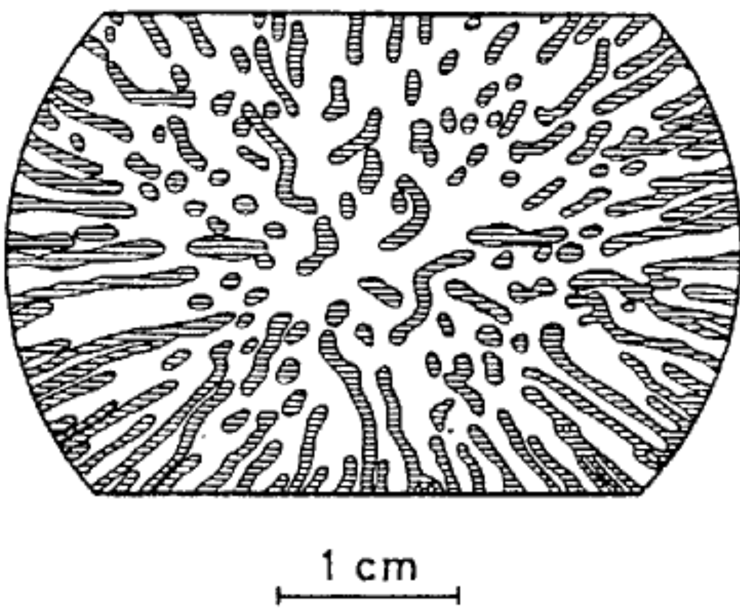


Intermediate state in type I superconductors

For a disk of the infinite radius, the transition to the *intermediate state* occurs in an infinitesimally small field H_0

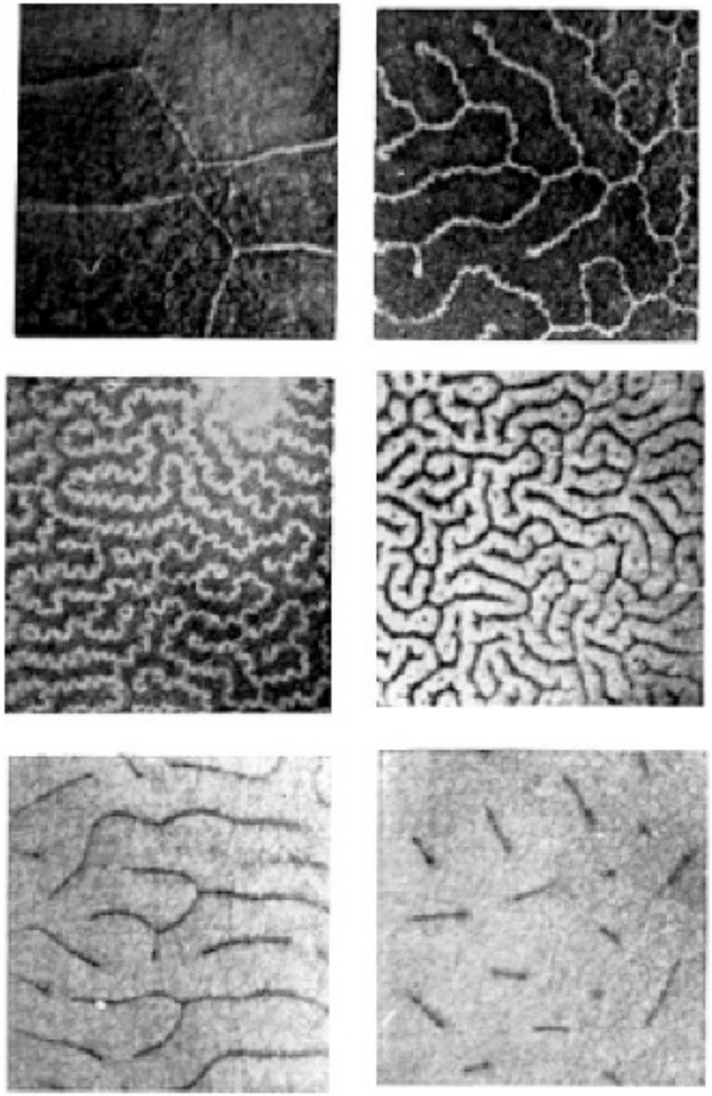
Intermediate state consists of regions of normal material carrying a magnetic field mixed with regions of superconducting material containing no field.





Distribution of the superconducting and normal regions in a tin sphere.
Shaded regions are superconducting

Intermediate state in type I superconductors



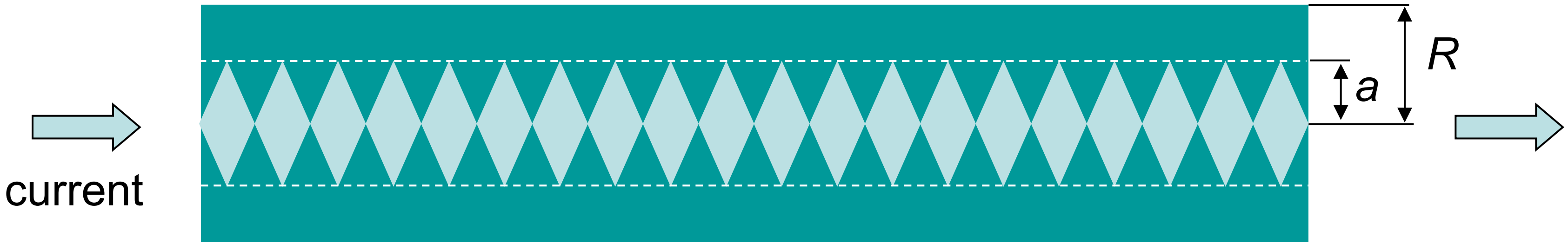
Meandering laminar structure with alternating normal and superconducting regions, typical for type I superconductors with the magnetic field applied normal to a flat slab.

Dark areas are superconducting regions.

T. E. Faber
Proc. Roy. Soc. **A248**, 460 (1958)

Intermediate state of a current-carrying wire

Intermediate state of a wire carrying an electric current, which value is larger than critical



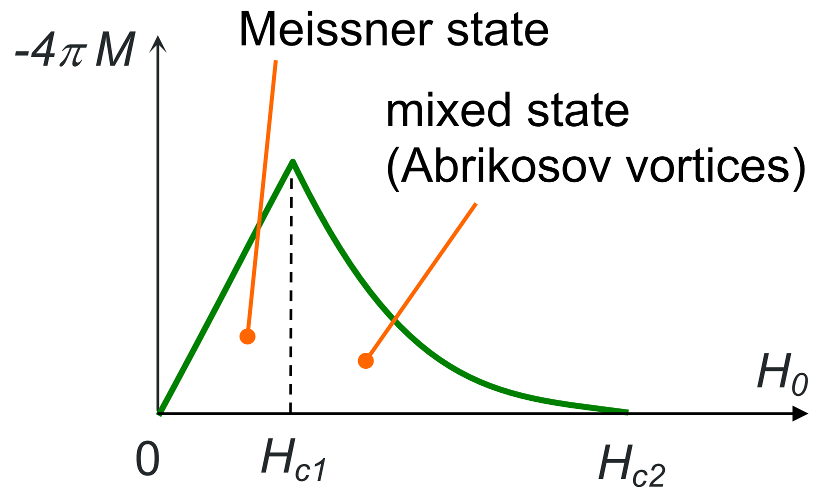
A sketch of the distribution of normal ( ) and superconducting ( ) regions in such a wire is shown for a current exceeding the critical value. A normal layer of thickness (*R* - *a*) forms at the surface of the wire, and its thickness grows in proportion to the excess current (over the critical value) .

Magnetic properties of type II superconductors

Magnetization curve $\vec{B} = \vec{H}_0 + 4\pi\vec{M}$

The first critical field H_{c1}

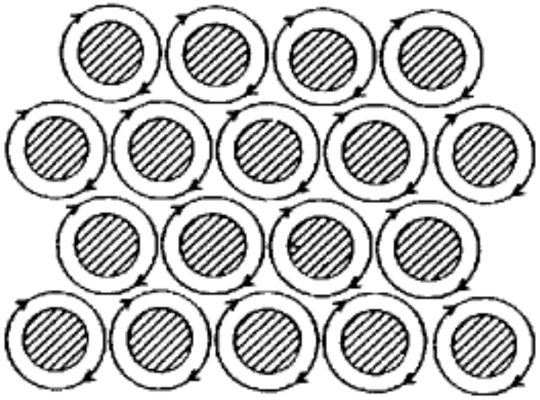
The second critical field H_{c2}



Even at $H_0 > H_{c2}$, in a thin surface layer the superconductivity remains up to $H_0 = 1.69\, H_{c2}$

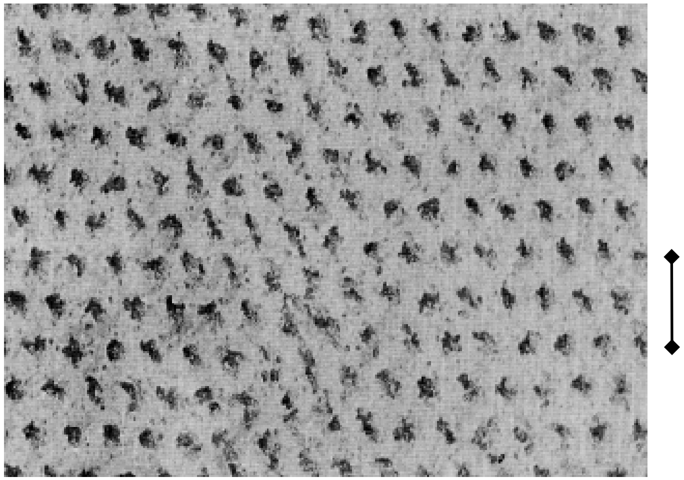
It is called the third critical field H_{c3}

Mixed state of a type II superconductor



Mixed state (Shubnikov phase) of a type II superconductor consists of a regular lattice of Abrikosov vortices.

Magnetic decoration image of a vortex lattice

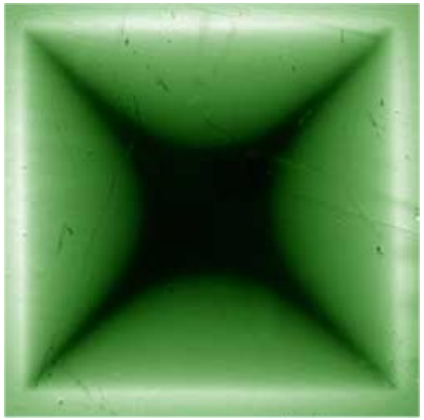


Field penetration in type II superconductors

<u>Meissner effect</u> with a small perpendicular field H_0 applied. The image shows that the field does not penetrate the superconductor (black corresponds to zero field). The magnetic field lines have to bend around the superconductor, and thus concentrate near the edges which are the brightest areas on the image.

$H_m < H_{cm}$

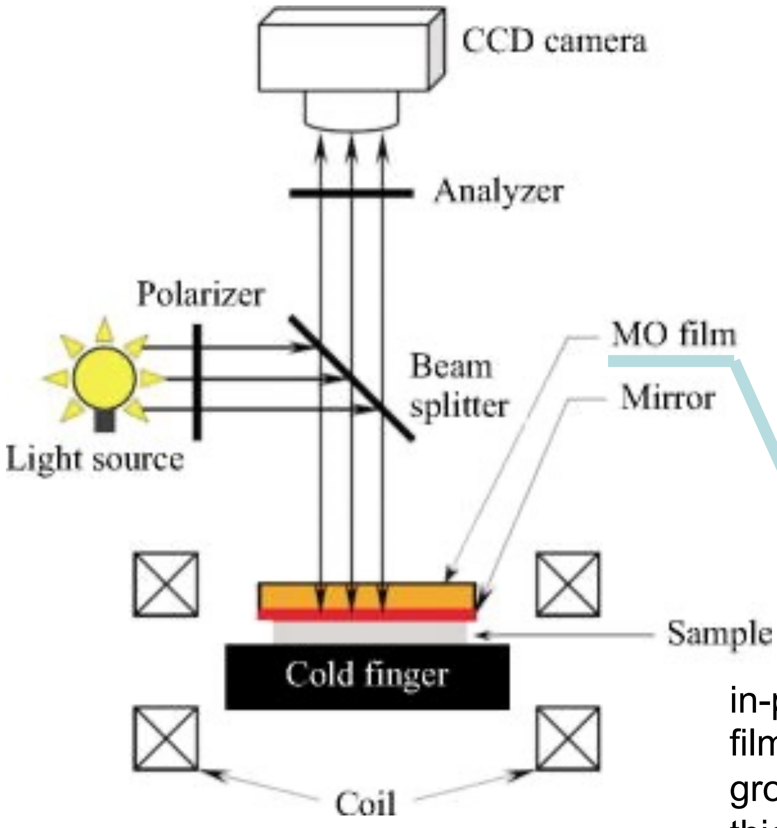
<u>Critical state:</u> The image shows flux penetration at $H_m > H_0$. Similar to the above state, the brightest areas are found at the edges where the expelled flux concentrates. At the same time, the flux already penetrated deep inside the superconductor from the sides of the square. Only the corners and the central part remain flux free (completely black).



$H_m > H_{cm}$

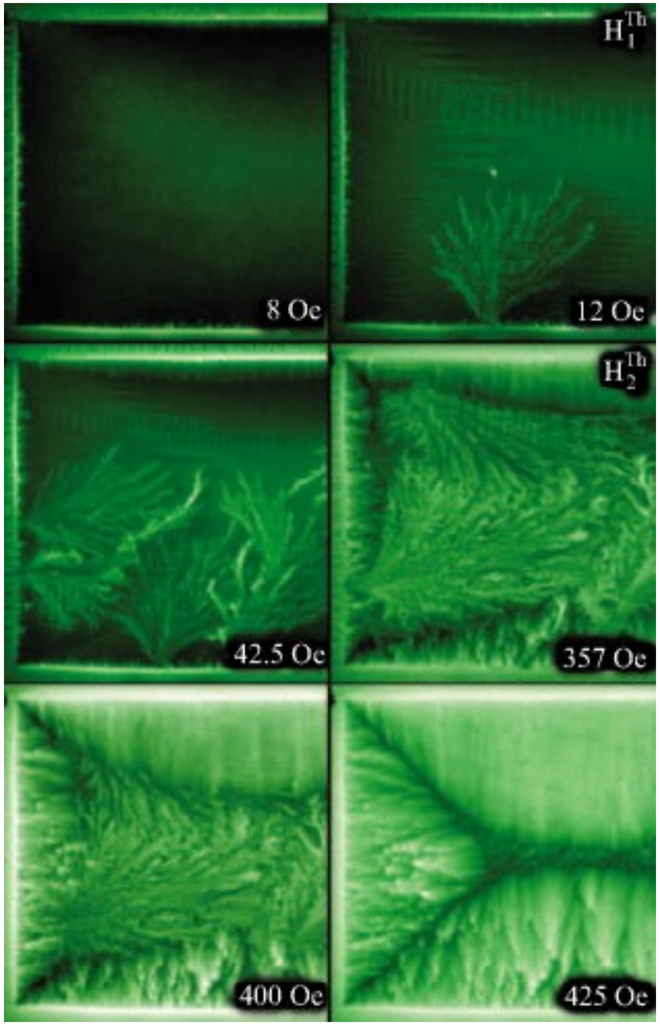
magneto-optical imaging of the field penetration in a superconductor

Magneto-optical imaging



Faraday rotation

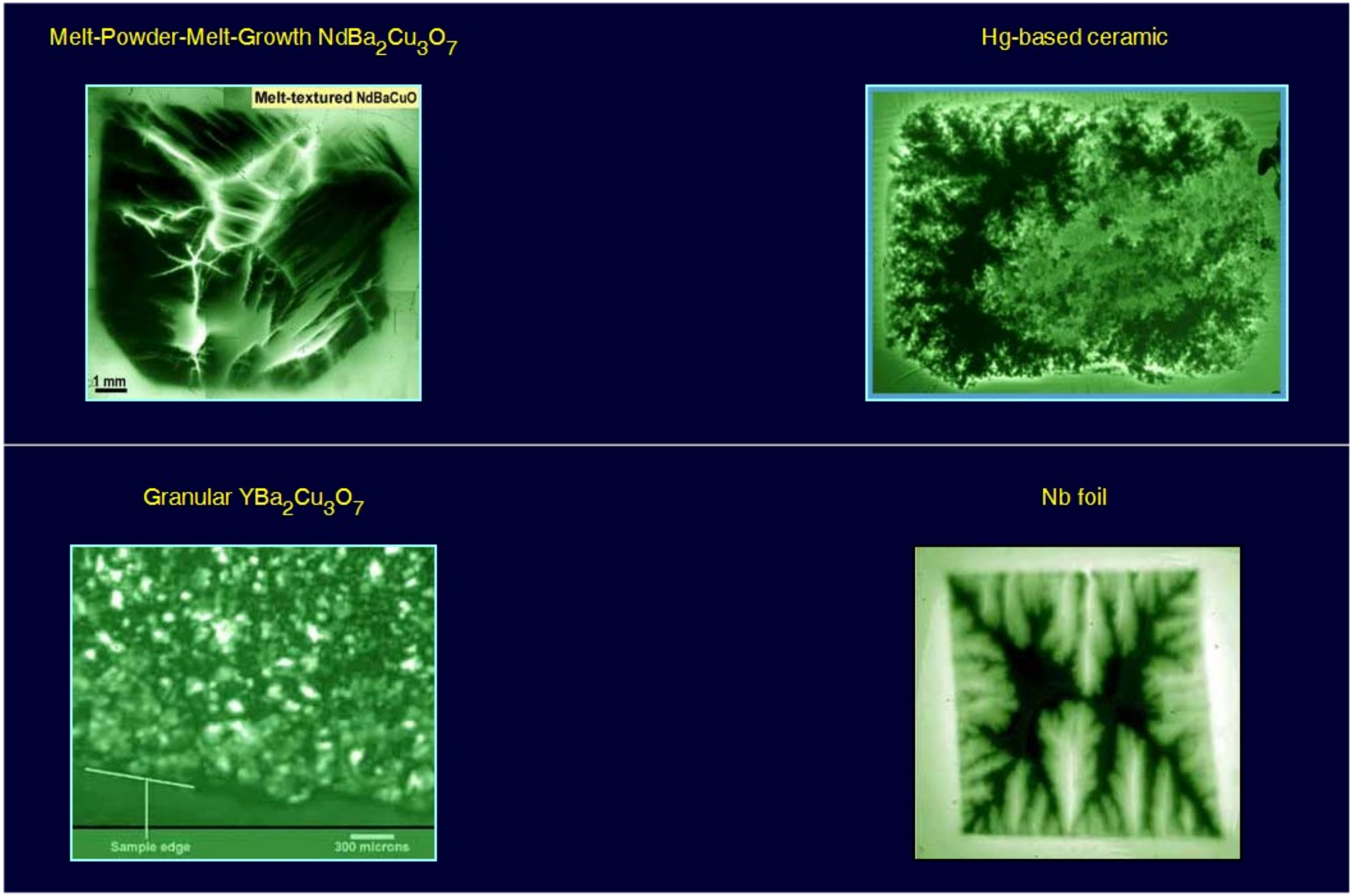
Magneto-optical images of a NbN film at 4K



in-plane magnetized garnet films $(Lu,Bi)_2(Fe,Ga)_5O_{12}$, grown as a few-microns-thick epitaxial layer on gadolinium gallium garnet (transparent) substrates

Low Temp. Phys. **35**, 619 (2009) : DOI:10.1063/1.3224713

More field penetration pictures



Dendritic flux avalanches in superconducting films: DOI:10.1063/1.3224713

Thermodynamics of superconductors

- Thermodynamic critical field
- Entropy
- Phase transitions
- Specific heat

Thermodynamic critical field

Let us find the value of the field H_{cm} which destroys the superconductivity (type I).

At $H_0 < H_{\mathrm{cm}}$ magnetic moment of the unit volume

$$\vec{M} = -\vec{H}_0/4\pi$$

$$\Rightarrow \quad -\vec{M}\,\mathrm{d}\vec{H}_0 = \vec{H}_0\,\mathrm{d}\vec{H}_0/4\pi$$

When the field changes from 0 to H_0

$$-\int_0^{H_0} \vec{M}\,\mathrm{d}\vec{H}_0 = {H_0}^2/8\pi$$

This work is stored in the free energy of the superconductor.

Free energy density

$$F_{\mathrm{s}H} = F_{\mathrm{s}0} + {H_0}^2/8\pi$$

At $H_0 = H_{\mathrm{cm}}$

$$\Rightarrow \quad F_{\mathrm{n}} - F_{\mathrm{s}0} = H_{\mathrm{cm}}^2/8\pi$$

The critical field H_{cm} measures the difference in free energy between the normal and superconducting states

Entropy of a Superconductor

The first law of thermodynamics

$$\delta Q = \delta R + \delta U$$

δU: internal energy

δR: work done by unit volume

δQ: thermal energy density

Free energy density $F = U - TS$

$$\Rightarrow \delta F = \delta U - T\delta S - S\delta T$$

for a reversible process $\delta Q = T\delta S$

$$\delta U = T\delta S - \delta R$$

$$\Rightarrow \delta F = -\delta R - S\delta T$$

assuming that work *R* is done by field *H*

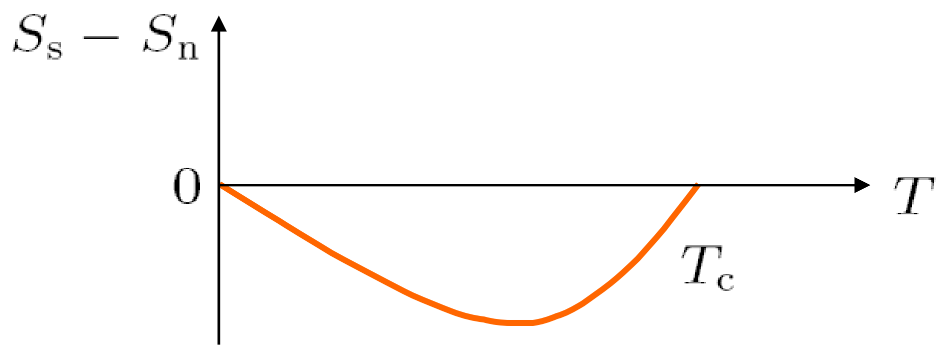
Thus $S = -(\partial F / \partial T)_H$

differentiating $F_{\mathrm{n}} - F_{\mathrm{s0}} = H_{\mathrm{cm}}^2 / 8\pi$

we get $S_{\mathrm{s}} - S_{\mathrm{n}} = \dfrac{H_{\mathrm{cm}}}{4\pi} \left(\dfrac{\partial H_{\mathrm{cm}}}{\partial T} \right)_H$

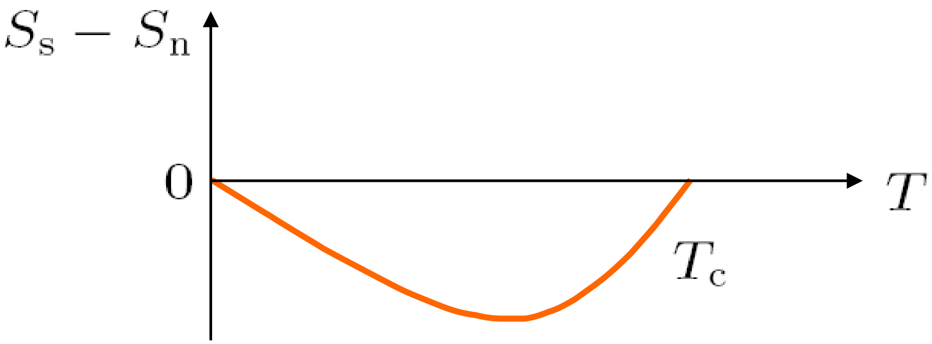
Nernst theorem: $(\partial H_{\mathrm{cm}} / \partial T)_{T=0} = 0$

At $0 < T < T_{\mathrm{c}}$ always $\partial H_{\mathrm{cm}} / \partial T < 0$



Entropy and phase transitions

The superconducting state is a more ordered state compared to the normal one because it is characterized by a lower entropy.



The transition at $T = T_c$ is a second-order phase transition

All the transitions at $T < T_c$ are first-order phase transitions.

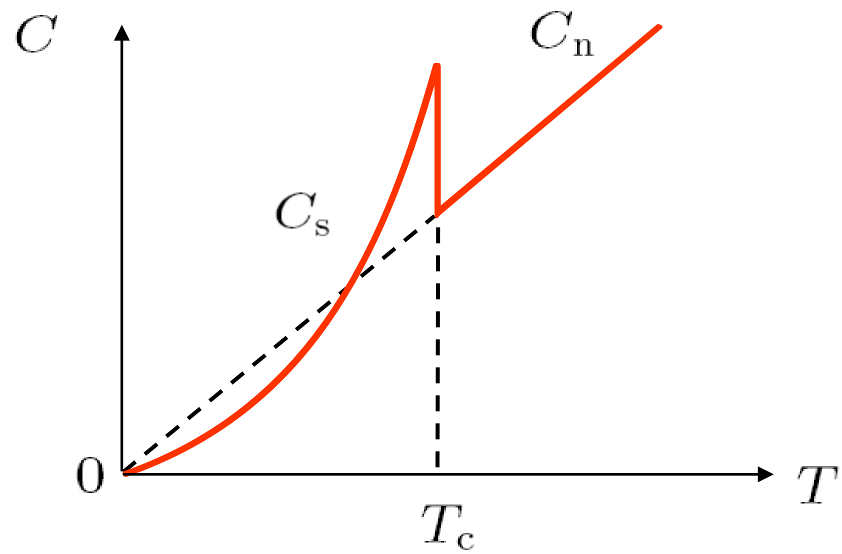
Specific heat

$$C = T\,(\partial S/\partial T)$$

From $S_\mathrm{s} - S_\mathrm{n} = \frac{H_\mathrm{cm}}{4\pi}\left(\frac{\partial H_\mathrm{cm}}{\partial T}\right)_H$

we get

$$C_\mathrm{s} - C_\mathrm{n} = \frac{T}{4\pi}\left[\left(\frac{\partial H_\mathrm{cm}}{\partial T}\right)^2 + H_\mathrm{cm}\frac{\partial^2 H_\mathrm{cm}}{\partial T^2}\right]$$



Since $H_\mathrm{cm} = 0$ at $T = T_\mathrm{c}$, we have $C_\mathrm{s} - C_\mathrm{n} = \frac{T_\mathrm{c}}{4\pi}\left(\frac{\partial H_\mathrm{cm}}{\partial T}\right)^2_{T_\mathrm{c}}$

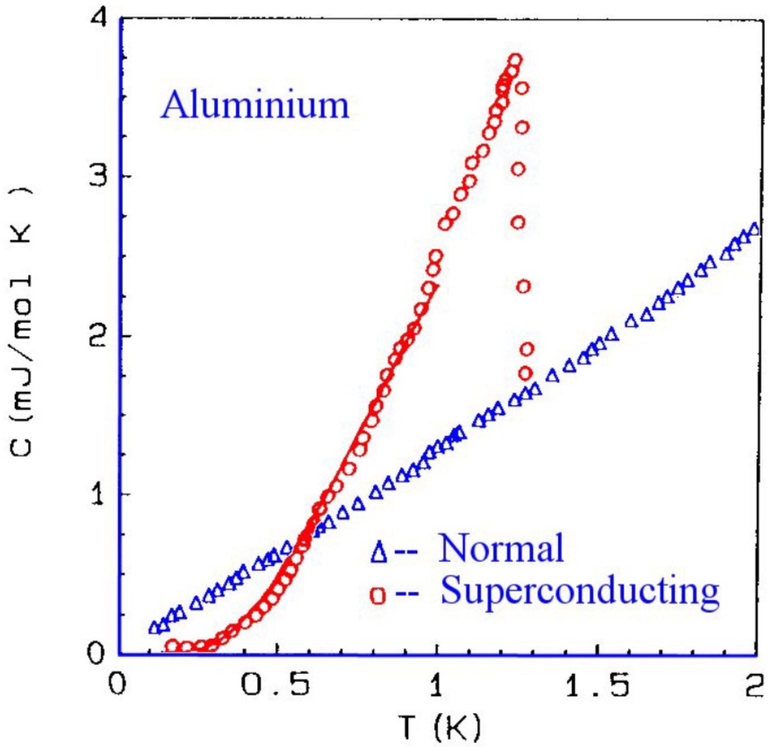
Specific heat

$$C = T\,(\partial S/\partial T)$$

From $S_{\rm s} - S_{\rm n} = \dfrac{H_{\rm cm}}{4\pi}\left(\dfrac{\partial H_{\rm cm}}{\partial T}\right)_R$

we get

$$C_{\rm s} - C_{\rm n} = \frac{T}{4\pi}\left[\left(\frac{\partial H_{\rm cm}}{\partial T}\right)^2 + H_{\rm cm}\frac{\partial^2 H_{\rm cm}}{\partial T^2}\right]$$



Since $H_{\rm cm} = 0$ at $T = T_{\rm c}$, we have $C_{\rm s} - C_{\rm n} = \dfrac{T_{\rm c}}{4\pi}\left(\dfrac{\partial H_{\rm cm}}{\partial T}\right)^2_{T_{\rm c}}$