Electronics for Physicists

Analog Electronics

Chapter 1; Lecture 03

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Karlsruhe Institute of Technology

KIT, Winter 2023/24

Institute for
Data Processing
and Electronics

Chapter 1

Basics

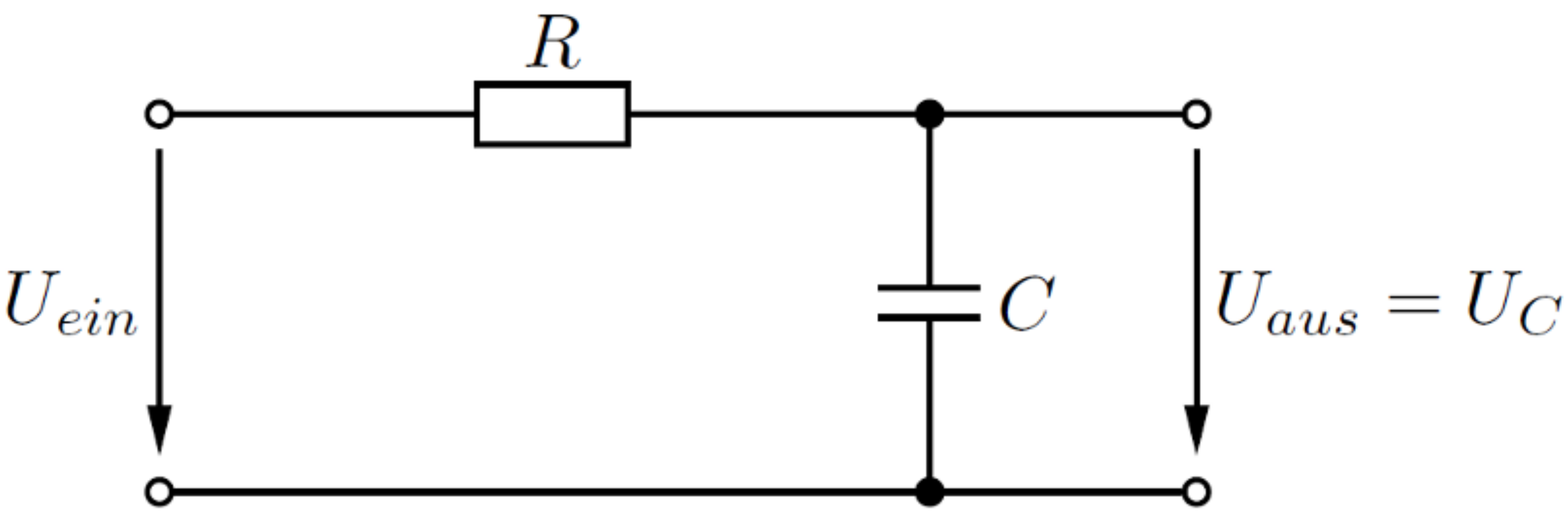
Part 2

- Simple RC Filters
- Inductors
- RC and RL Circuits
- Linear Networks

Low Pass / Integrator

Tiefpass / Integrator

- Behavior for time-dependent voltages



$$\frac{dU_C}{dt} + \cancel{\frac{U_C}{RC}} = \frac{U_{ein}}{RC}$$

First approximation: U_C is small

$$U_C = U_{aus} = \frac{1}{RC}\int U_{ein}\,dt$$

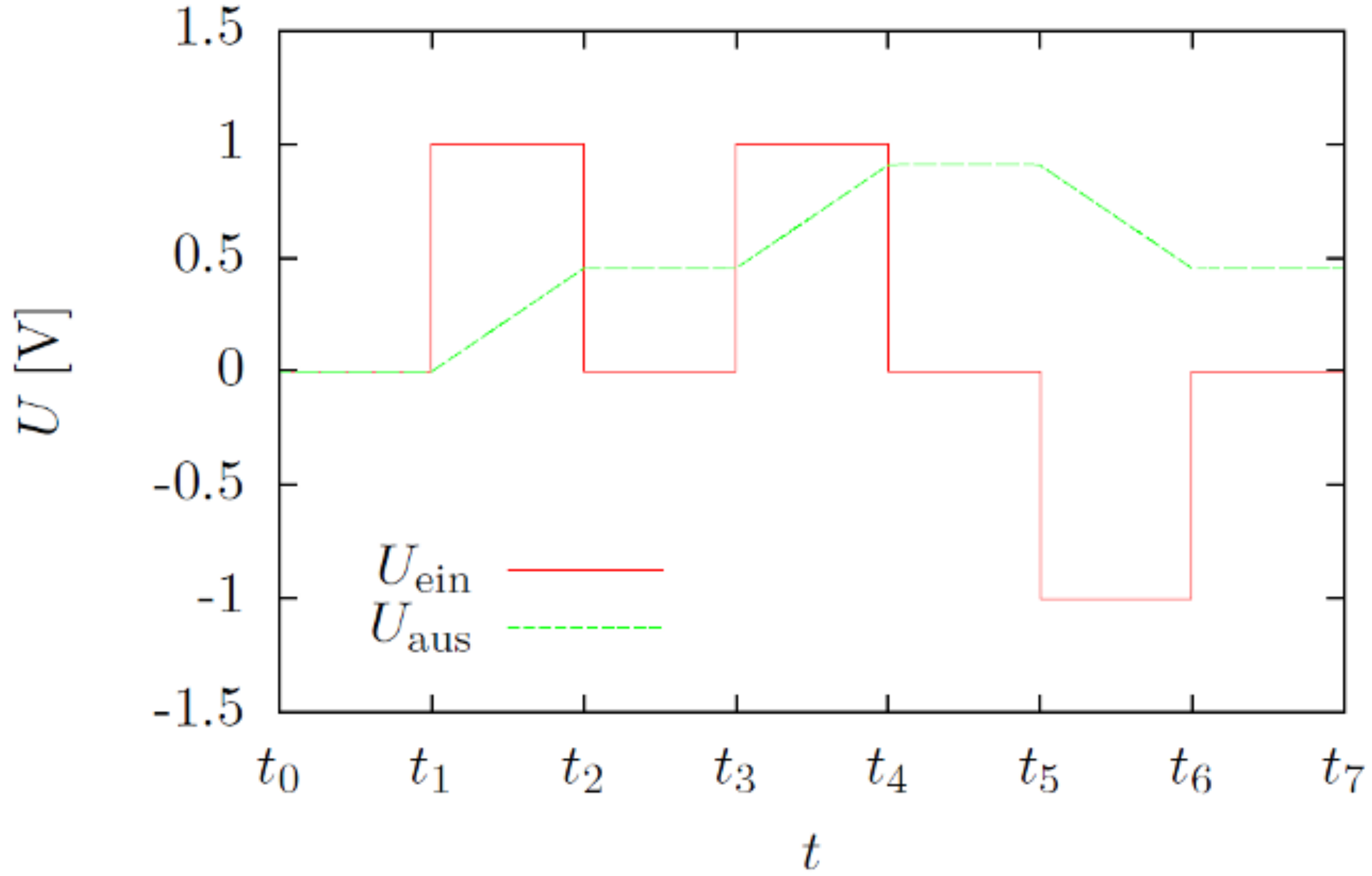
Integrator circuit!

Kirchhoff's Laws:

$$I_C = I_R \qquad \text{KCL}$$

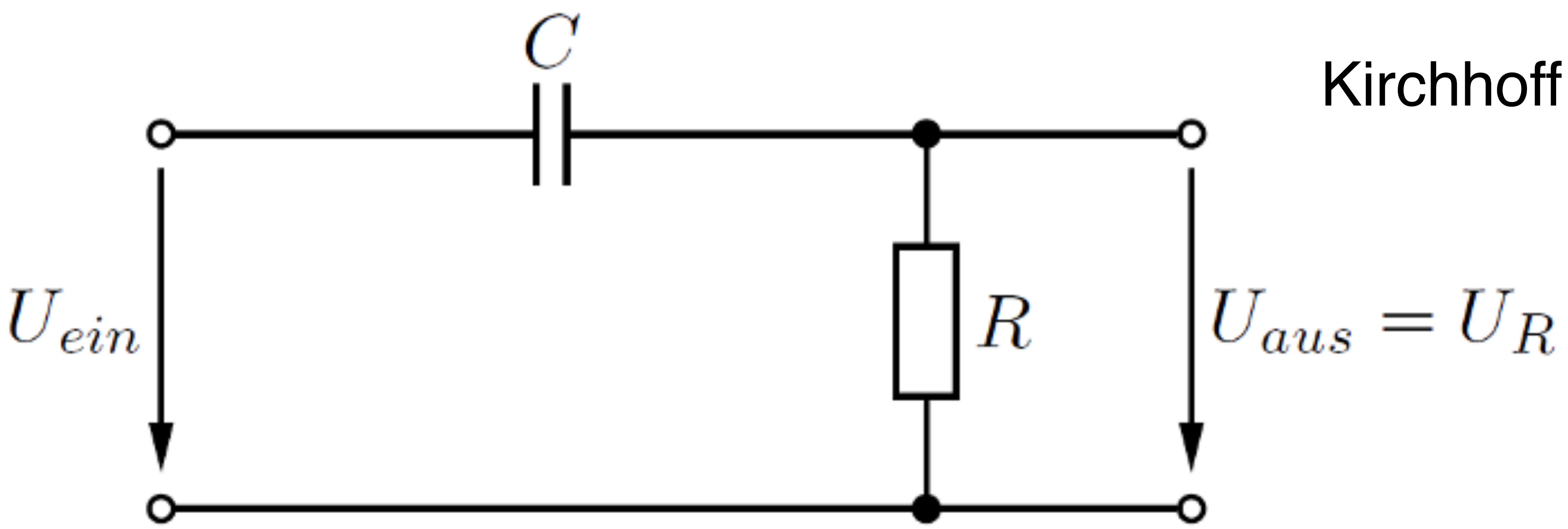
$$U_{ein} - U_C - U_R = 0 \qquad \text{KVL}$$

$$I_C = I_R \rightarrow C\frac{dU_C}{dt} = \frac{U_{ein} - U_C}{R}$$



High Pass / Differentiator

Hochpass / Differenzierer



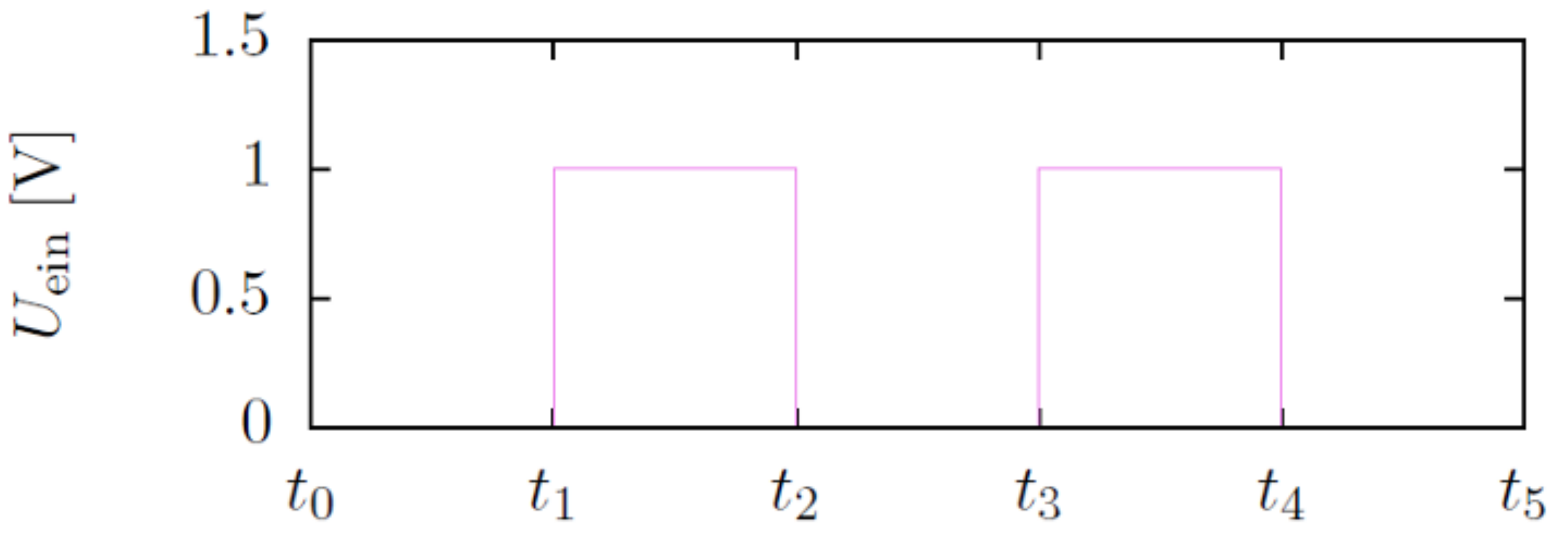
Kirchhoff: $I_C = I_R$ $U_{ein} - U_C - U_R = 0$

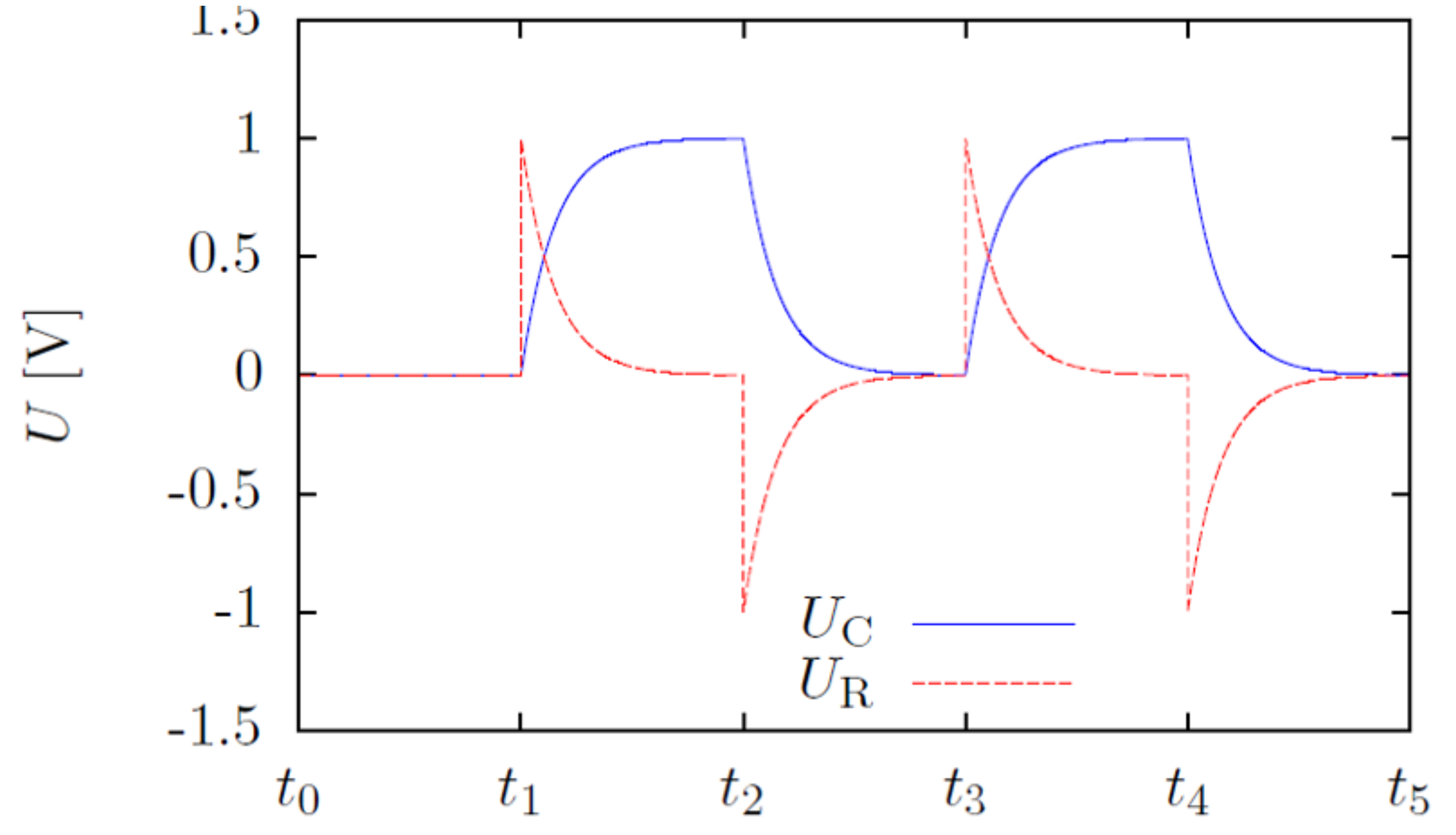
$$U_{ein} = U_C + RC\frac{dU_C}{dt}$$

$$U_{ein} - U_C = RC\frac{d(U_{ein} - U_{aus})}{dt} = U_{aus}$$

For small R: $U_{ein} - U_{aus} \sim U_{ein}$ (only small voltage drop)

$$U_{aus} = RC\frac{dU_{ein}}{dt}$$

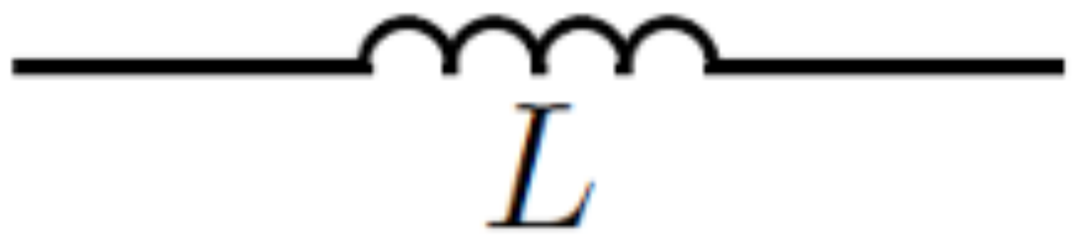




Inductors

Spulen

- Symbol:



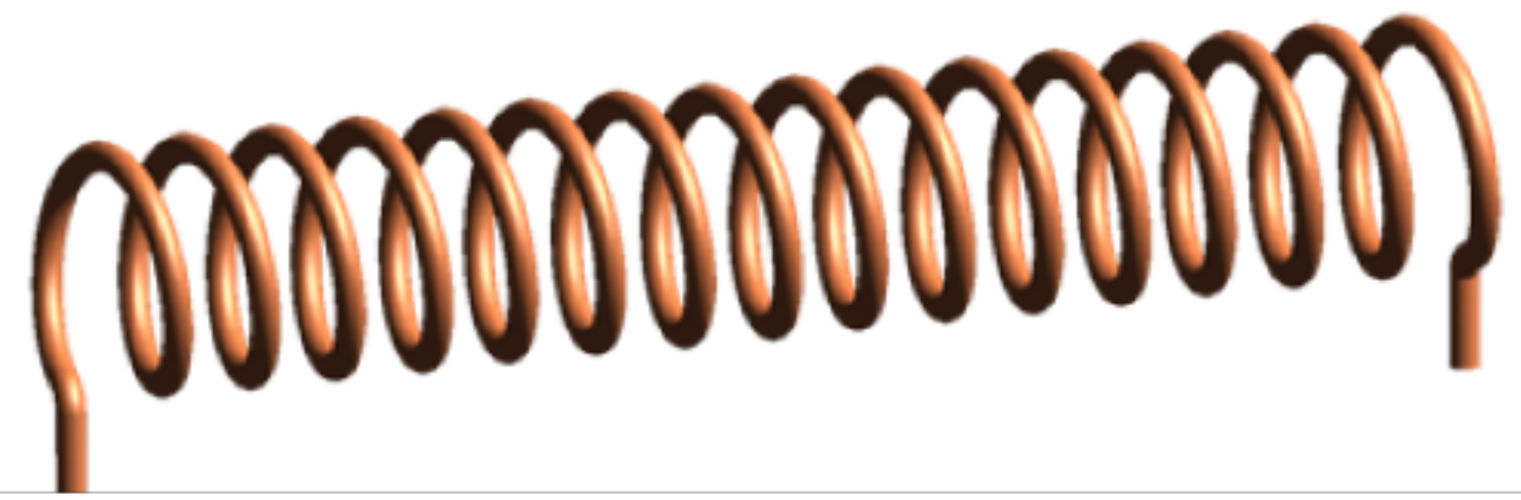
L: Inductance in Henry [H = Vs/A] *Induktivität*

$$U = L \frac{dI}{dt}$$

$$E = \frac{1}{2} L I^2$$

“classic” inductor: Solenoid - “coil”

$$\mu_0 = 4\,\pi\,10^{-7}\,[\tfrac{\mathrm{H}}{\mathrm{m}}] = 12,566\ldots\,10^{-7}\,[\tfrac{\mathrm{H}}{\mathrm{m}}]$$



$$L \approx \mu_0\,\mu_r\,N^2\,\frac{A}{l}\,\frac{1}{1+0,45\,\frac{D}{l}}$$

Inductivity of a coil with finite length

N: Number of windings

A: cross section area of the coil [m^2]

D: diameter [m]

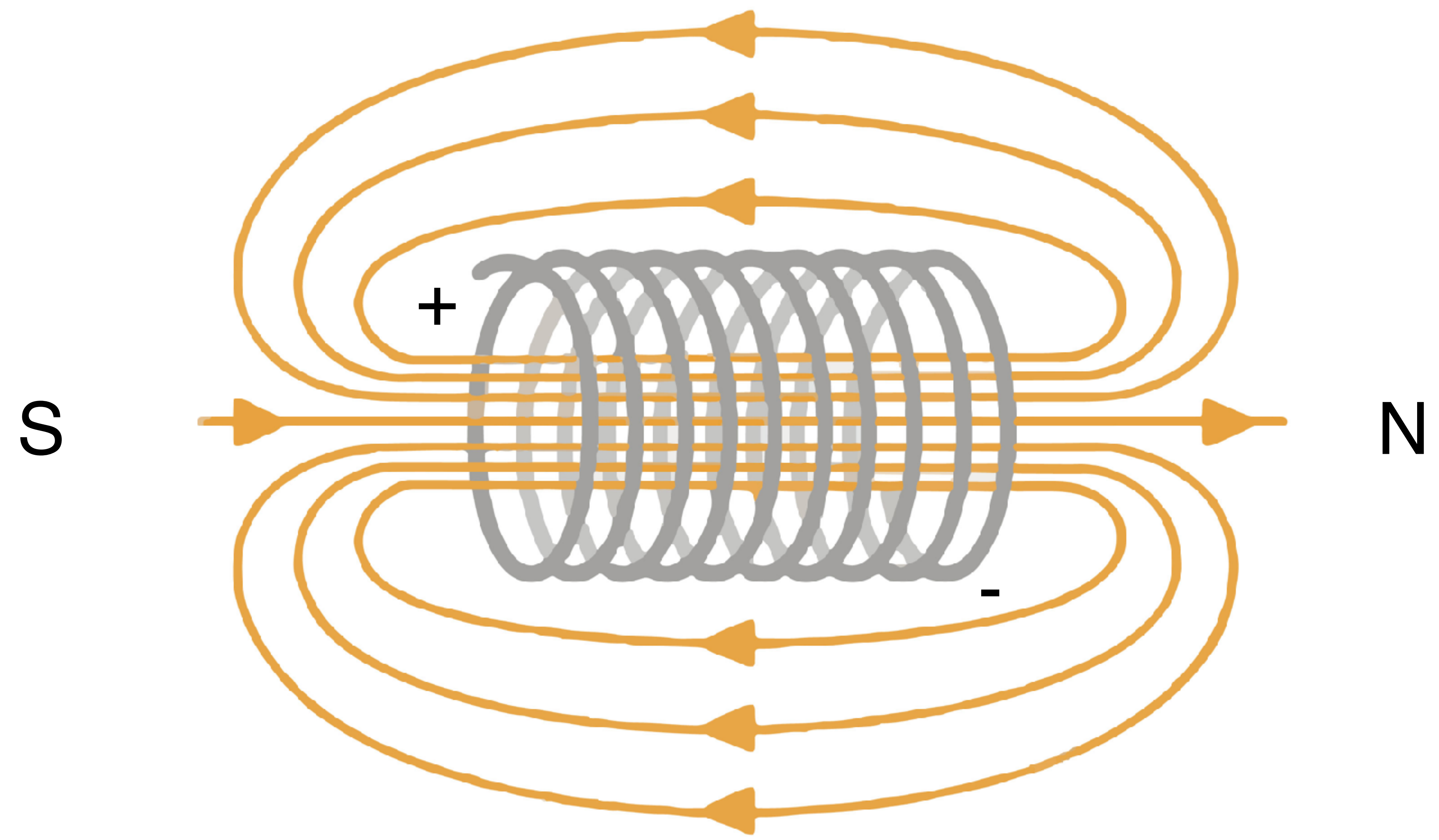
l: length [m]

μ_r: permeability of the core (1 für Luft/Vakuum;

~ 2000+ for Fe, up to ~ 10^5 for specialized alloys)

Magnetic Field of a Coil

Magnetfeld einer Spule



$$B = \mu_0 \frac{N}{l} I$$

B: Magnetic field [T]

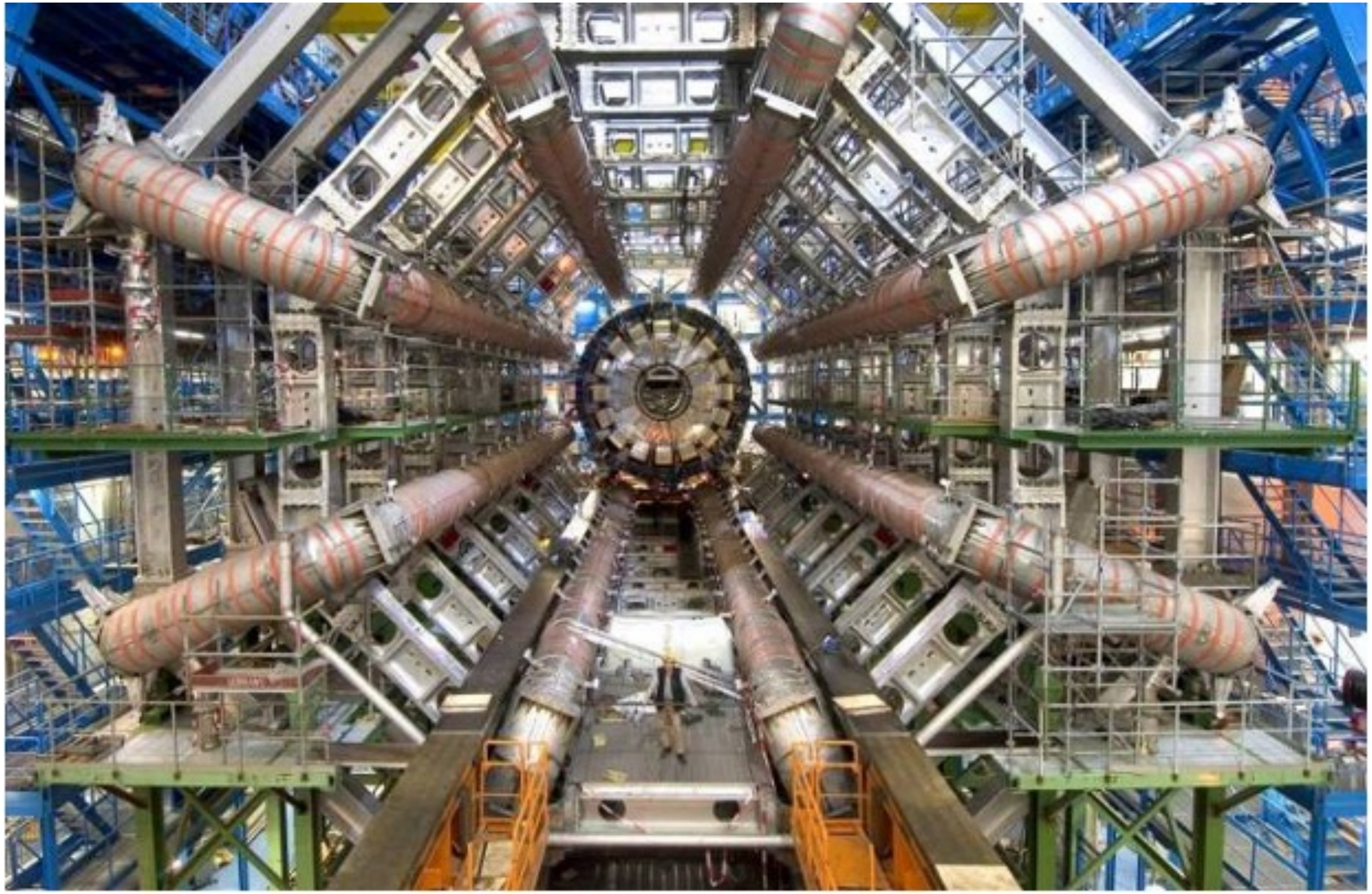
N: Number of windings

l: Length of the coil

- For applications in electronics normally not relevant - but interesting, and helpful for in-depth understanding.

Very large Coils

An Example



- ATLAS main magnet: Air core toroid

Overview: Behavior of passive Components

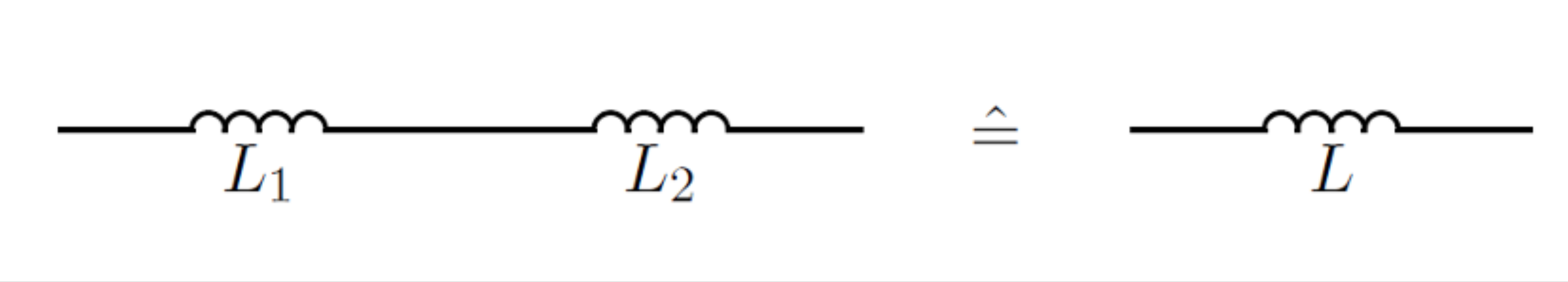
Resistor, Capacitor, Inductor

	Voltage	Current
Resistor	$U = RI$	$I = \frac{U}{R} = GU$
Capacitor	$U = \frac{1}{C}\int I dt$	$I = C\frac{dU}{dt}$
Inductor	$U = L\frac{dI}{dt}$	$I = \frac{1}{L}\int U dt$

Inductor Circuits

Series, Parallel

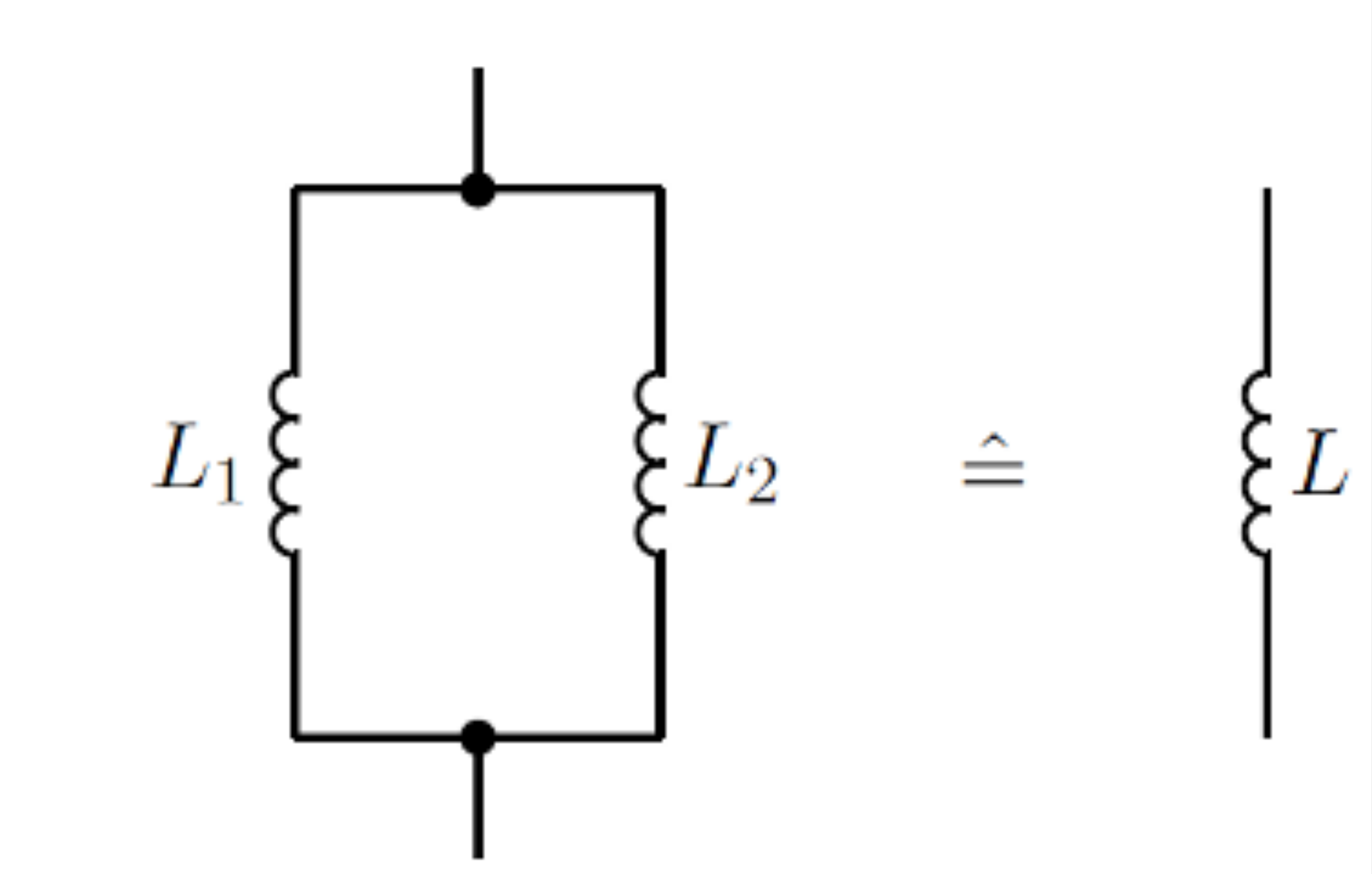
- Inductors in series



$$L = L_1 + L_2$$

Same current through all inductors: $I_{L1} = I_{L2}$

- Inductors in parallel



The same voltage across all inductors

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

$U_{L1} = U_{L2}$

(but the current is split over all)

$$L = \frac{L_1 L_2}{L_1 + L_2}$$

Overview: Circuits with passive Components

Resistor, Capacitor, Inductor

	Series	Parallel
Resistor	$R = R_1 + R_2$ increases	$R = \frac{R_1 R_2}{R_1 + R_2}$ decreases
Capacitor	$C = \frac{C_1 C_2}{C_1 + C_2}$ decreases	$C = C_1 + C_2$ increases
Inductor	$L = L_1 + L_2$ increases	$L = \frac{L_1 L_2}{L_1 + L_2}$ decreases

Inductance

Examples & Orders of Magnitude

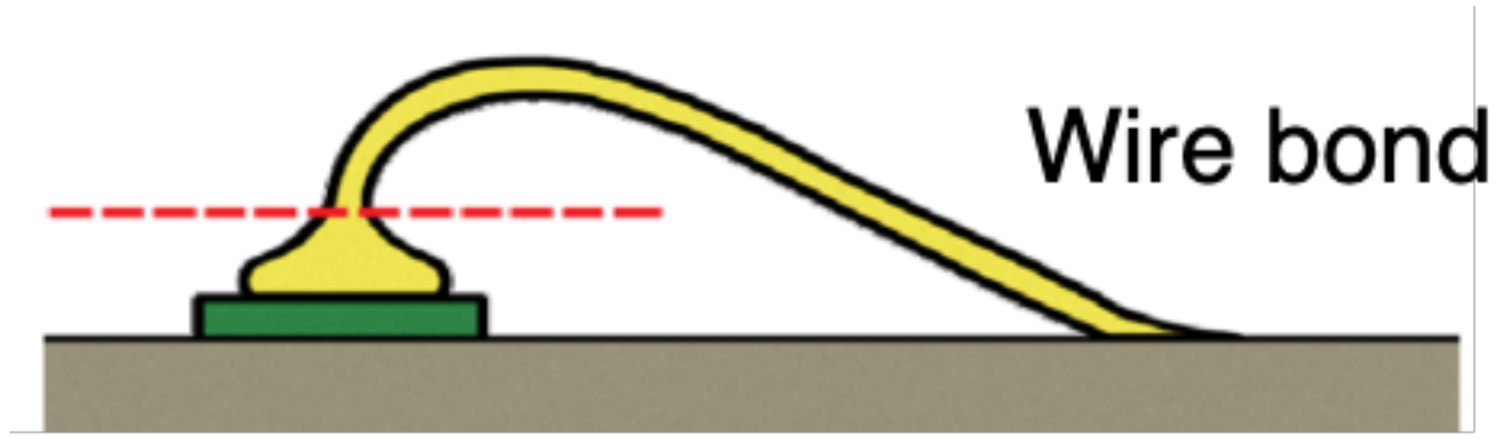
- Depending on the application a very wide spectrum of numerical values - in general: H (Henry) is a “large” unit

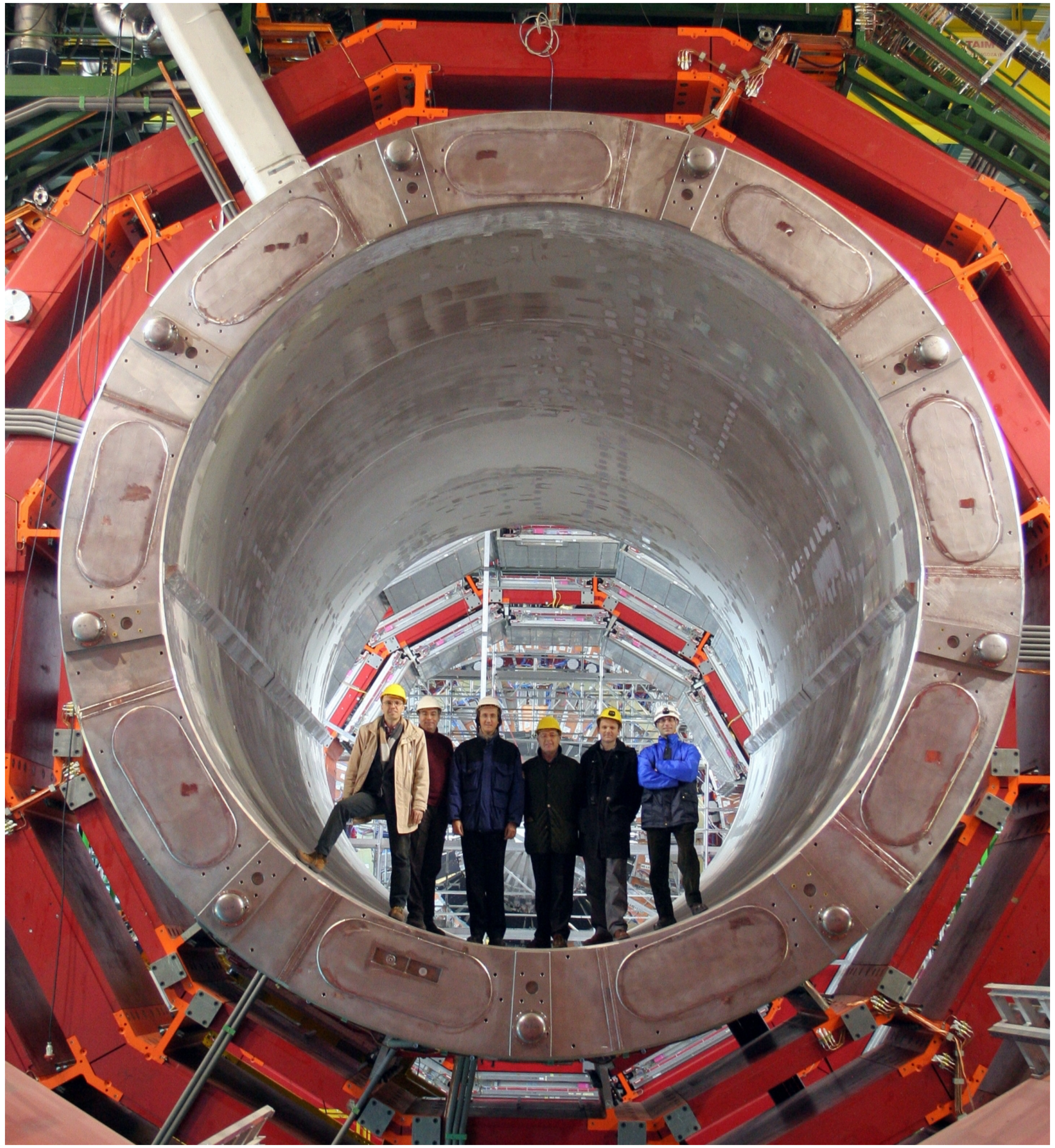
Example 1: The CMS Solenoid

- B = 3.8 T as standard, design for 4T
- Stored energy: 2.3 GJ (at 3.8 T)
- Current: 18.2 kA (at 3.8 T)
- Inductance L: 14 H
- Length: 13 m; Diameter 6 m

Example 2: A single wire bond

- Length: a few mm
- Inductance L: a few nH

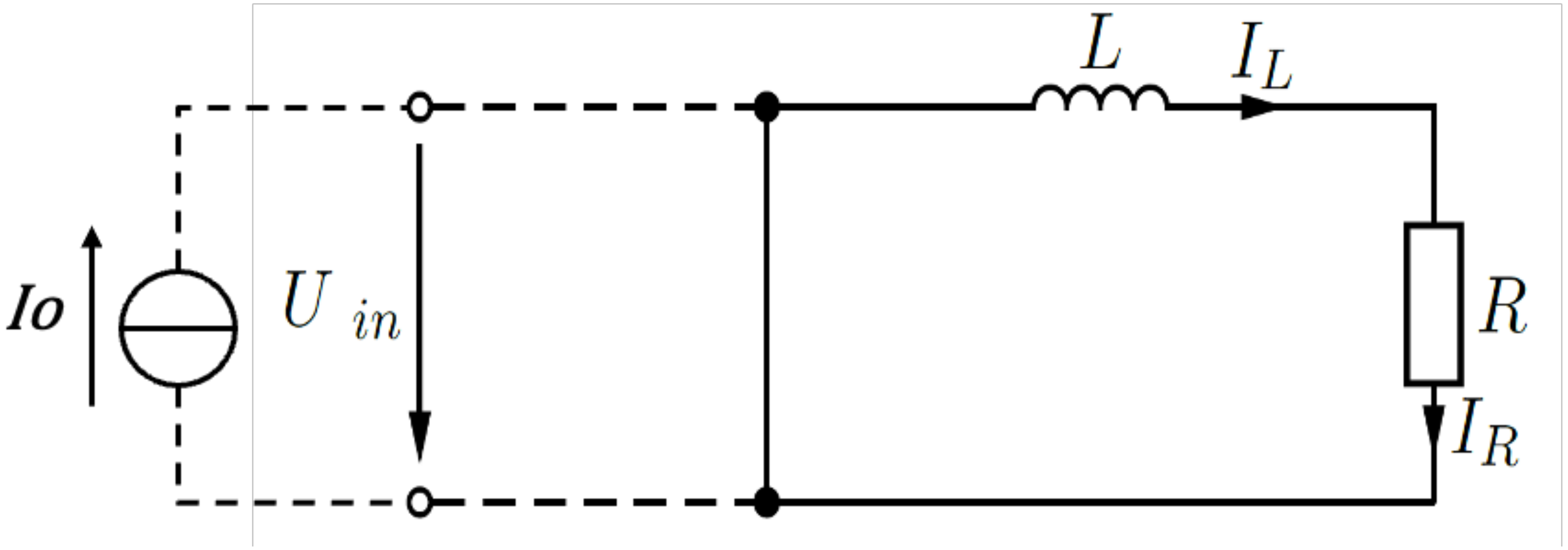




Discharging an Inductor

Entladen

- Discharging of an inductor through a resistor



Initial conditions at t=0: $I_L = I_0$

$$U_L + R\,I = 0 \Leftrightarrow L\,\frac{dI}{dt} = -R\,I$$

Solving the differential equation

$$I = I_0 e^{-\frac{t}{L/R}} = I_0 e^{-\frac{t}{\tau}} \qquad \tau = \frac{L}{R}$$

for t > 0: Exponential decay of the current

=> $$U_L = L\,\frac{dI}{dt} = -R\,I_0\,\mathrm{e}^{-\frac{t}{\tau}}$$

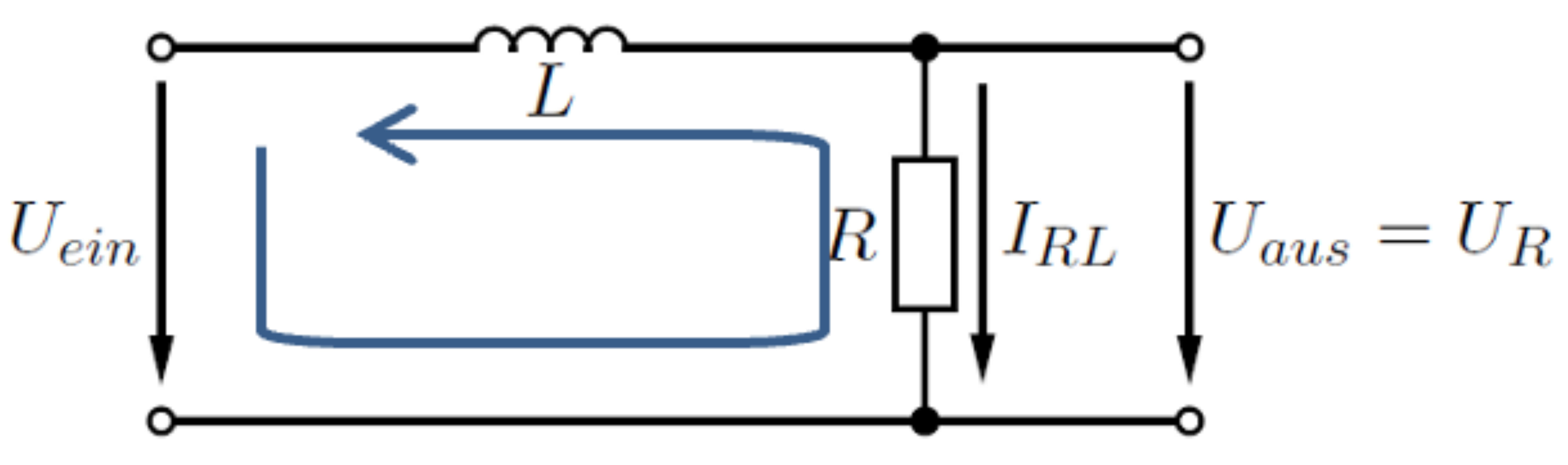
Comparing to a capacitor:

Exponential decay of the voltage

Magnetizing an Inductor

Magnetisieren

- An electric circuit with a voltage source, inductor and resistor



Kirchhoff:

$$I_L = I_{RL}$$

$$U_{ein} - U_L - U_R = 0$$

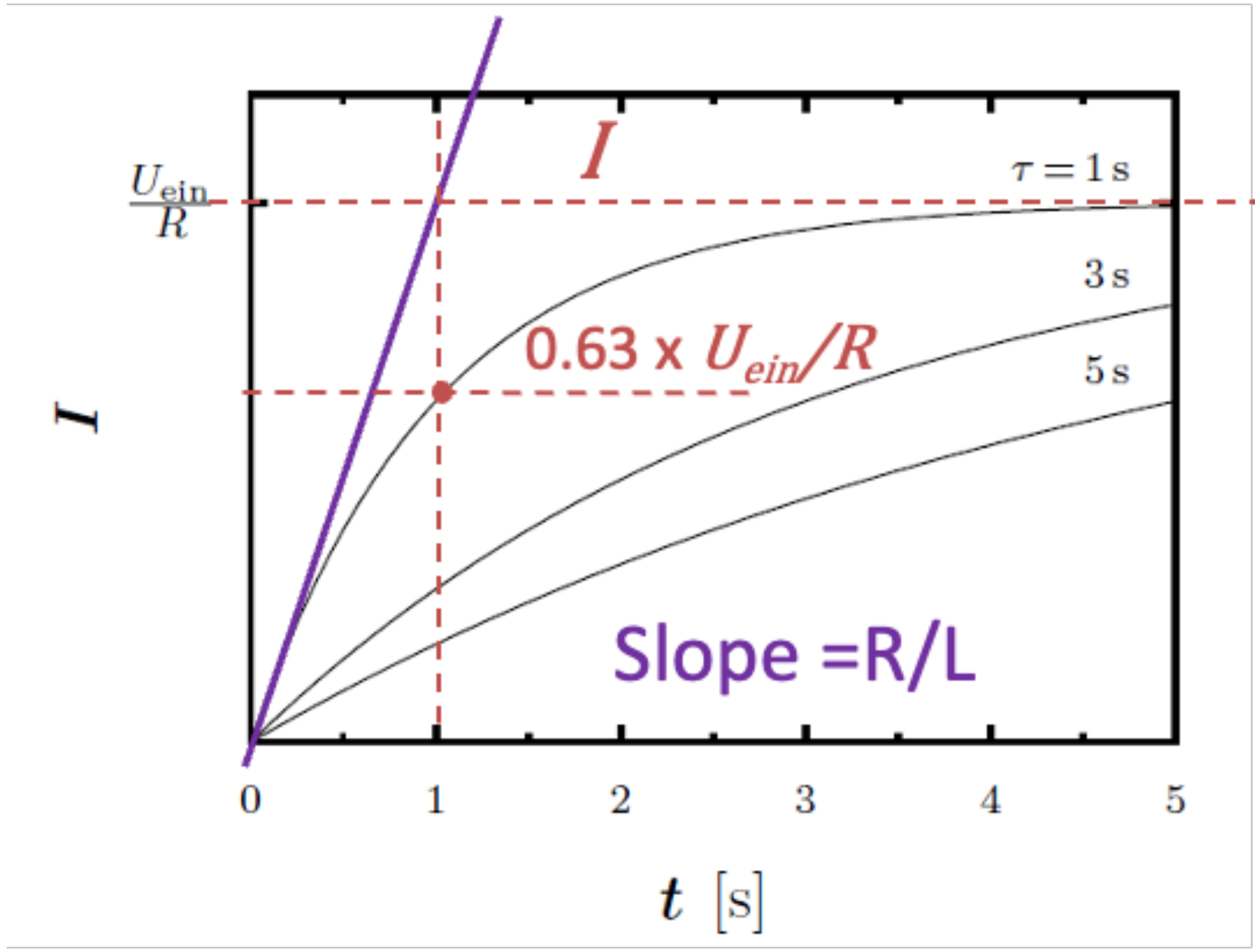
$$=> \quad U_{ein} = L\frac{dI}{dt} + RI$$

$$=> \quad \frac{dI_{RL}}{dt} + \frac{R}{L} I_{RL} = \frac{U_{ein}}{L} \qquad => \quad I_{RL} = \frac{U_{ein}}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\tau = \frac{L}{R} \quad , \quad U_R = U_{in}\left(1 - e^{-\frac{t}{\tau}}\right) \quad , \quad U_L = U_{in}\, e^{-\frac{t}{\tau}}$$

For t = 0: A non-magnetized inductor behaves like an “open” circuit

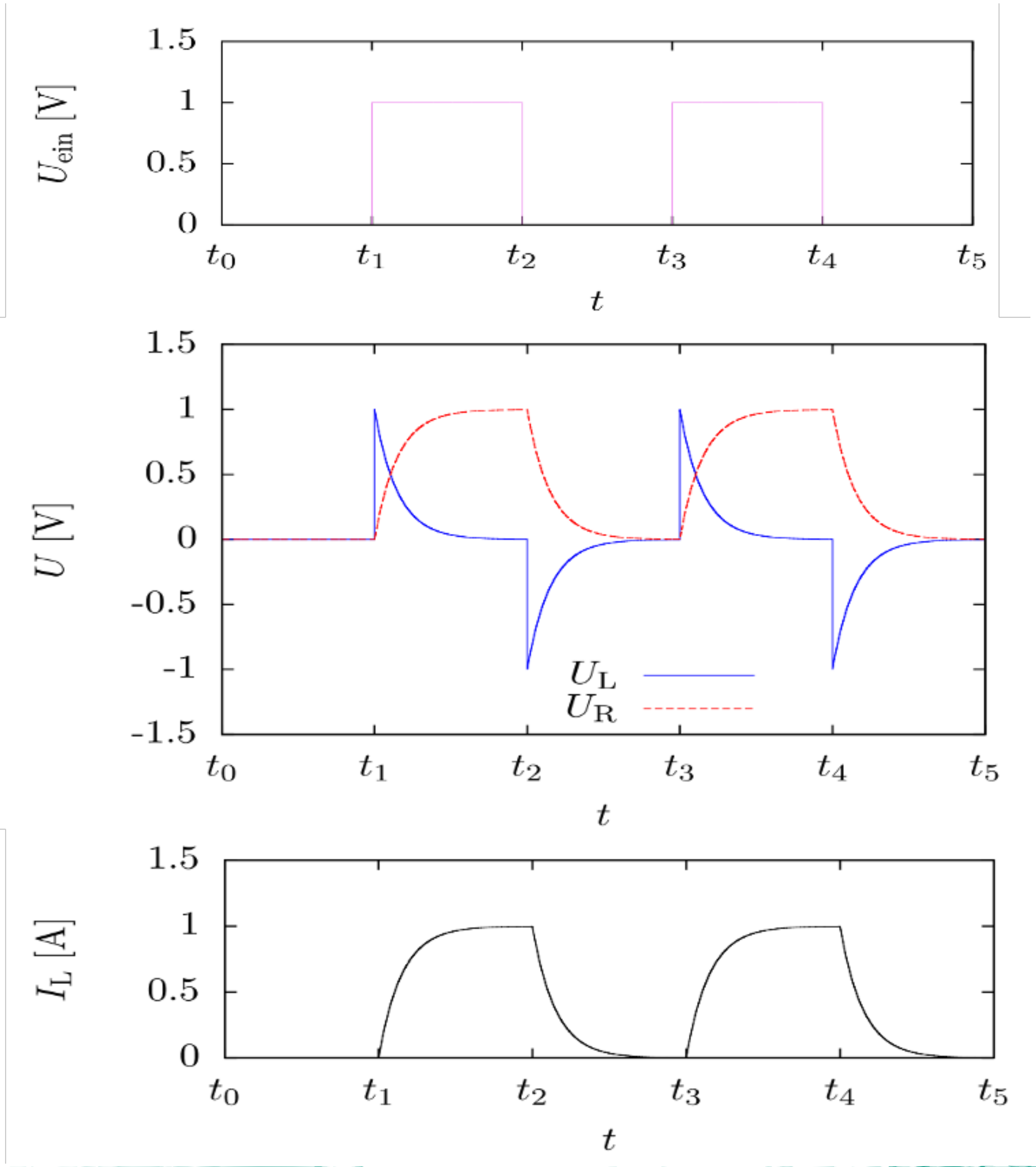
For t = ∞: A magnetized inductor behaves like a short circuit

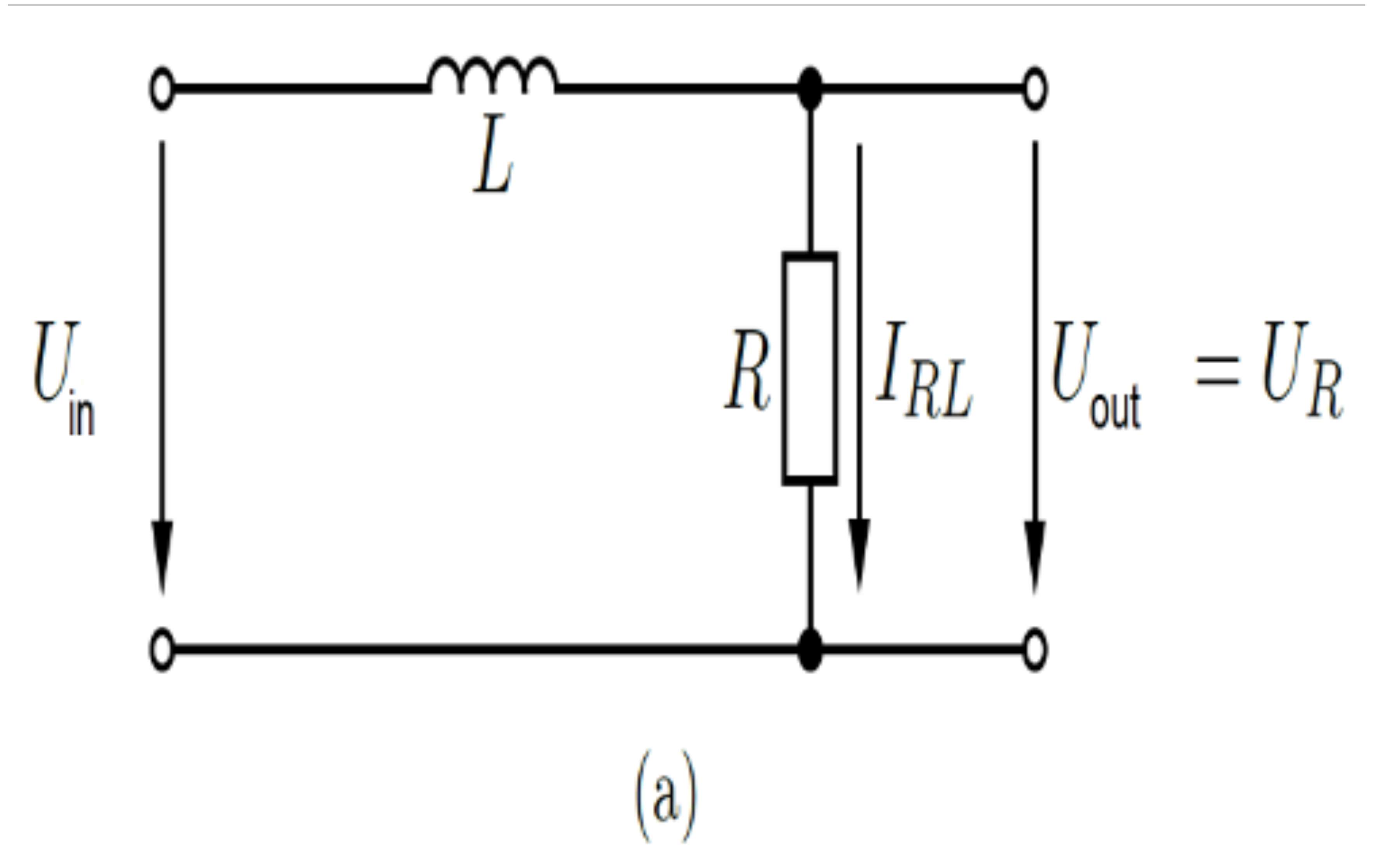


Time-Dependent Voltage

Behavior for a sequence of rectangular pulses

- Comparable to the behavior of a capacitor (but with swapped integration / differentiation)





RL low pass: integrator

To discuss: Can a high pass be constructed as well?

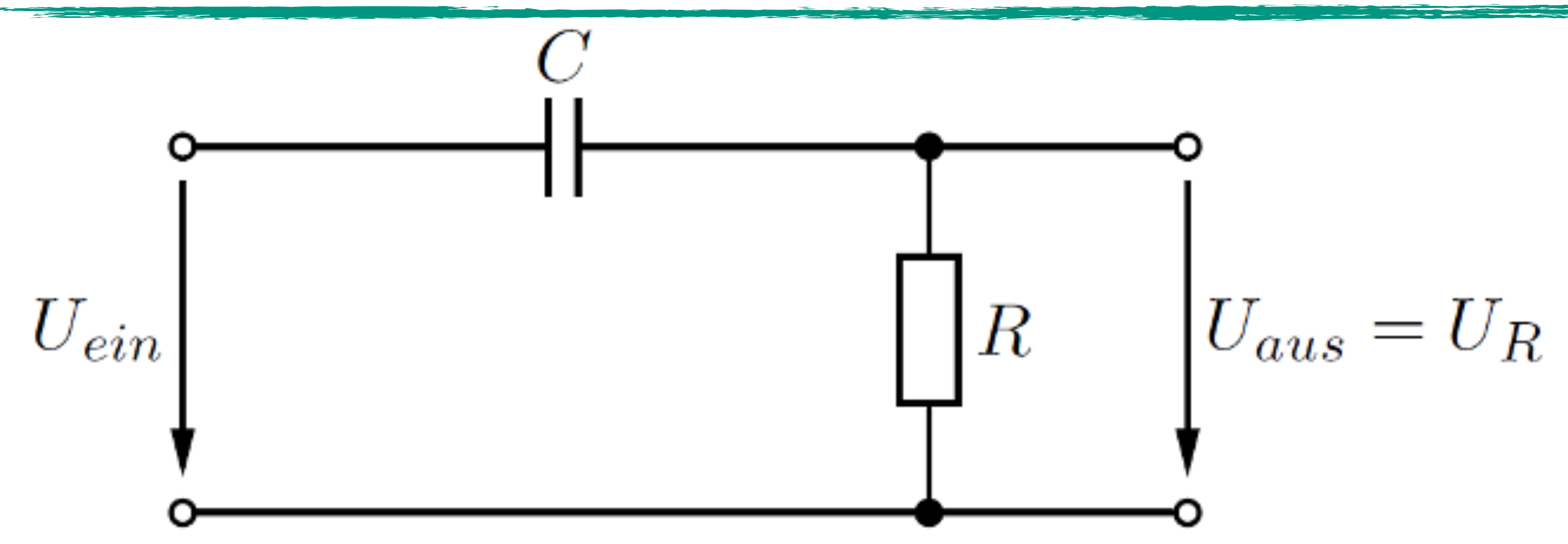
Overview: Time behavior of passive Components

Resistor, Capacitor, Inductor

	Stored Energy	t = 0 (when switching on)	t -> ∞ (in equilibrium)
Resistor	-	time independent	
Capacitor	electric charge Q: -> voltage	Q = 0; U = 0: -> ***short circuit*** Q ≠ 0; U ≠ 0: -> **voltage source**	Q konstant dQ/dt = I = 0: -> ***open circuit***
Inductor	magnetic field B: -> current	B = 0; I = 0: -> ***open circuit*** B ≠ 0; I ≠ 0: -> **current source**	B konstant dI/dt = U = 0: -> ***short circuit***

RC and RL Circuits

A Comparison

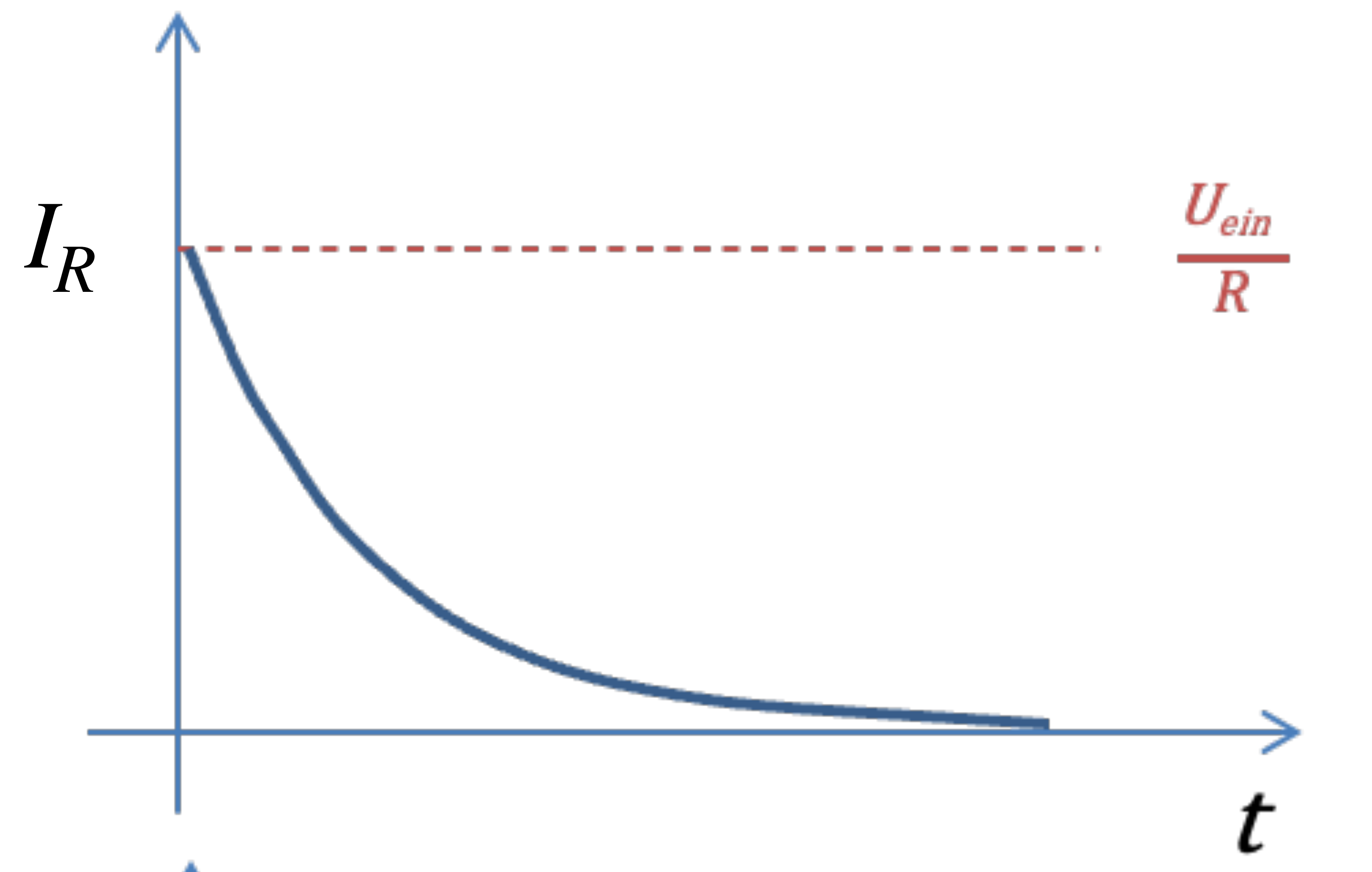


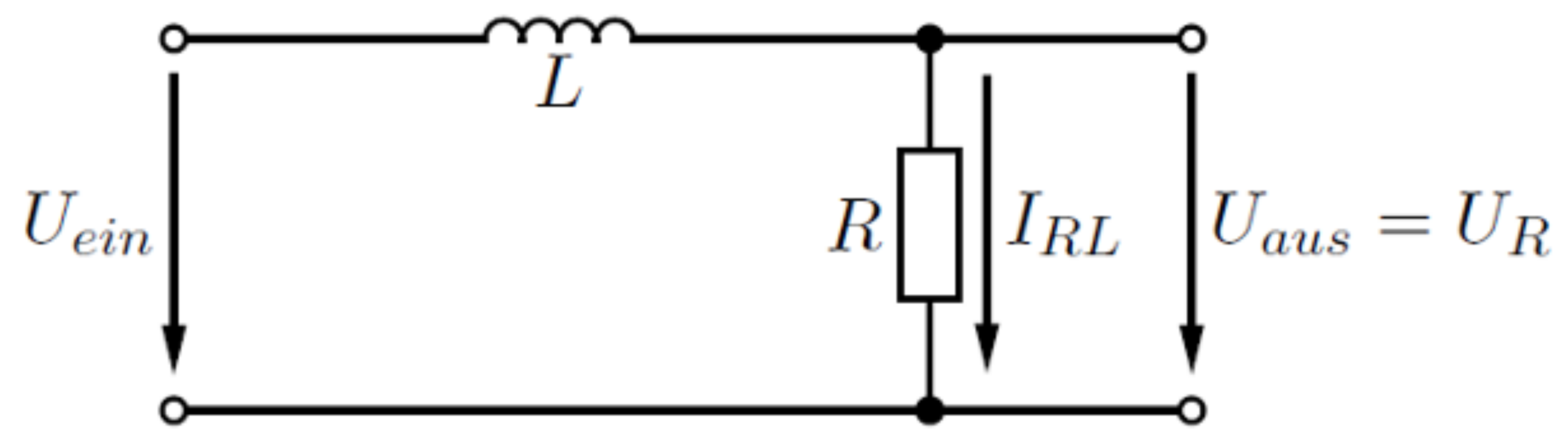
t = 0 -> A capacitor behaves like a short circuit

t = ∞ -> Capacitor as open circuit

$$I_R = \frac{U_{ein}}{R}$$

$$I_R = 0$$



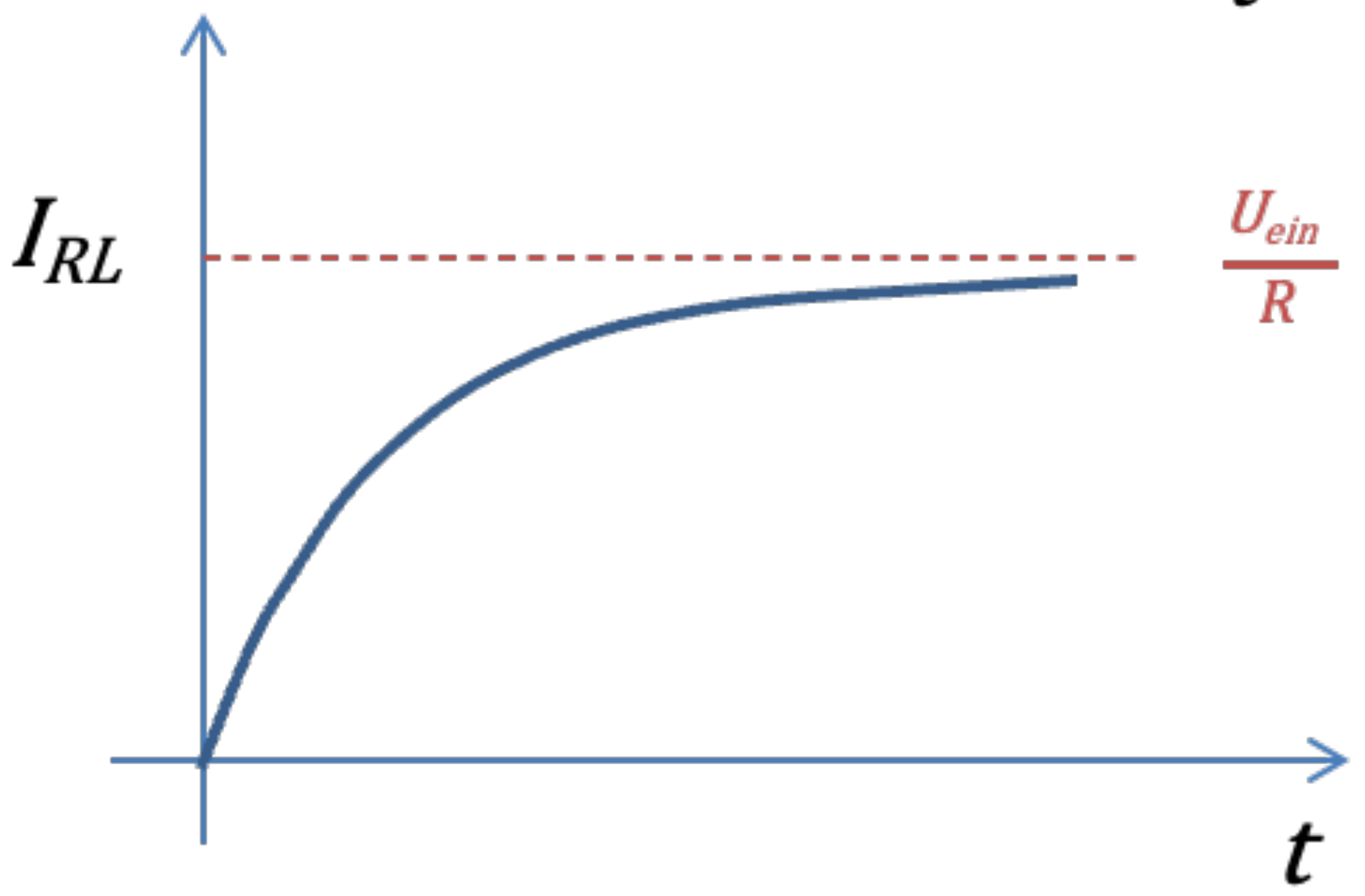


t = 0 -> An inductor behaves like an open circuit

t = ∞ -> An inductor behaves like a short circuit

$$I_{RL} = 0$$

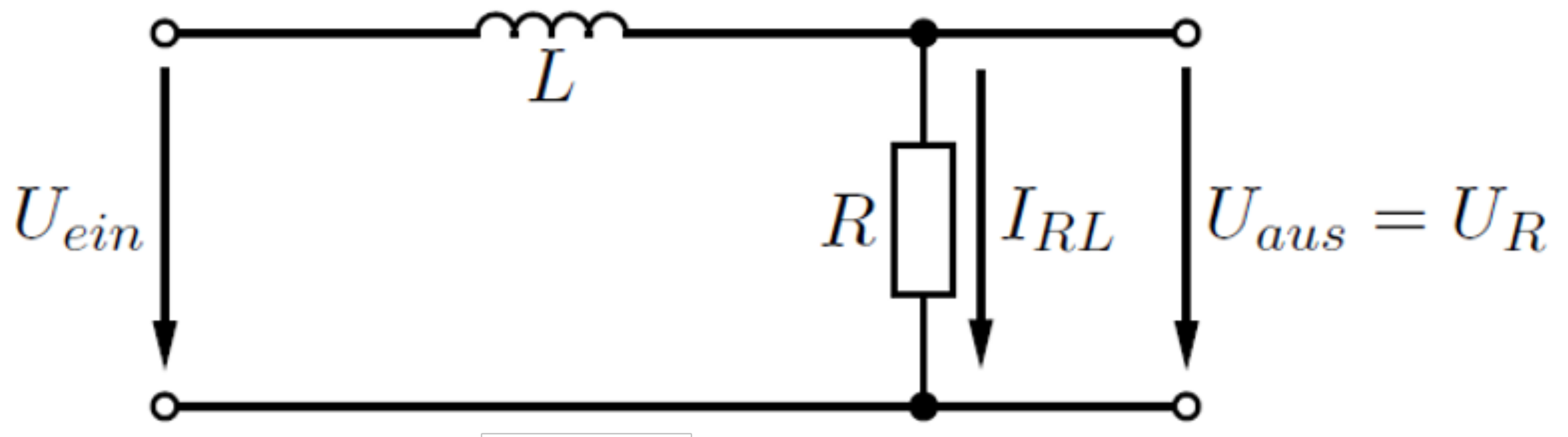
$$I_{RL} = \frac{U_{ein}}{R}$$



High and Low Pass

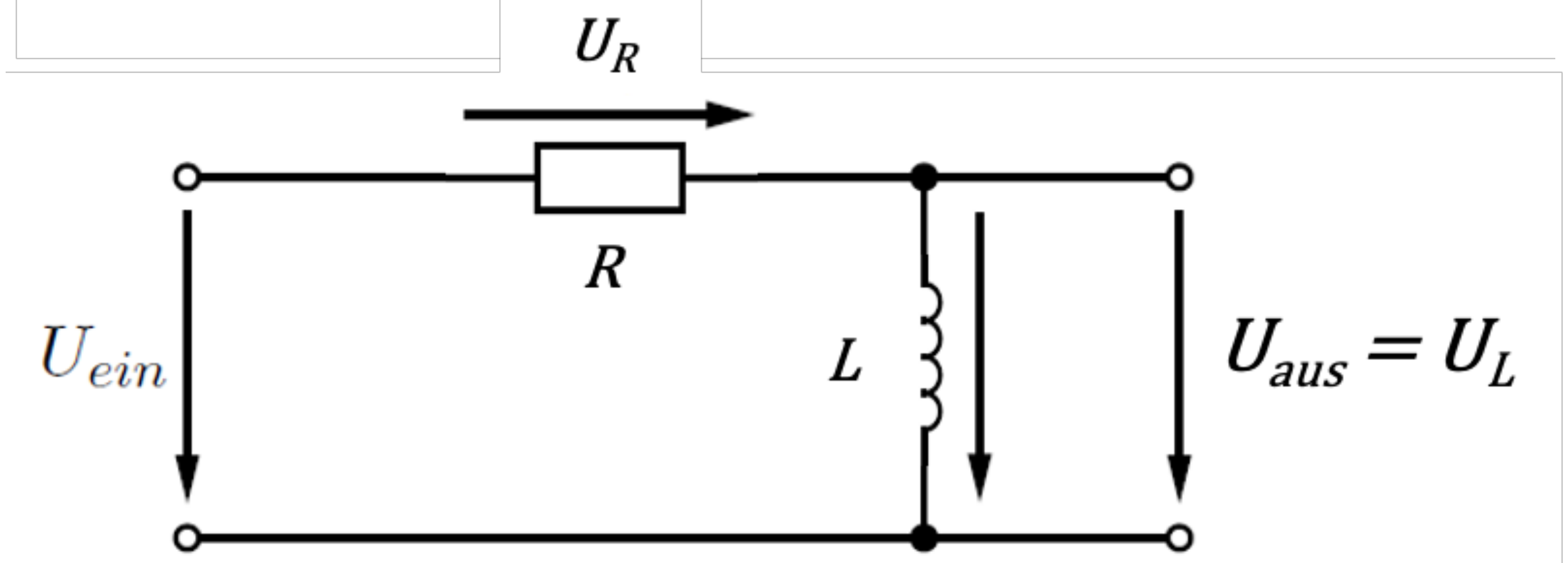
To follow in detail at home

- Building on slide 43, analogous to slides 30, 31:
  Try to follow the math



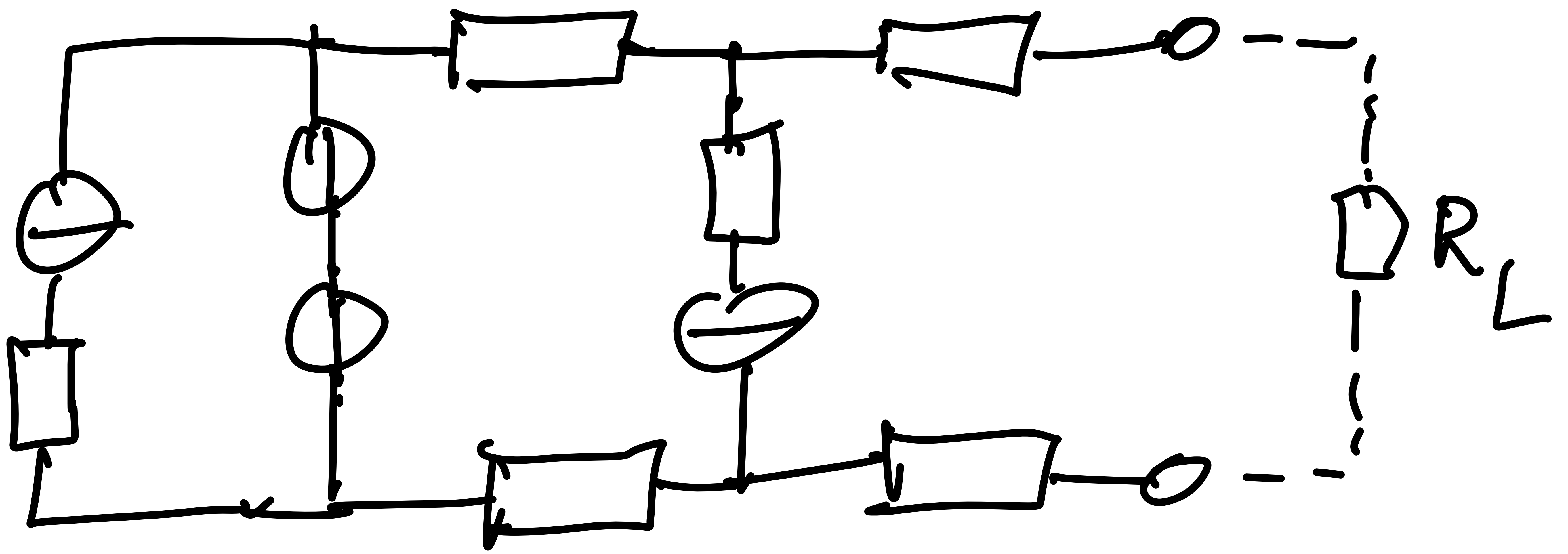
Transfer from capacitor case

Low pass / integrator



High pass / differentiator

- Circuits with resistors, capacitors, inductors and an arbitrary number of current and voltage sources: No limit to complexity!



U_L ?

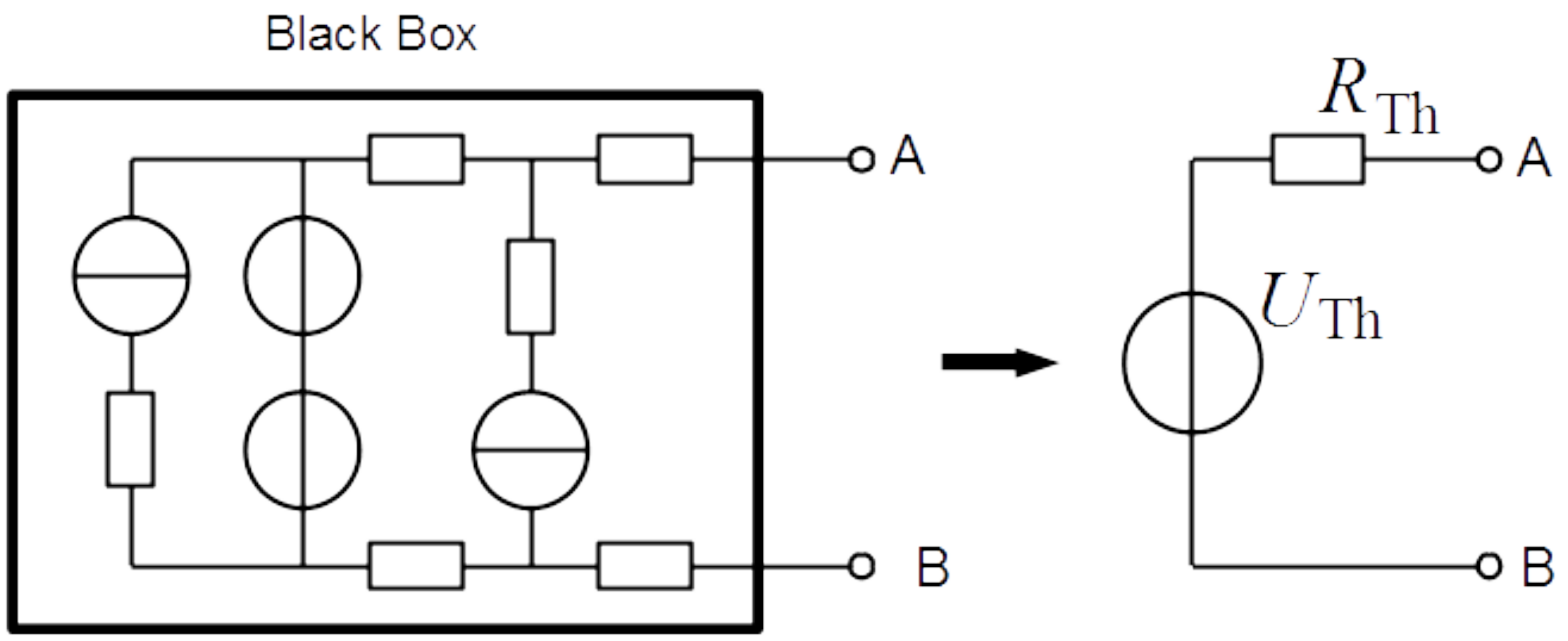
I_L ?

Thévenin Theorem

also known as: Helmholtz Theorem

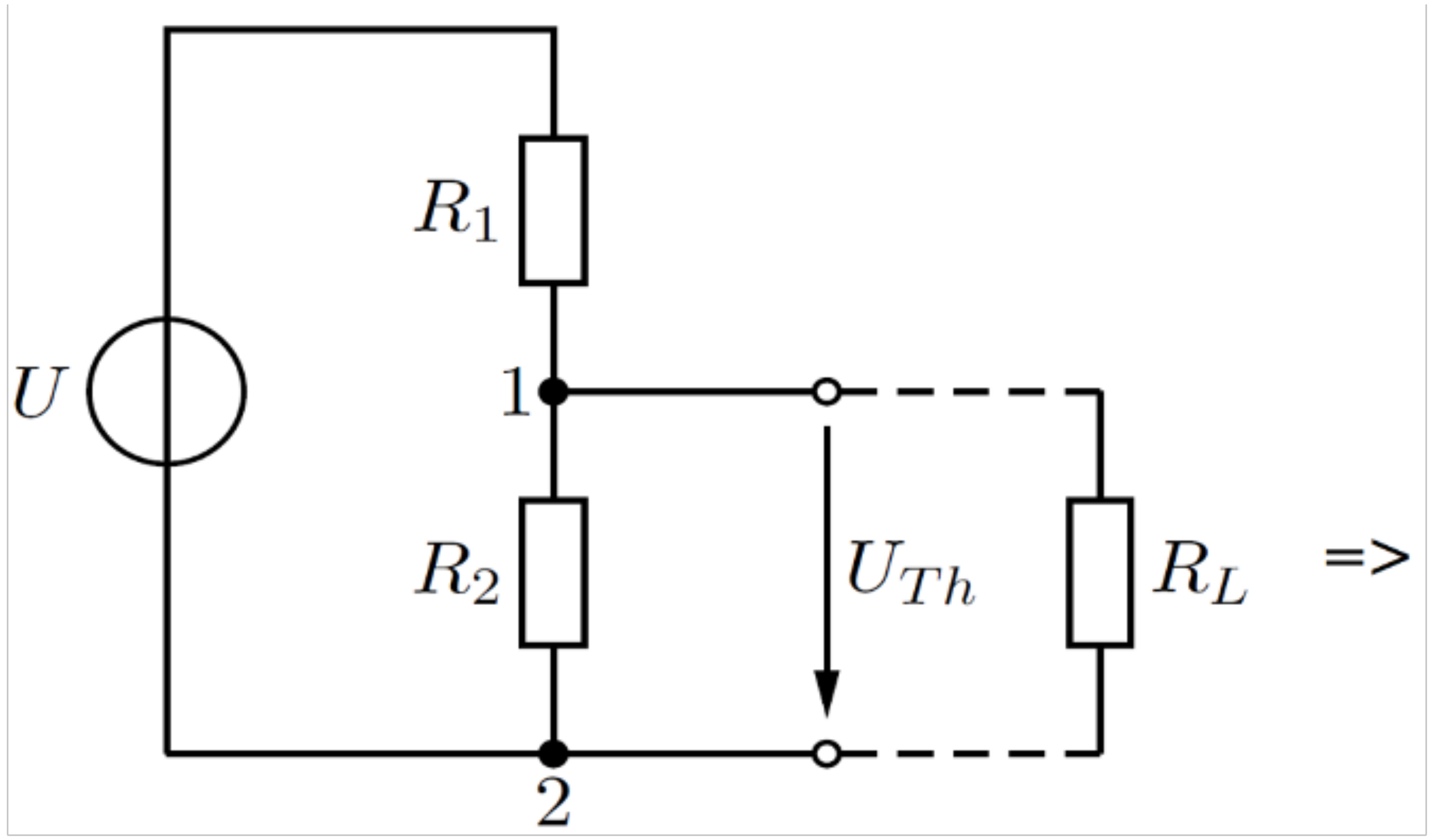
- Every linear network can be described by one ideale voltage source $\mathbf{U_{Th}}$ and one (internal) resistance $\mathbf{R_{Th}}$ in series:
  - Answers the following question:
    What is the voltage across and the current through a load resistor connected between any 2 nodes of the net?
    (NB: the power consumption may not be predicted correctly!)

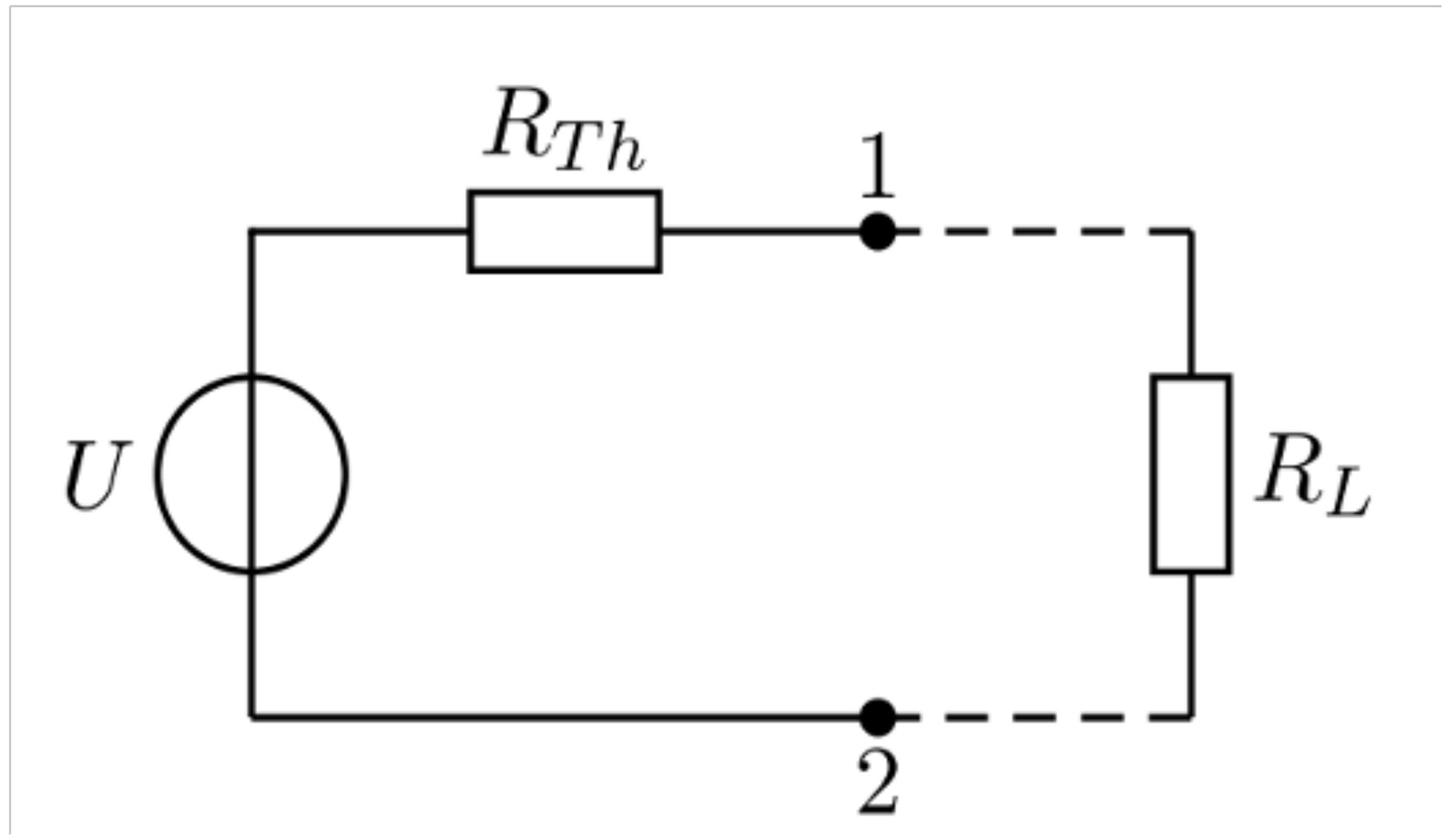
=> Thévenin equivalent, with U_{Th} and R_{Th}



Example: Voltage Divider

Determining the Thévenin Equivalent





- How are U_{Th} and R_{Th} calculated?

1. **U_{Th}** corresponds to the output voltage of the divider for the case of no load ($R_L = \infty$)

$$U_{Th} = \frac{UR_2}{R_1 + R_2}$$

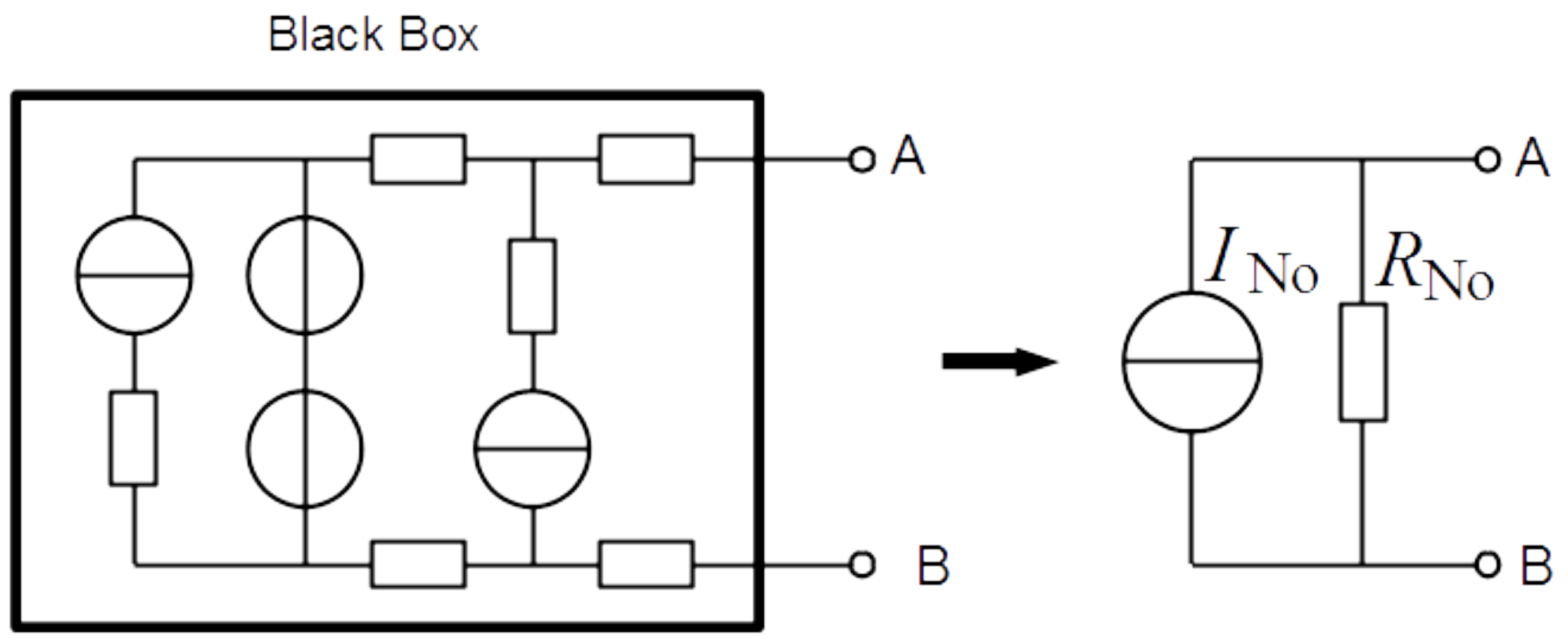
2. **R_{Th}** is determined for the case where all voltage sources are replaced by a short circuit and all current sources are replaced with an open circuit.

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2}$$

Northon's Theorem

Northon Theorem

- Every linear network can be described by an ideal current source $\mathbf{I_N}$ and a parallel internal resistor $\mathbf{R_N}$:



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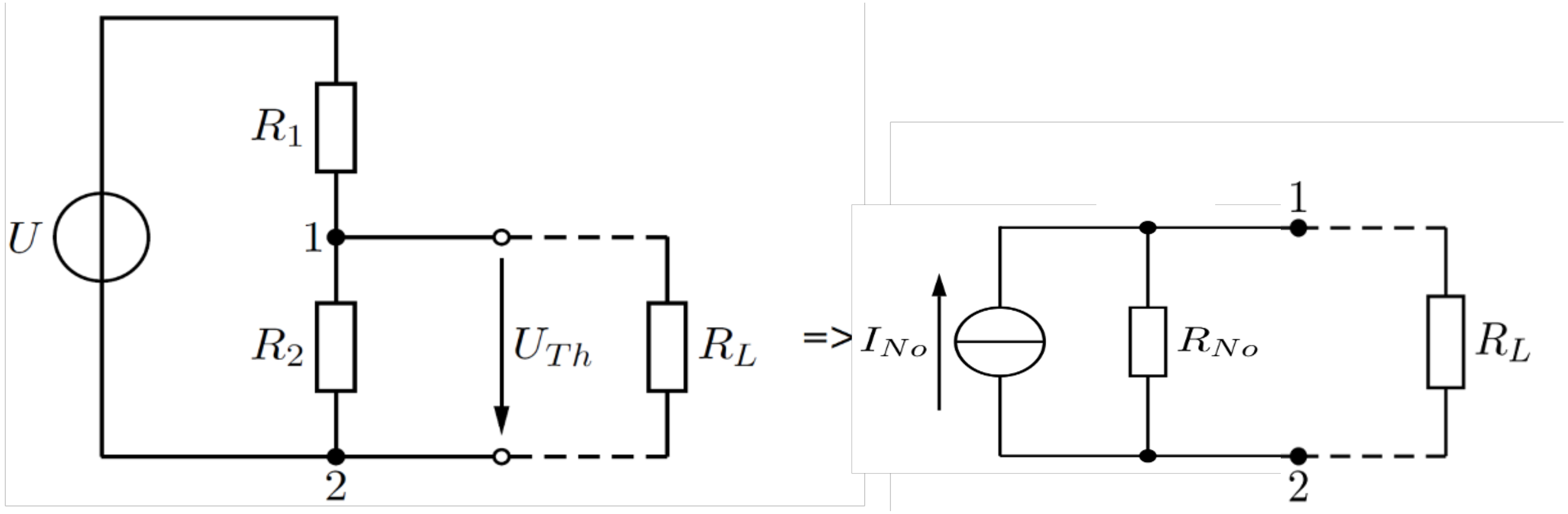
=> Northon Equivalent, with I_{No} and R_{No}

Example: Voltage Divider

Determining the Northon Equivalent



- Same procedure as for the Thévenin equivalent

1. **I_{No}** corresponds to the output current of the voltage divider for a short circuit ($R_L = 0$).

$$I_{No} = \frac{U}{R_1}$$

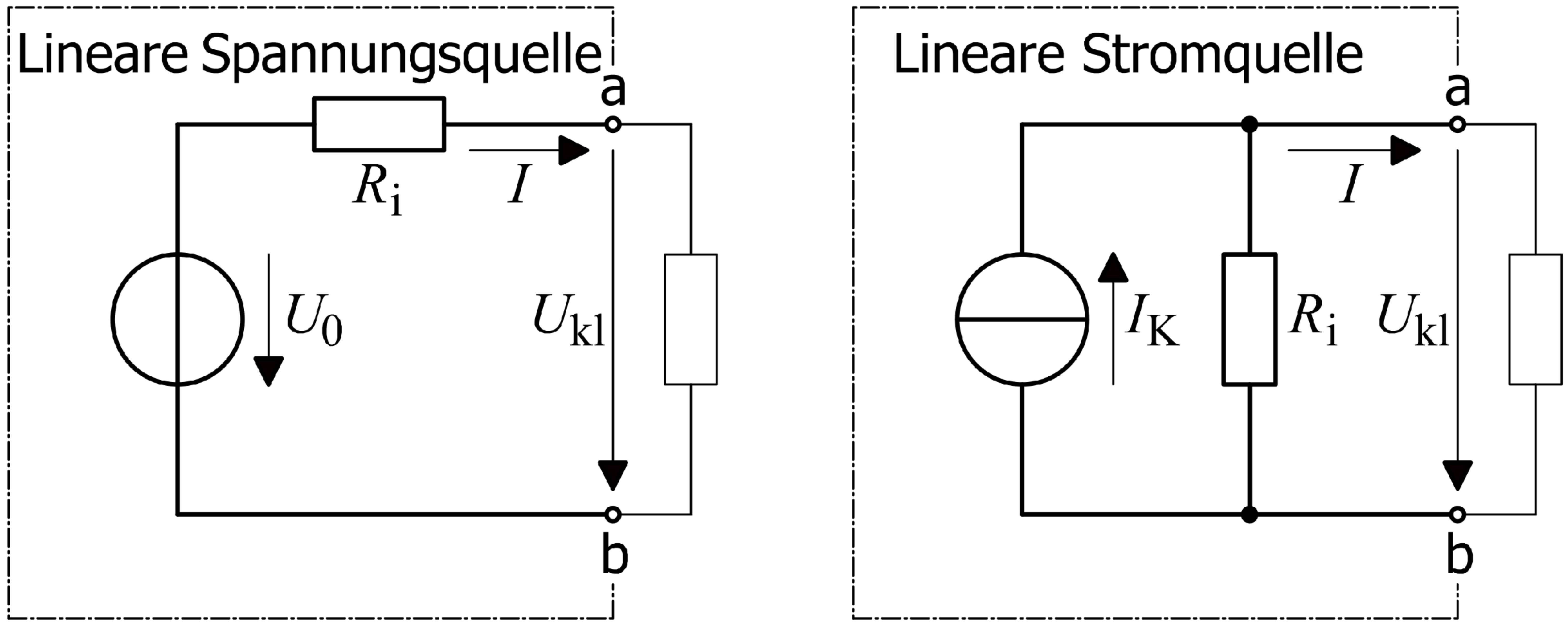
2. **R_{No}** is calculated by replacing all voltage sources by a short circuit and all current sources by an open circuit.

$$R_{No} = \frac{R_1 R_2}{R_1 + R_2}$$

Thévenin and Northon

Conversions between the two schemes

- A net can by represented both by a Thévenin and a Northon equivalent



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- Conversion:

  $R_{No} = R_{Th}$

  $I_{No} = U_{Th}/R_{No}$

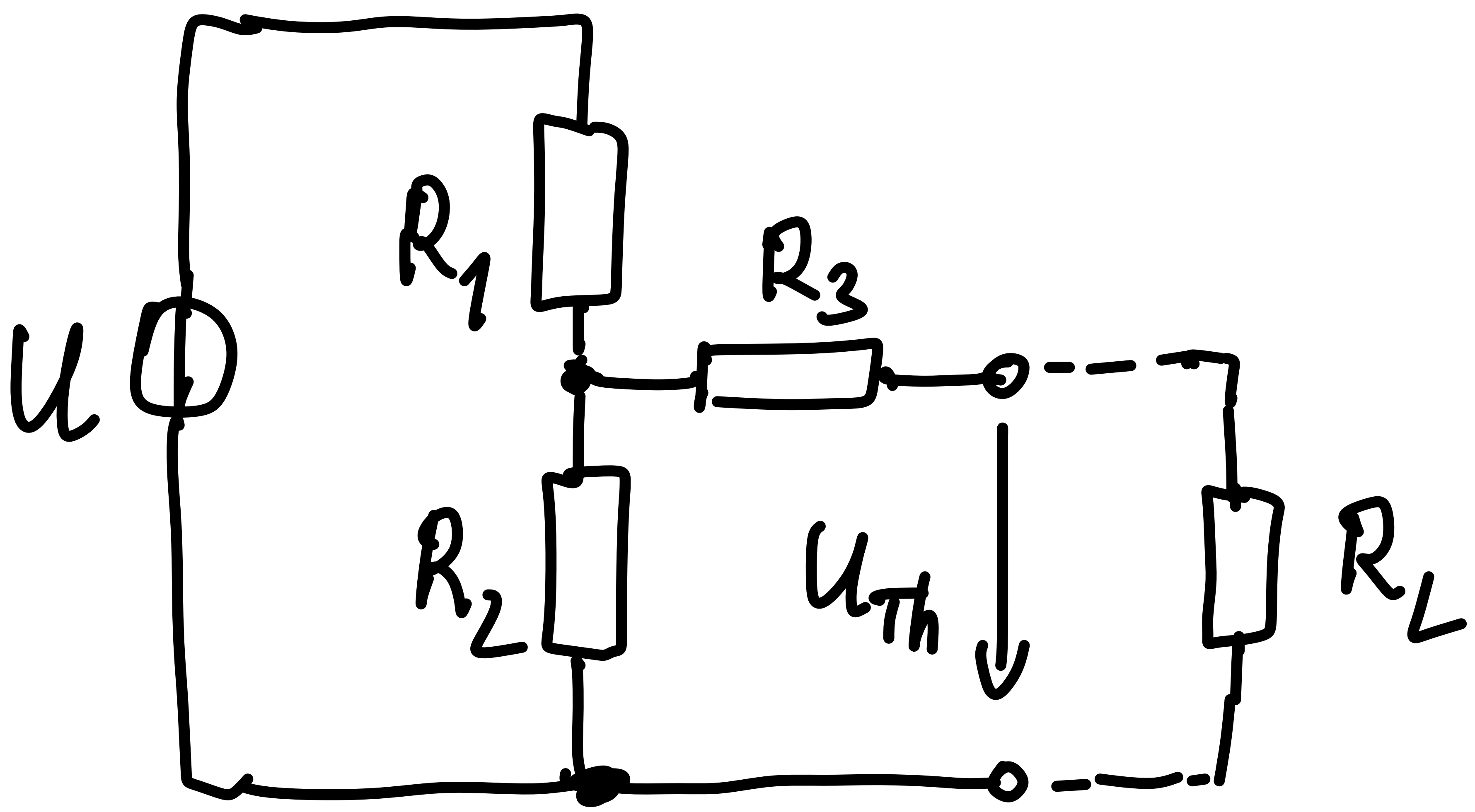
Food for thought:

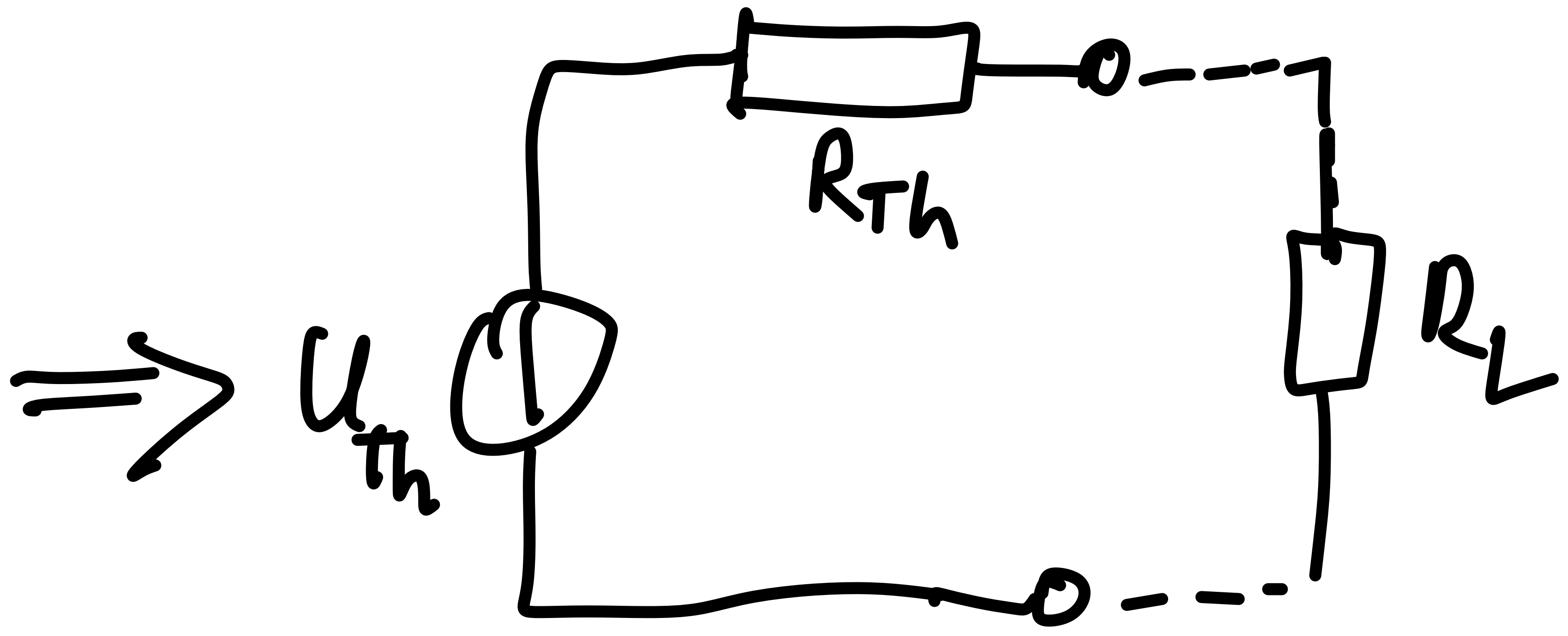
Seen from the outside both options are identical - where are the differences?

Food for Thought

Extension of the Voltage Divider

- Slight increase in complexity: Additional resistor R_3.
  What happens to U_{Th}, R_{Th} ?





For U_{Th}: Without load this is identical to the "simple" voltage divider!

$$U_{Th} = \frac{UR_2}{R_1 + R_2}$$

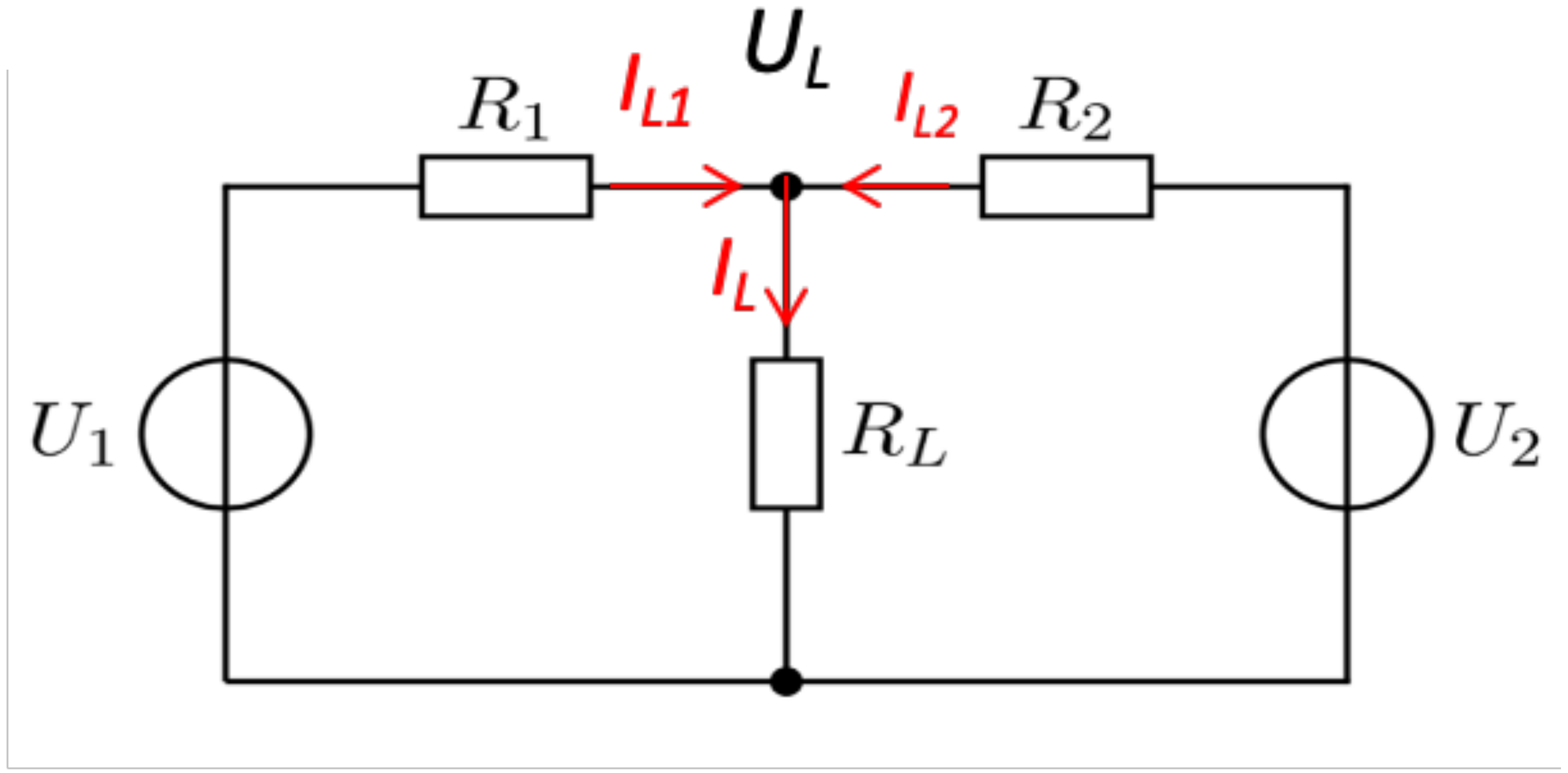
For R_{Th}: In addition R_3 in series:

$$R_{Th} = R_1 \quad R_2 + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

Analyzing Linear Networks

The Superposition Principle I

The Superposition Principle - for a simple example ("T-pad"):



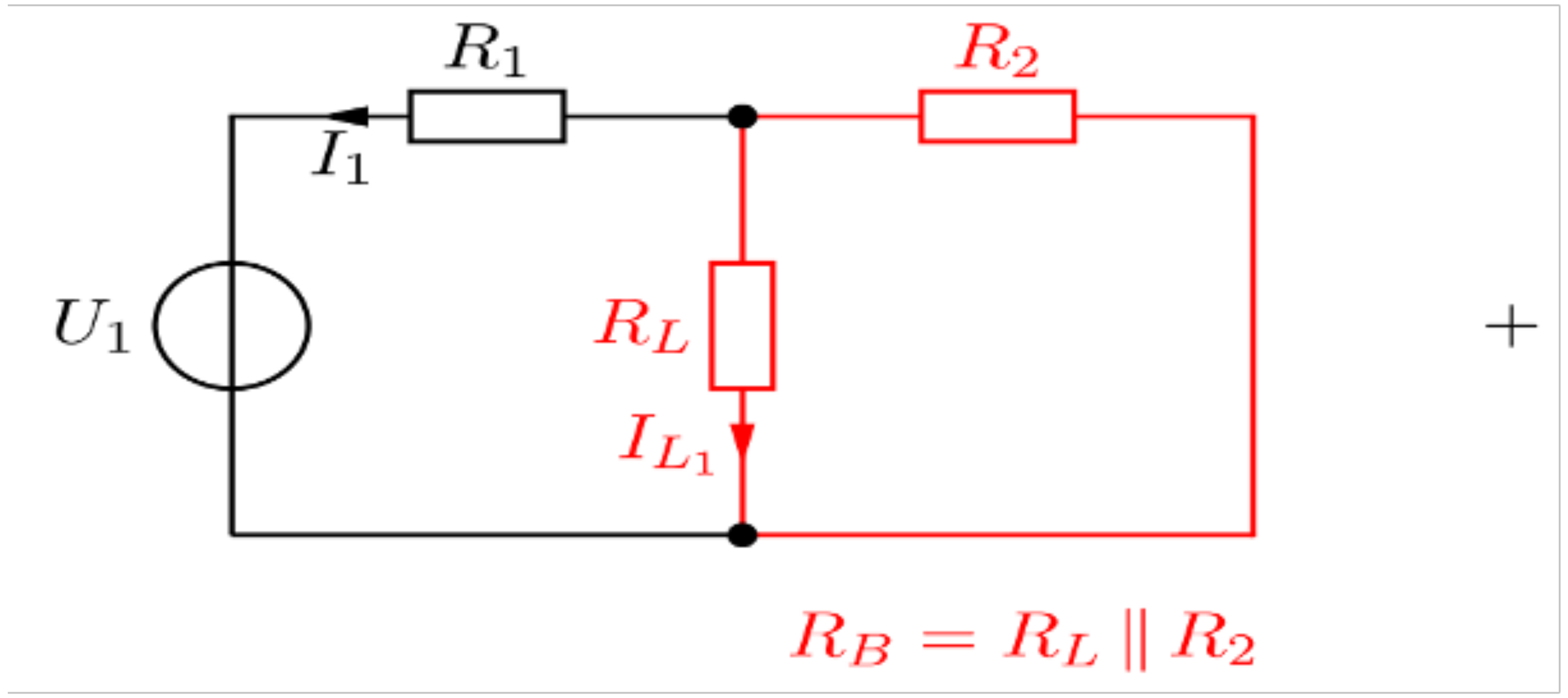
Circuit with two voltage sources and 3 resistors.

What is the load current through R_L?

A superposition of two circuits across R_L.

As usual Kirchhoff's laws are very helpful:

$I_L = I_{L1} + I_{L2}$ => $U_L = U_{L1} + U_{L2}$



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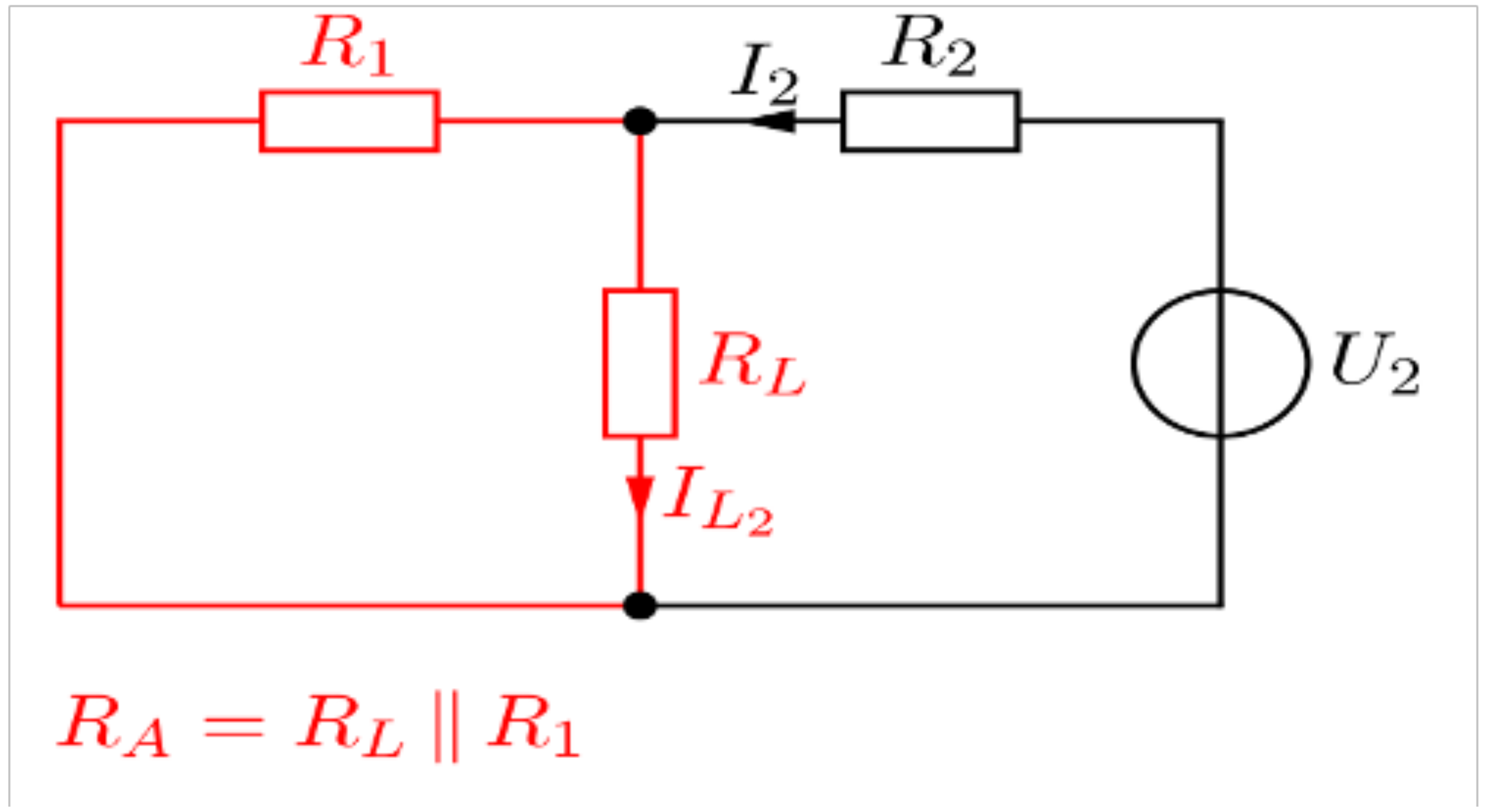

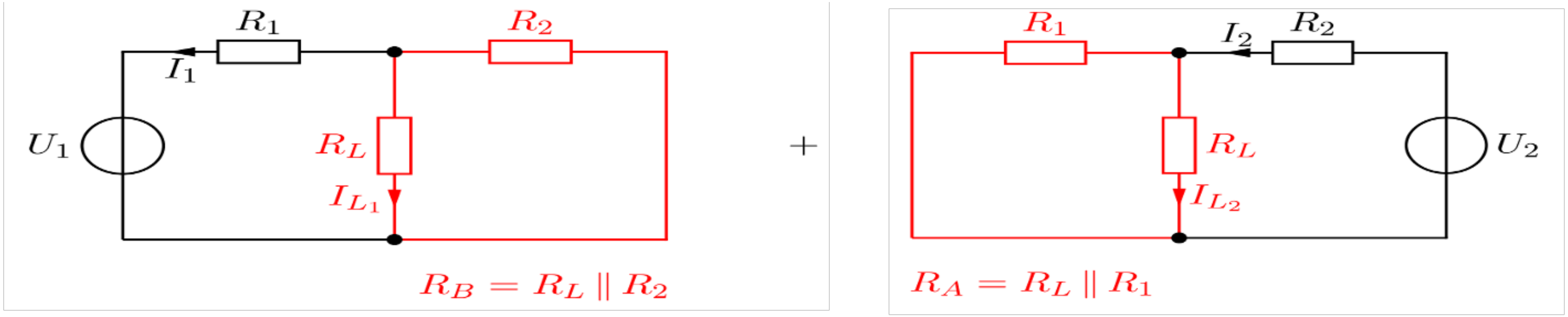

- To fully analyze the net, all but one current or voltage sources are replaced by the internal resistance in turn (here $R_I = 0$) and the resulting partial circuits are analyzed.

$$U_{L_1} = \frac{U_1 R_B}{R_1 + R_B} \qquad R_B = \frac{R_L R_2}{R_L + R_2} \qquad \text{and} \qquad U_{L_2} = \frac{U_2 R_A}{R_1 + R_A} \qquad R_A = \frac{R_L R_1}{R_L + R_1}$$

$$I_{L_1} = \frac{U_{L_1}}{R_L} \qquad \text{and} \qquad I_{L_2} = \frac{U_{L_2}}{R_L}$$

resulting in:

$$I_L = I_{L_1} + I_{L_2} = \frac{U_1 R_2 + U_2 R_1}{R_1 R_2 + R_L (R_1 + R_2)}$$

Solving Linear Networks

A system of linear equations

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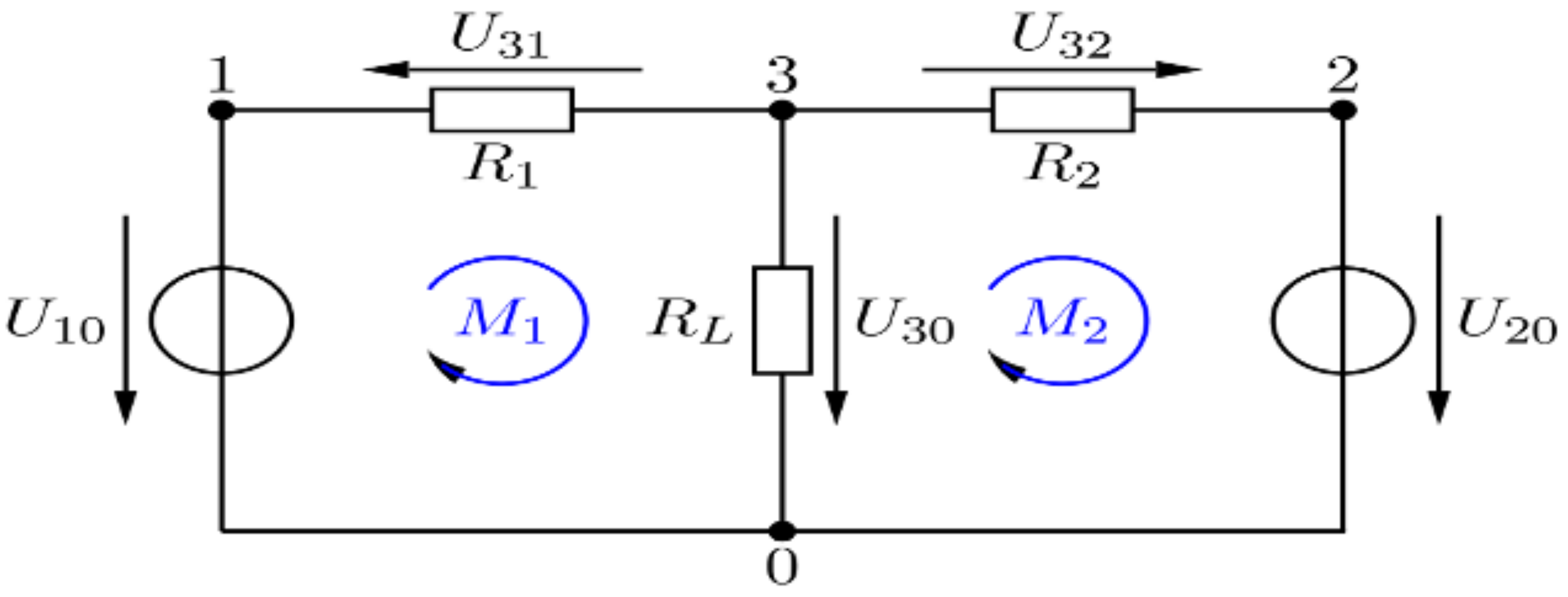

- The question: What is U_{30}, and what is the current through R_L?

$$I_L = \frac{U_{30}}{R_L}$$

- Ansatz: Rigorous use of Kirchhoff's laws for all relevant nodes and loops.

Sum of currents in node 3 = 0:

$$\frac{U_{31}}{R_1} + \frac{U_{32}}{R_2} + \frac{U_{30}}{R_L} = 0$$

Sum of voltages over loops M 1 & 2 = 0:

$$-U_{31} + U_{30} - U_{10} = 0 \quad (M_1)$$
$$-U_{30} + U_{32} + U_{20} = 0 \quad (M_2)$$

3 unknown voltages: U_{30}, U_{31}, $U_{32;}$ 3 independent equations: That can be solved!

Solving Linear Networks

Matrix Notation

- A compact way of writing and solving the system of equations:
  As a matrix

$$\frac{U_{31}}{R_1} + \frac{U_{32}}{R_2} + \frac{U_{30}}{R_L} = 0$$

$$-U_{31} + U_{30} - U_{10} = 0$$

$$-U_{30} + U_{32} + U_{20} = 0$$

To simplify notation:

$$G_i = \frac{1}{R_i}$$

$$\begin{pmatrix} G_L & G_1 & G_2 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} U_{30} \\ U_{31} \\ U_{32} \end{pmatrix} = \begin{pmatrix} 0 \\ U_{10} \\ U_{20} \end{pmatrix}$$

Can be solved using Cramer's rule:

$$U_{30} = \det \begin{pmatrix} 0 & G_1 & G_2 \\ U_1 & -1 & 0 \\ U_2 & 0 & -1 \end{pmatrix} \cdot \frac{1}{\det \begin{pmatrix} G_L & G_1 & G_2 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}}$$

result: $$U_{30} = \frac{G_1 U_{10} + G_2 U_{20}}{G_L + G_1 + G_2}$$

Next Lectures:

Digital - Thursday, November 2

Analog 04 - Chapter 02 - Tuesday, November 7

Electronics for Physicists

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31.10.2023

KIT, Winter 2023/24

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