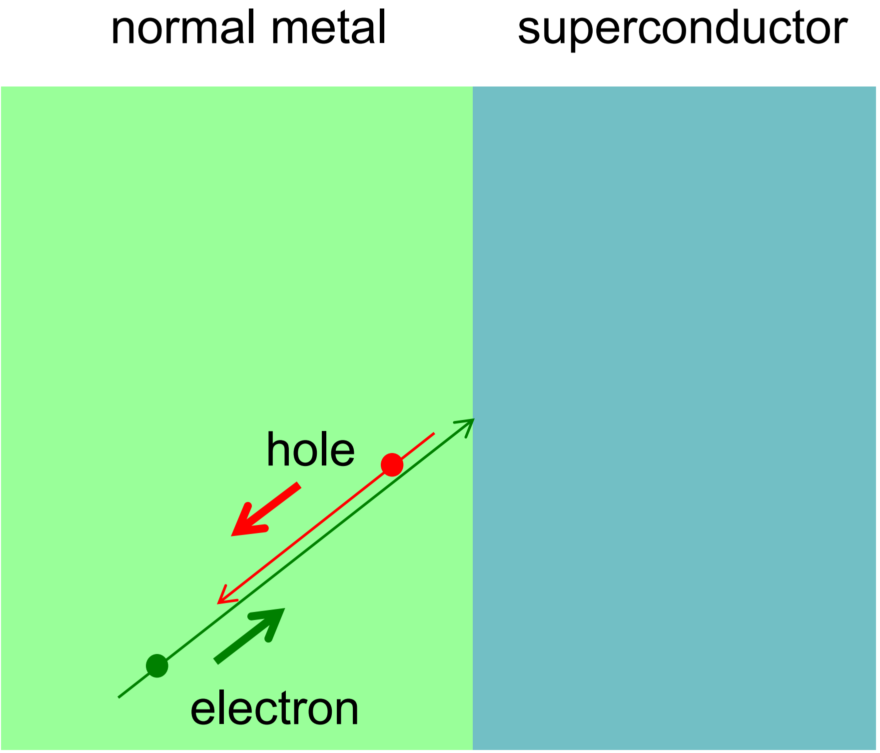
Superconductivity
Lecture 7



Elementary excitation (quasiparticle)

Ground-state energy is $W = E_{\mathrm{s}} - E_{\mathrm{n}} = -\frac{1}{2} N(0)\, \Delta_0^2 \; .$

The energy contribution of a single Cooper pair $(\vec{q},\, -\vec{q})$

$$w_{\vec{q}} = \underbrace{2\varepsilon_{\vec{q}} v_{\vec{q}}^2}_{\text{kinetic energy}} - \underbrace{2V\, v_{\vec{q}} u_{\vec{q}} \sum_{\vec{k}}{}' v_{\vec{k}} u_{\vec{k}}}_{\text{interaction energy}} =$$

$$2\varepsilon_{\vec{q}} \frac{1}{2} \left(1 - \frac{\varepsilon_{\vec{q}}}{E_{\vec{q}}}\right) - 2\left[\frac{1}{4}\left(1 - \frac{\varepsilon_{\vec{q}}^2}{E_{\vec{q}}^2}\right)\right]^{1/2} \Delta_0 \; = \; \varepsilon_{\vec{q}} - \frac{\varepsilon_{\vec{q}}^2}{E_{\vec{q}}} - \frac{\Delta_0^2}{E_{\vec{q}}} = \varepsilon_{\vec{q}} - E_{\vec{q}} \; .$$

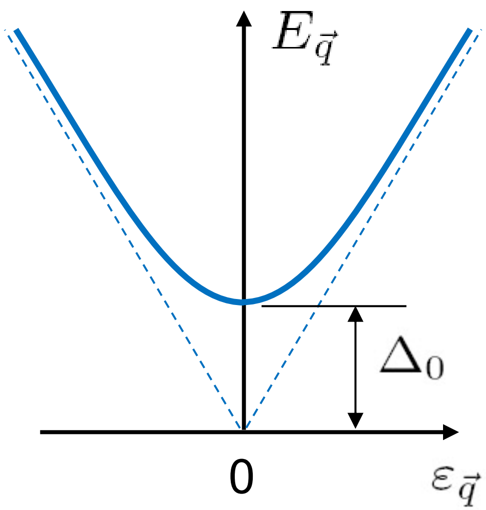
The energy of the superconductor with one 'extra' electron in the state $\vec{q}$ ⇨ $W_{\vec{q}} = W - w_{\vec{q}} + \varepsilon_{\vec{q}} \; .$

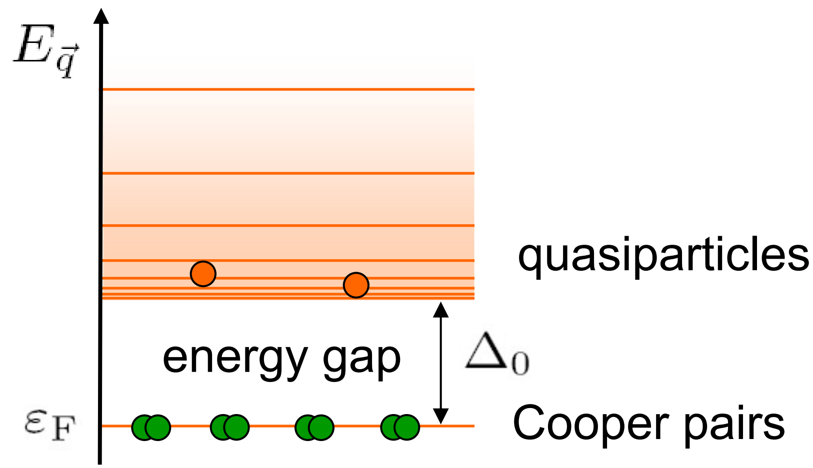
We shall refer to such uncoupled electron as an **elementary excitation** of the superconductor, and shall call it **quasiparticle**.

Energy gap

The energy of an elementary excitation: $W_{\vec{q}} = W - w_{\vec{q}} + \varepsilon_{\vec{q}} = W + E_{\vec{q}}$

where $E_{\vec{q}} = \sqrt{\varepsilon_{\vec{q}}^2 + \Delta_0^2}$





$$\varepsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m} - \frac{\hbar^2 k_F^2}{2m}$$

The „price" of breaking one Cooper pair is $2\Delta_0$.

Quasiparticle density of states

$$E_{\vec{k}} = \sqrt{\varepsilon_{\vec{k}}^2 + \Delta_0^2} = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \frac{\hbar^2 k_F^2}{2m}\right)^2 + \Delta_0^2}\,.$$

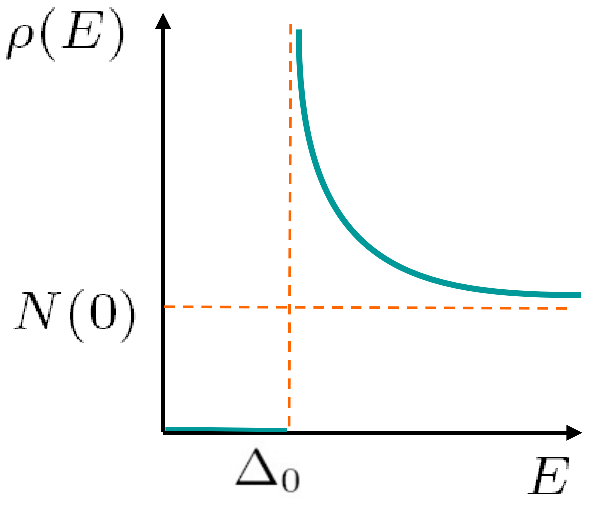
The density of states (the number of energy levels per unit energy and unit volume):

$$\rho(E) = \mathrm{d}\nu/\mathrm{d}E$$

where $\mathrm{d}\nu$ is the number of energy levels in the interval $\mathrm{d}E$ in the vicinity of E .

$$\rho(E) = \frac{\mathrm{d}\nu}{\mathrm{d}\varepsilon}\frac{\mathrm{d}\varepsilon}{\mathrm{d}E} = N(0)\,\frac{E}{\sqrt{E^2 - \Delta_0^2}}$$

At $E \rightarrow \Delta_0$ we have $\rho(E) \rightarrow \infty$.



Non-zero temperatures

$$f_{\vec{k}} = \frac{1}{\exp\left(E_{\vec{k}}/k_{\mathrm{B}}T\right) + 1}$$

⇒ the probability that the pair $(\vec{k},\ -\vec{k})$ state can participate in the scattering processes is $1 - 2f_{\vec{k}}$.

$$W = \sum_{\vec{k}} 2\,|\varepsilon_{\vec{k}}|\, f_{\vec{k}} \quad + \quad 2\sum_{\vec{k}} \varepsilon_{\vec{k}}(1 - 2f_{\vec{k}})\, v_{\vec{k}}^2 - $$

$$- \quad V \sum_{\vec{k}\,k'}{}' v_{\vec{k}} u_{\vec{k}} v'_{\vec{k}} u'_{\vec{k}}\, (1 - 2f_{\vec{k}})\,(1 - 2f_{\vec{k}'})$$

Free energy $F = W - TS$, in equilibrium $\dfrac{\partial F}{\partial (v_{\vec{q}}^2)} = 0$.

⇒ $$\frac{v_{\vec{q}} u_{\vec{q}}}{1 - 2v_{\vec{q}}^2} = \frac{\Delta}{2\varepsilon_{\vec{q}}}\,,$$ where $$\Delta = V \sum_{\vec{k}}{}' v_{\vec{k}} u_{\vec{k}}\,(1 - 2f_{\vec{k}})\,.$$

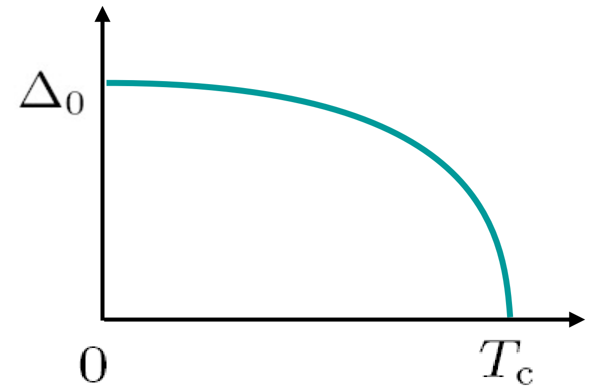
Temperature dependence of energy gap

Since $v_{\vec{q}}^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_{\vec{q}}}{E_{\vec{q}}}\right)$, with $E_{\vec{q}} = \sqrt{\varepsilon_{\vec{q}}^2 + \Delta^2(T)}$

$$\Rightarrow \quad \Delta = V \sum_{\vec{k}}{}' \frac{\Delta}{2E_{\vec{k}}} \left(1 - \frac{2}{\exp\left(E_{\vec{k}}/k_{\mathrm{B}}T\right) + 1}\right)$$

from sum to integration: $\frac{1}{N(0)\,V} = \int_0^{\hbar\omega_{\mathrm{D}}} \frac{\mathrm{d}\varepsilon}{\sqrt{\varepsilon^2 + \Delta^2(T)}} \tanh \frac{\sqrt{\varepsilon^2 + \Delta^2(T)}}{2k_{\mathrm{B}}T}$.

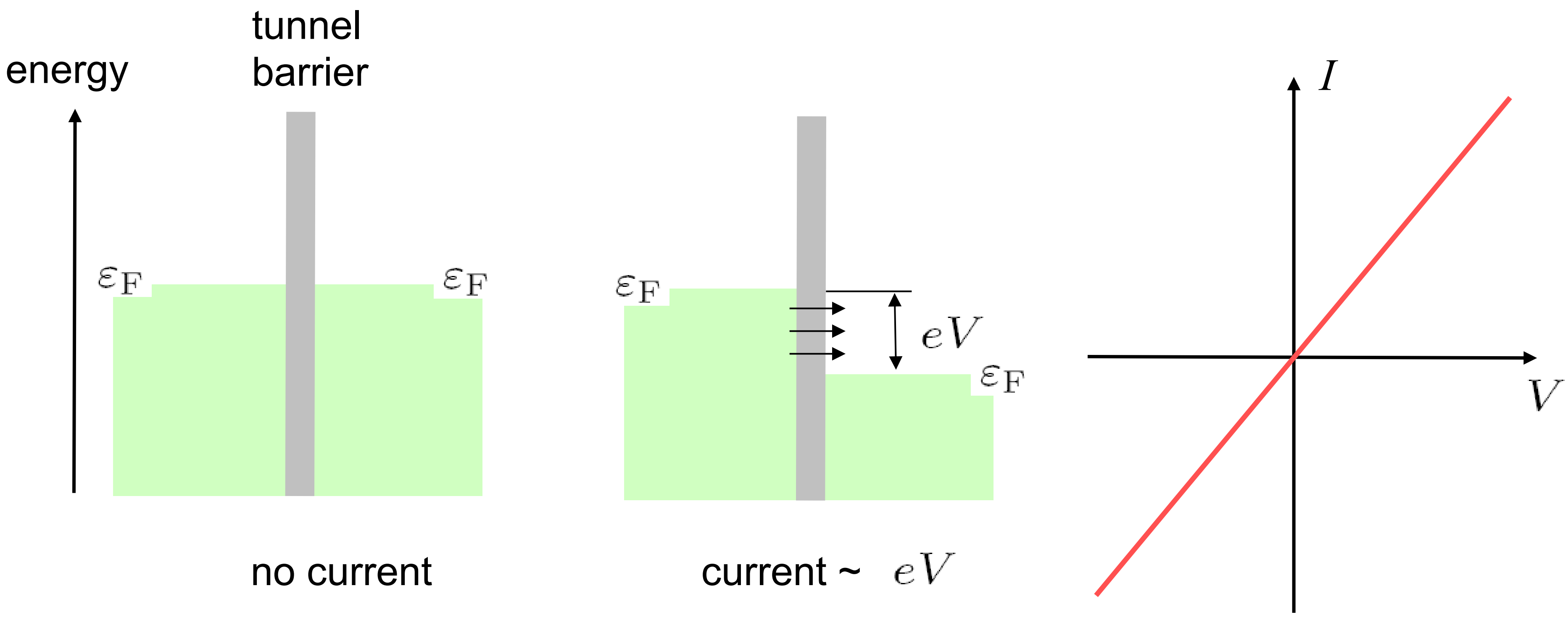
At $T = T_{\mathrm{c}}$, $\frac{1}{N(0)\,V} = \int_0^{\hbar\omega_{\mathrm{D}}} \frac{\mathrm{d}\varepsilon}{\varepsilon} \tanh \frac{\varepsilon}{2k_{\mathrm{B}}T_{\mathrm{c}}}$.



$$\Rightarrow \quad k_{\mathrm{B}}T_{\mathrm{c}} = 1.14\,\hbar\omega_{\mathrm{D}} \exp\left(-\frac{1}{N(0)\,V}\right)$$

Using $\Delta_0 = 2\,\hbar\omega_{\mathrm{D}} \exp\left(\frac{-1}{N(0)\,V}\right)$ we get $2\Delta_0 = 3.52\,k_{\mathrm{B}}T_{\mathrm{c}}$

Tunneling between two metals: NIN junction

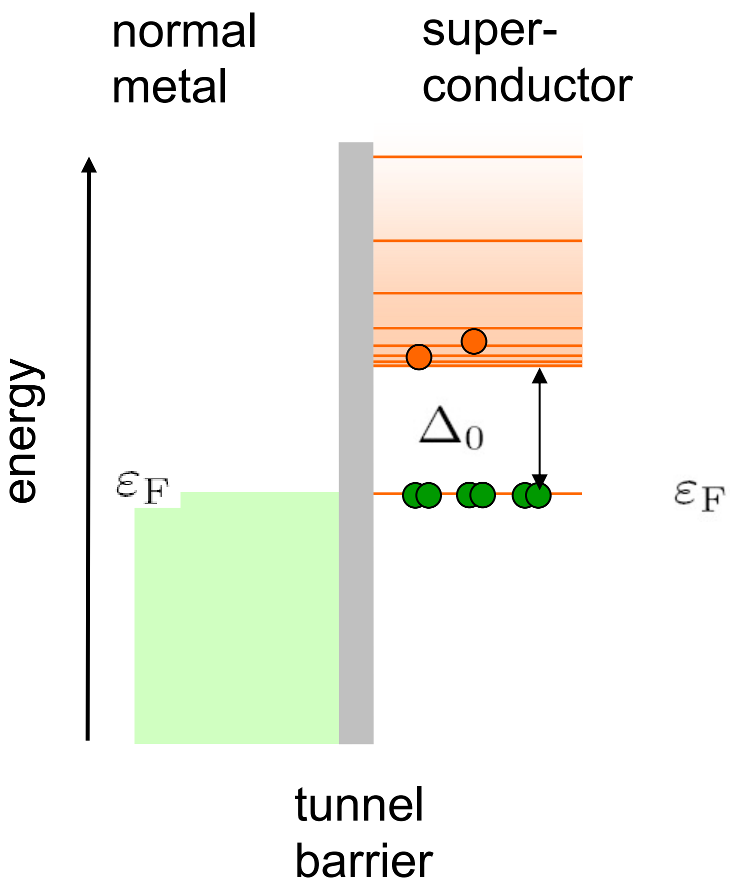


Tunneling of quasiparticles

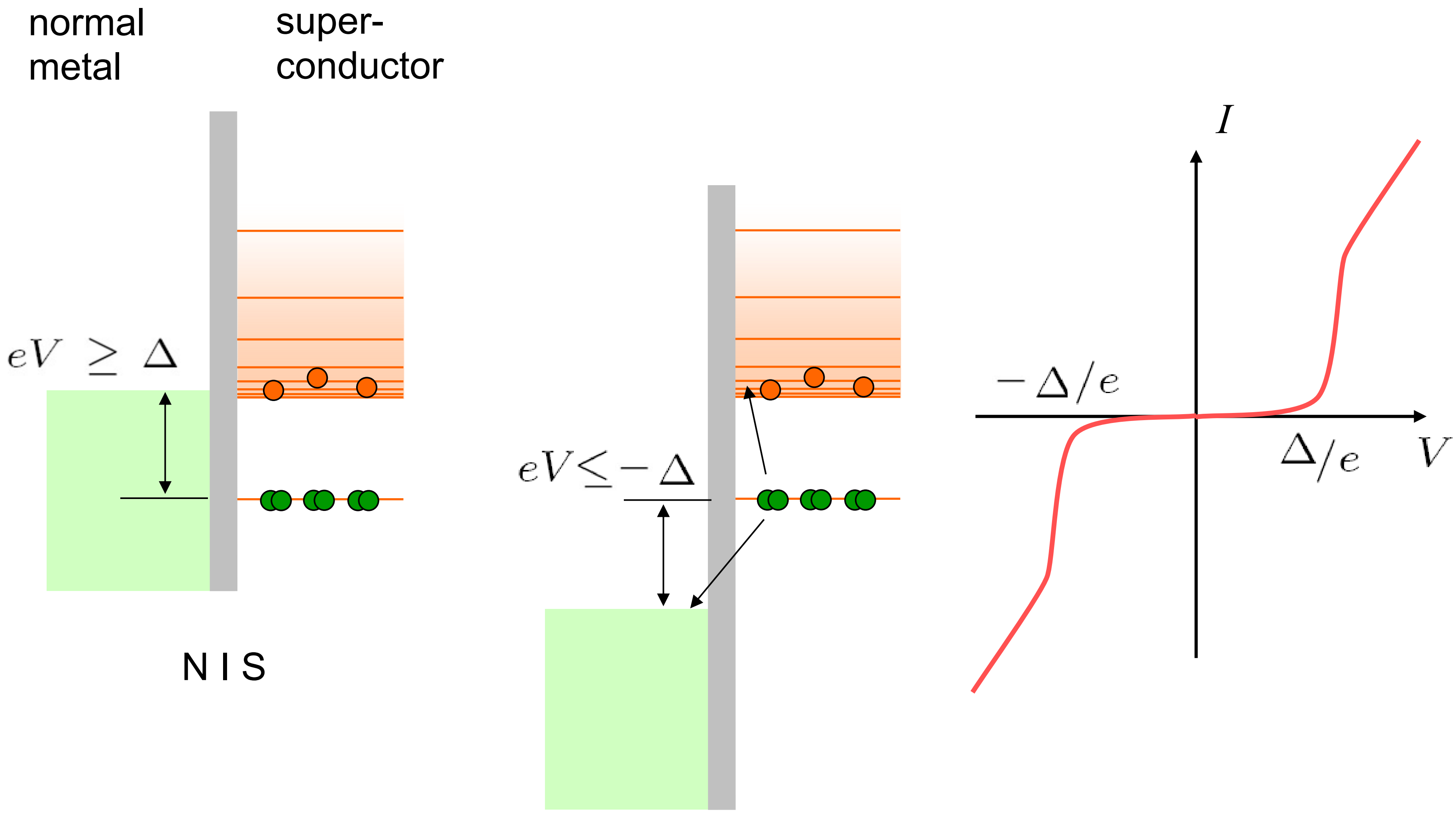
Giaever's experiments in 1960

Nobel Prize 1973

$$V = 0$$

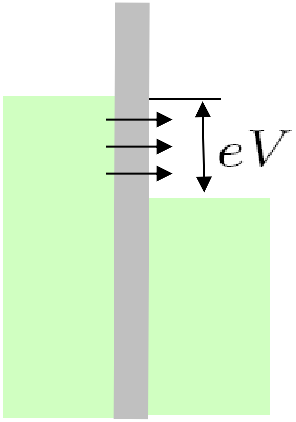


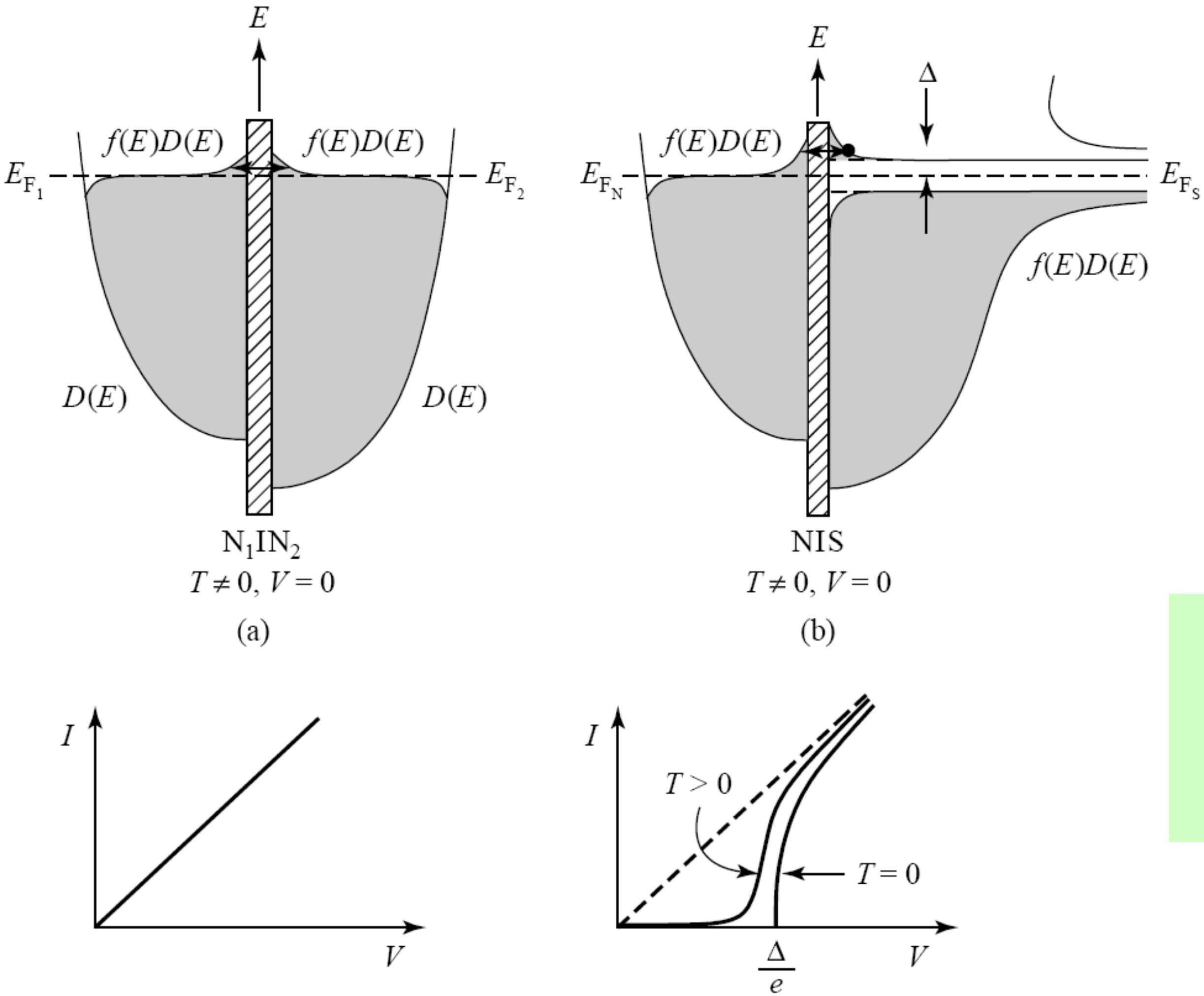
NIS tunnel junction



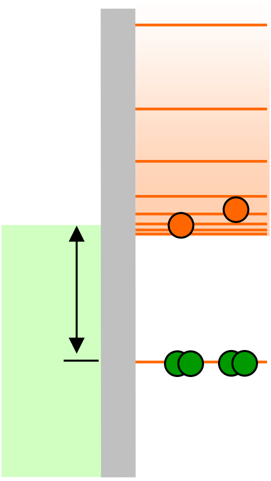
"Semiconductor-like" model for NIN and NIS junctions

N I N





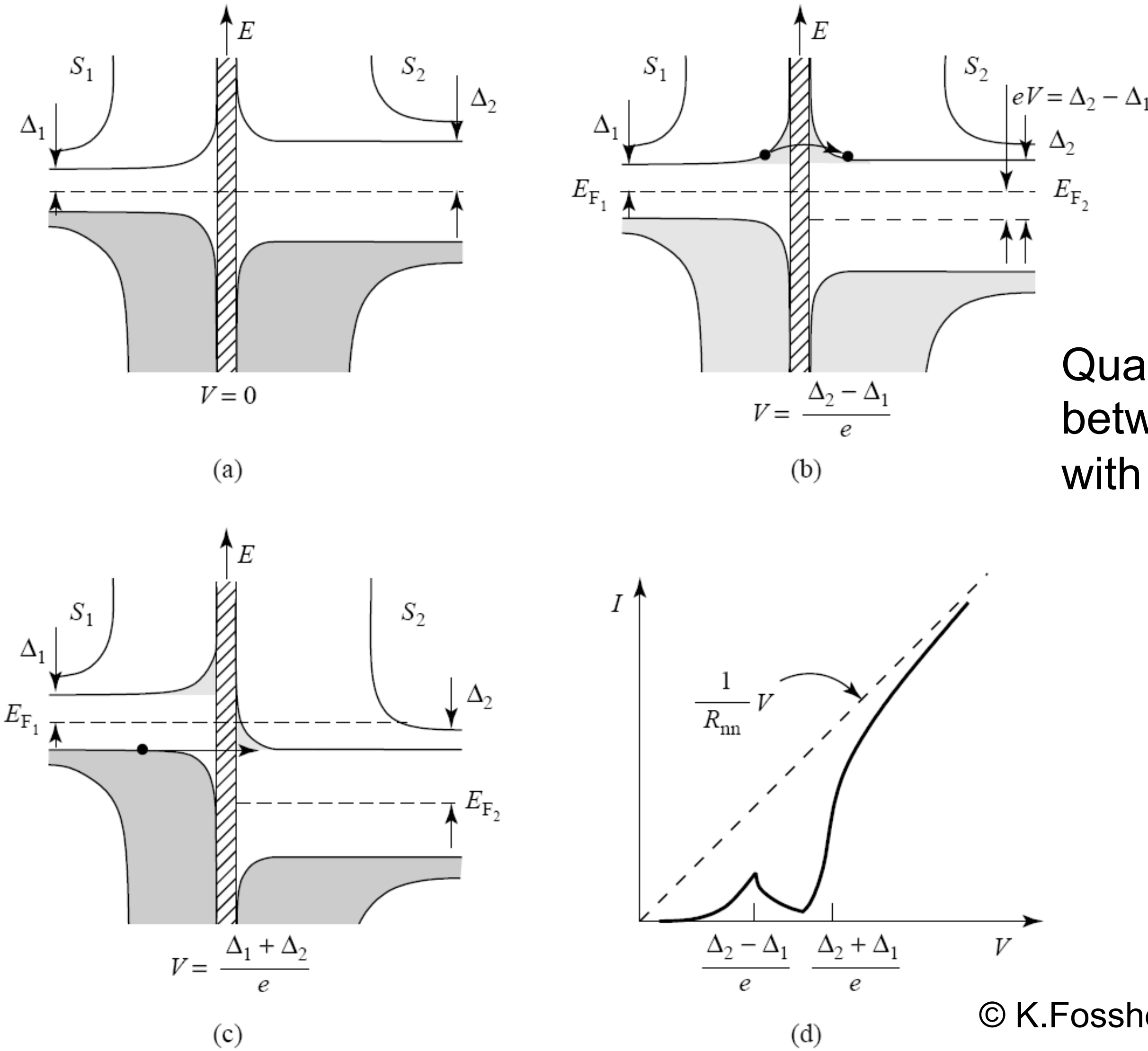
N I S



$eV \geq \Delta$

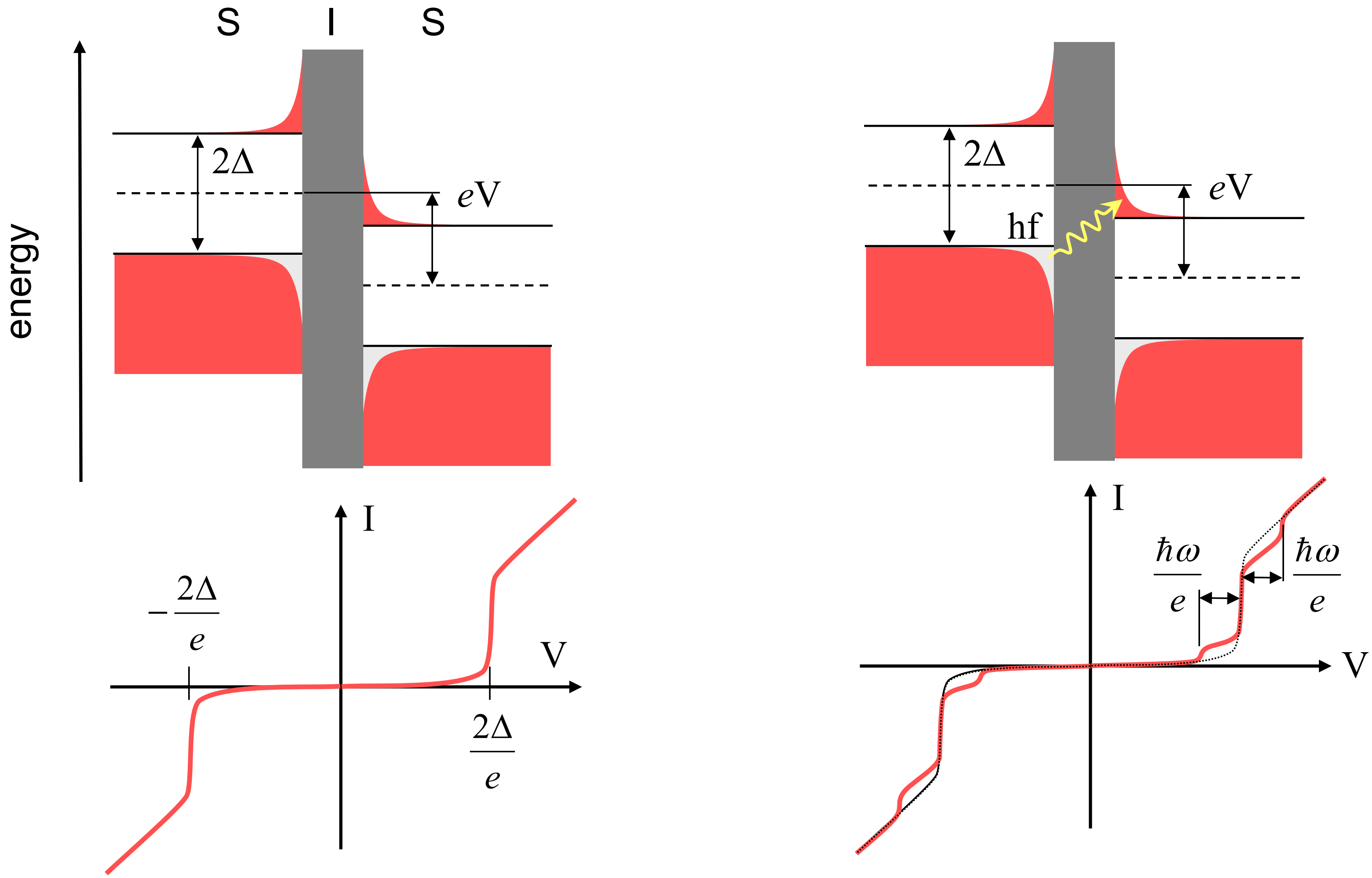
© K.Fossheim and A.Sudbø, "Superconductivity"

SIS junction

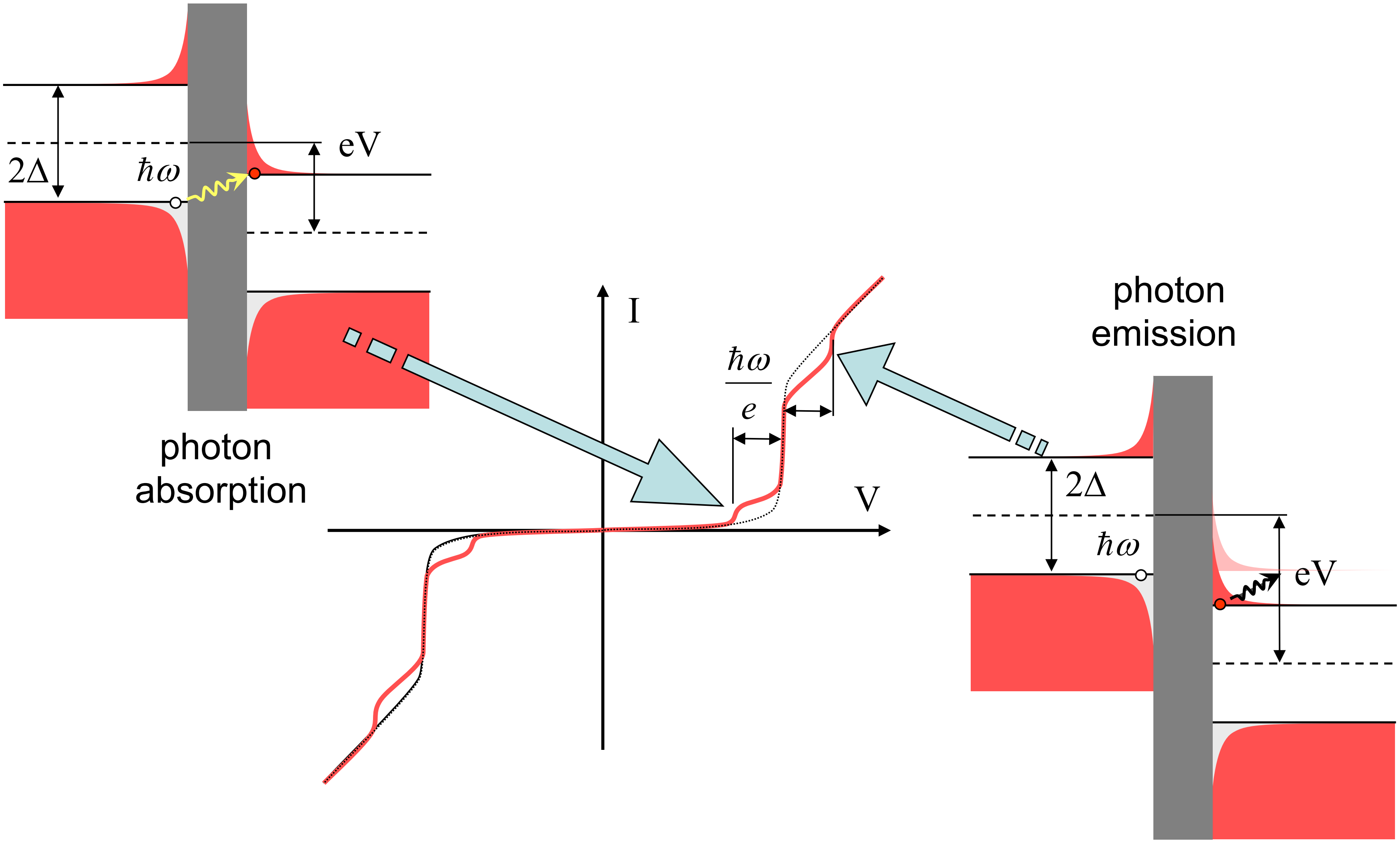


Quasiparticle tunneling between two superconductors with different gaps

Photon-assisted tunneling in SIS junction



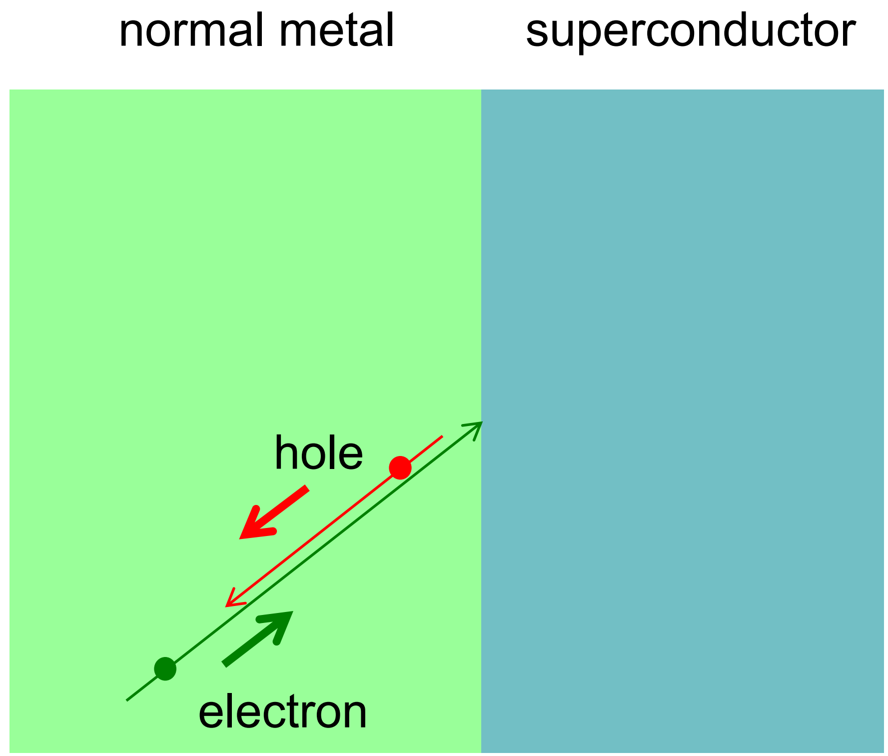
Absorption and emission of photons during tunneling



Andreev reflection

Alexander F. Andreev

"Thermal conductivity of the intermediate state of superconductors"
Sov. Phys. JETP **19**, 1228 (1964)



Andreev reflection

$$E_{\vec{q}} = \sqrt{\varepsilon_{\vec{q}}^2 + \Delta^2(T)}$$