# Electronics for Physicists

## *Analog Electronics*

*Chapter 2; Lecture 04*

**Frank Simon**
**Institute for Data Processing and Electronics**

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Karlsruhe Institute of Technology

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Institute for Data Processing and Electronics

*Chapter 2*

# Circuits with R, C, L

- AC behavior of R, C, L
- Complex Voltages & Currents
- Filters
- Oscillators

***Overview***

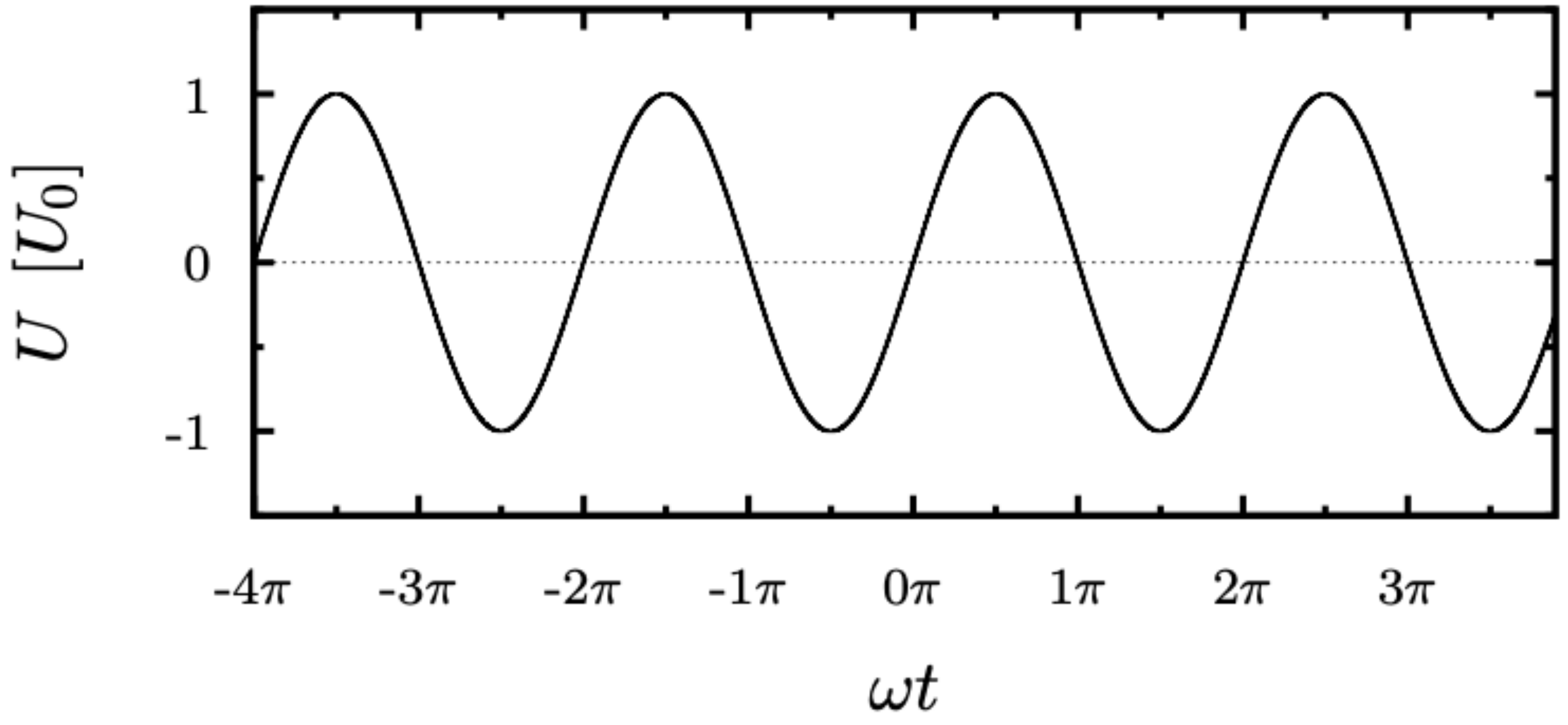
*Chapter 2*

# AC Behavior

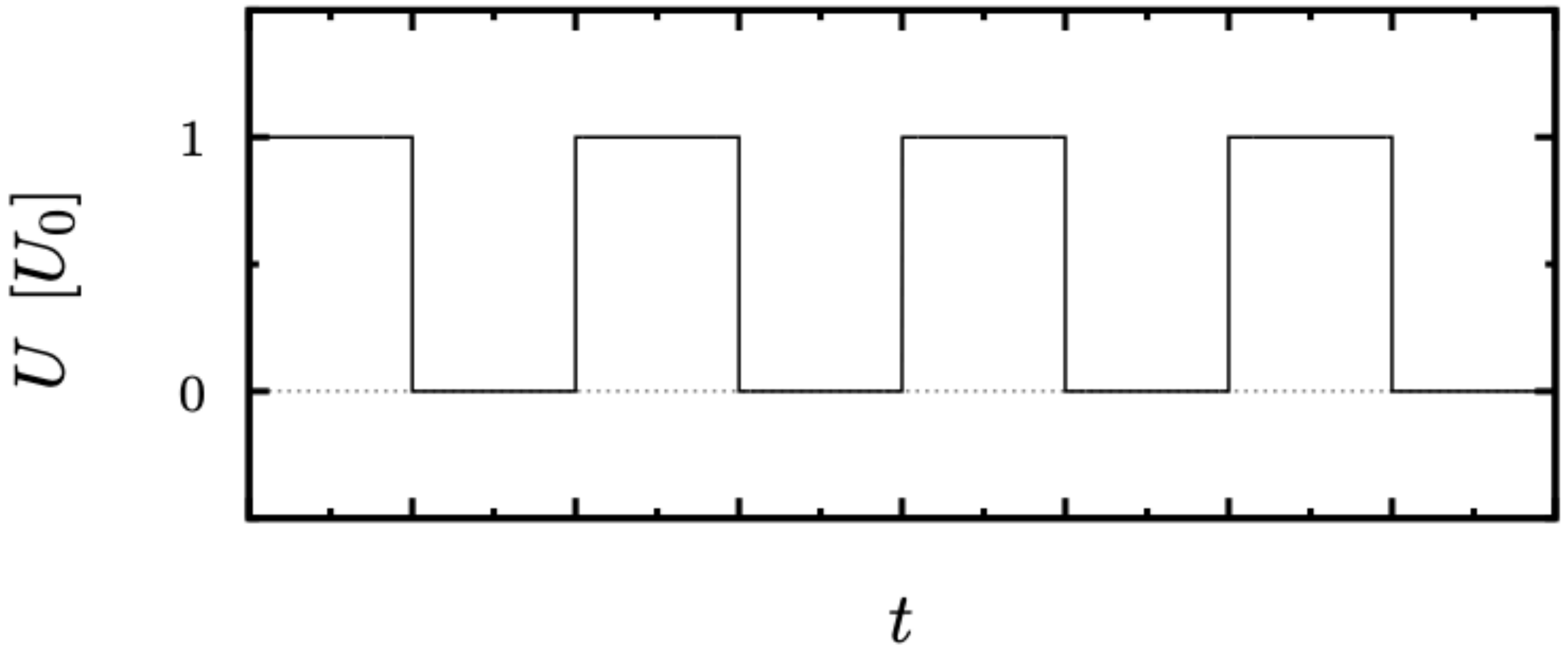
*In: Circuits with R, C, L*

# Waveforms

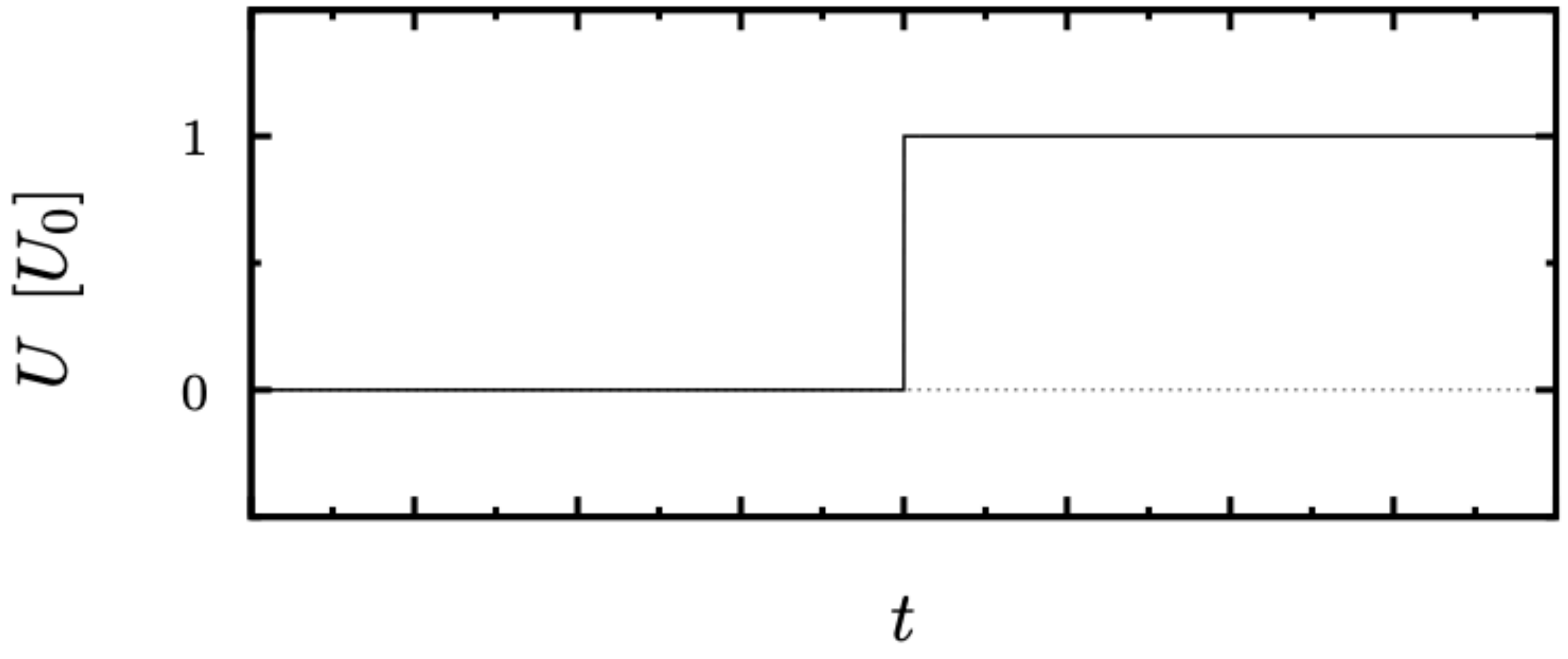
*Signalformen*



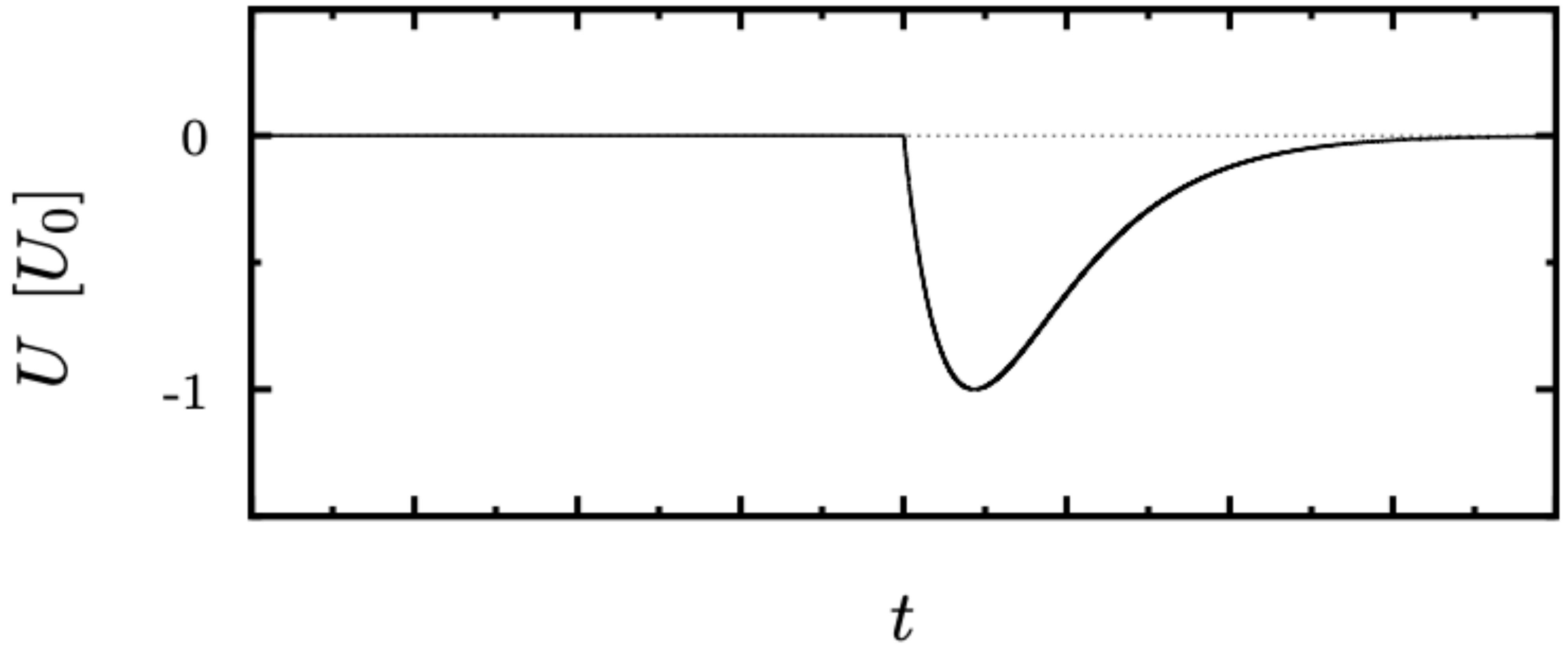

harmonic oscillation




square wave




step function

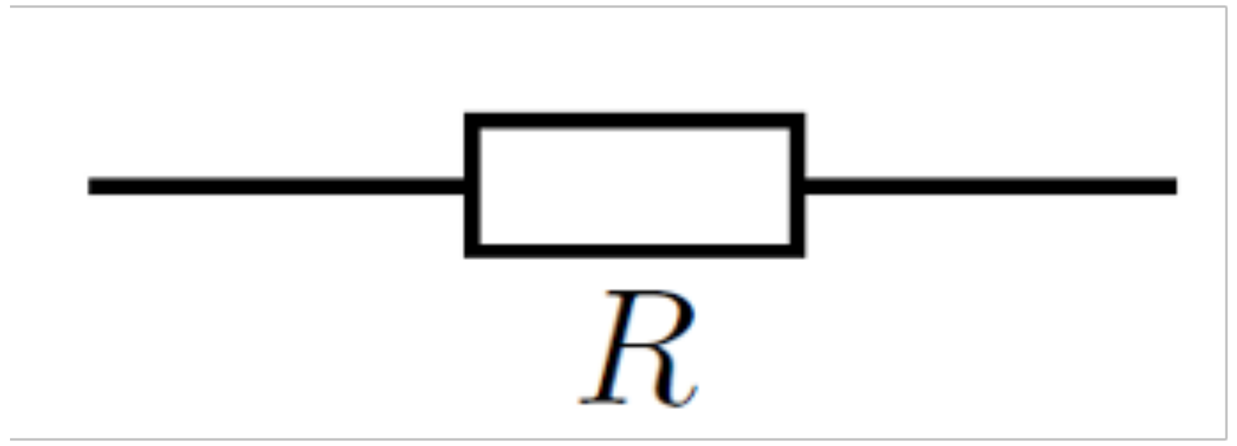



typical detector signal after RC-CR filtering

# Behavior for harmonic Signals

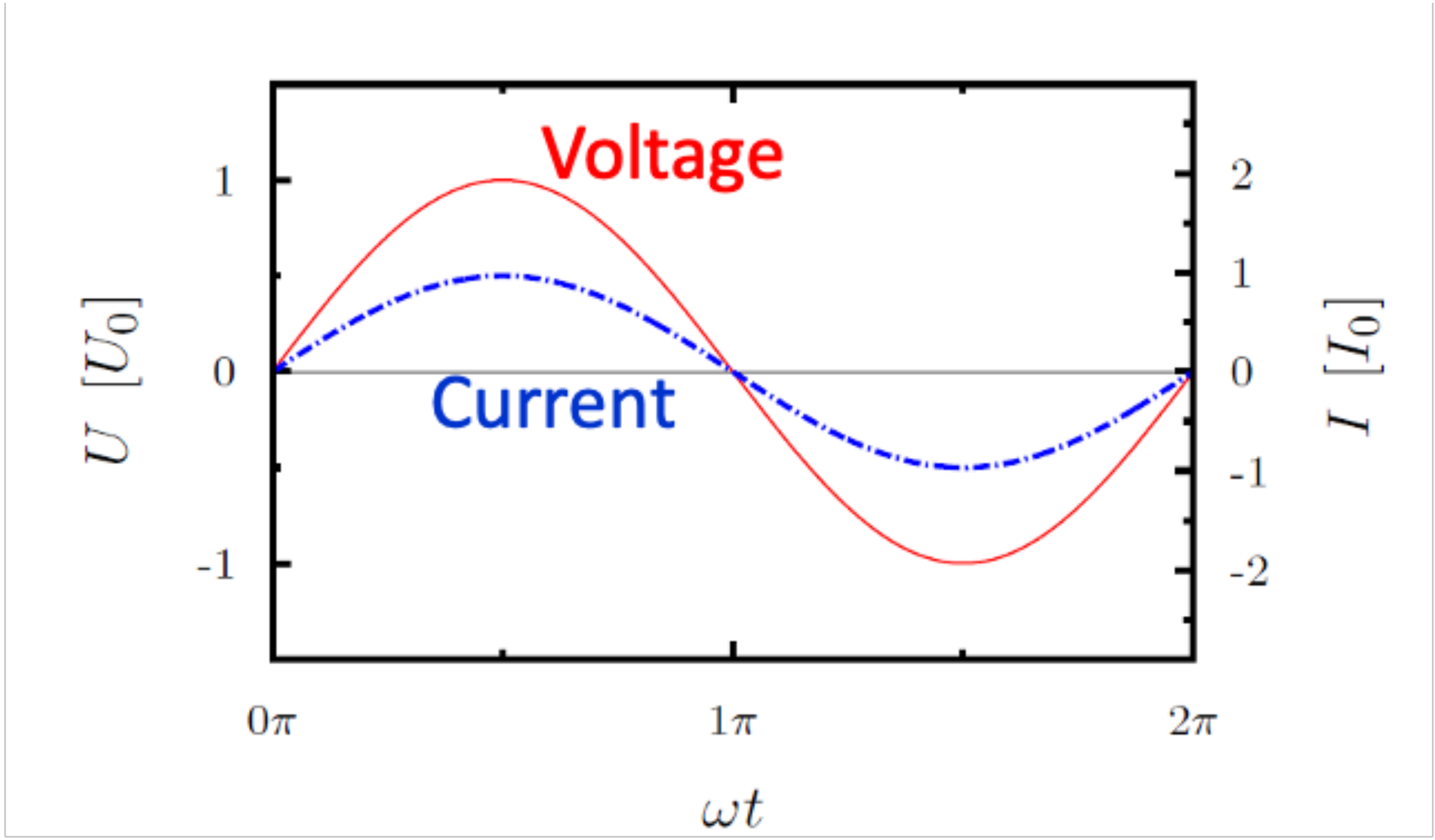
*Resistor*

- Harmonic signal $U(t) = U_0 \sin(\omega t + \varphi)$




$$U(t) = R\,I(t)$$

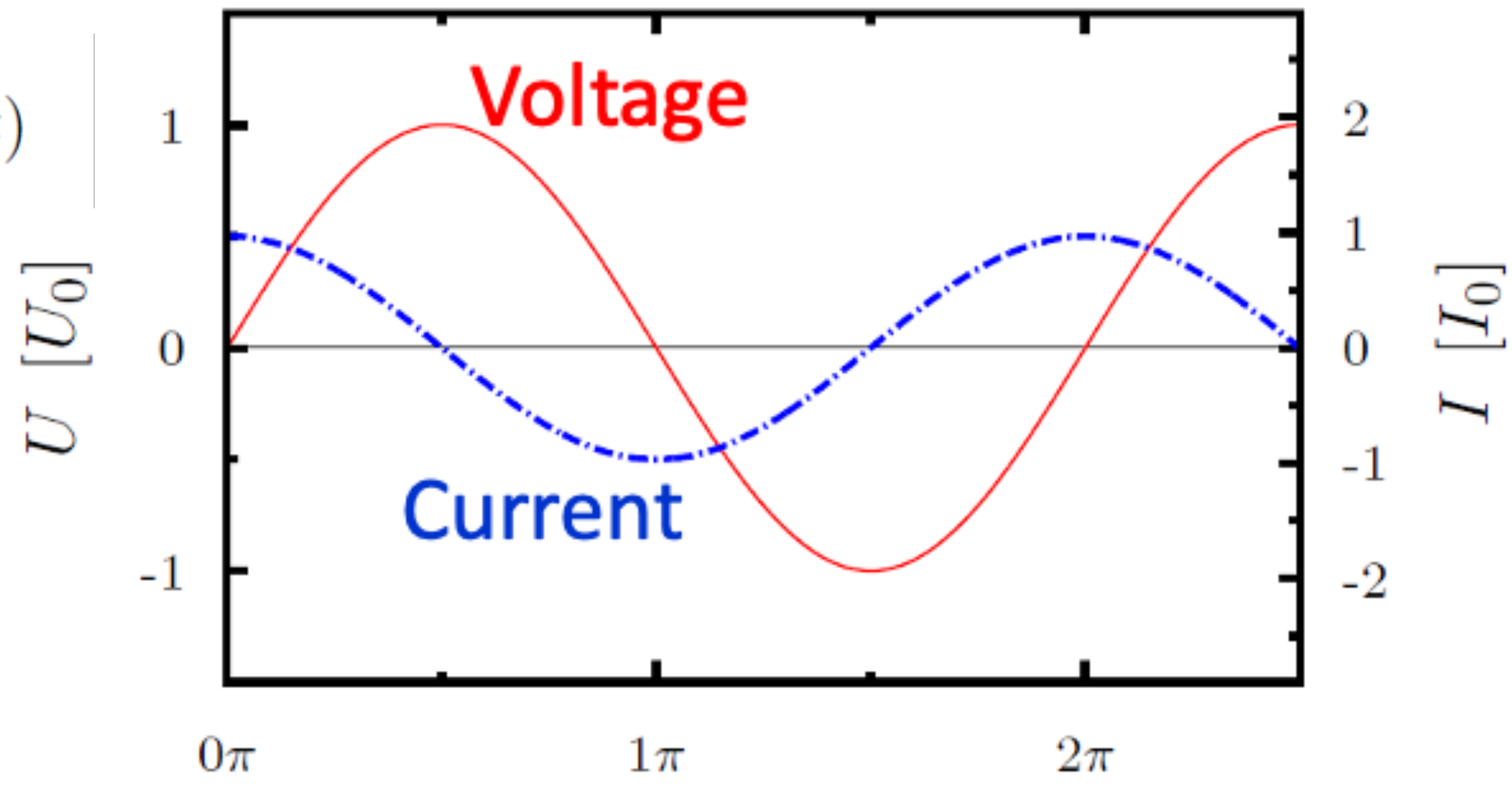
Voltage and current are in phase.



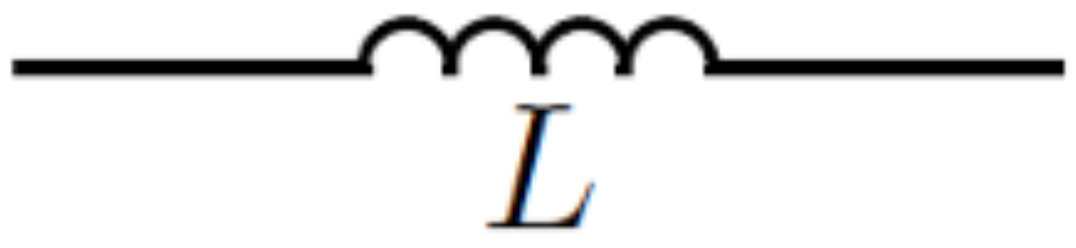
- Harmonic signal $U(t) = U_0 \sin(\omega t + \varphi)$

$$I = C \frac{dU}{dt} \quad \Rightarrow \quad I(t) = U_0 \, \omega \, C \cos(\omega t)$$

90° phase shift:
Voltage follows current!



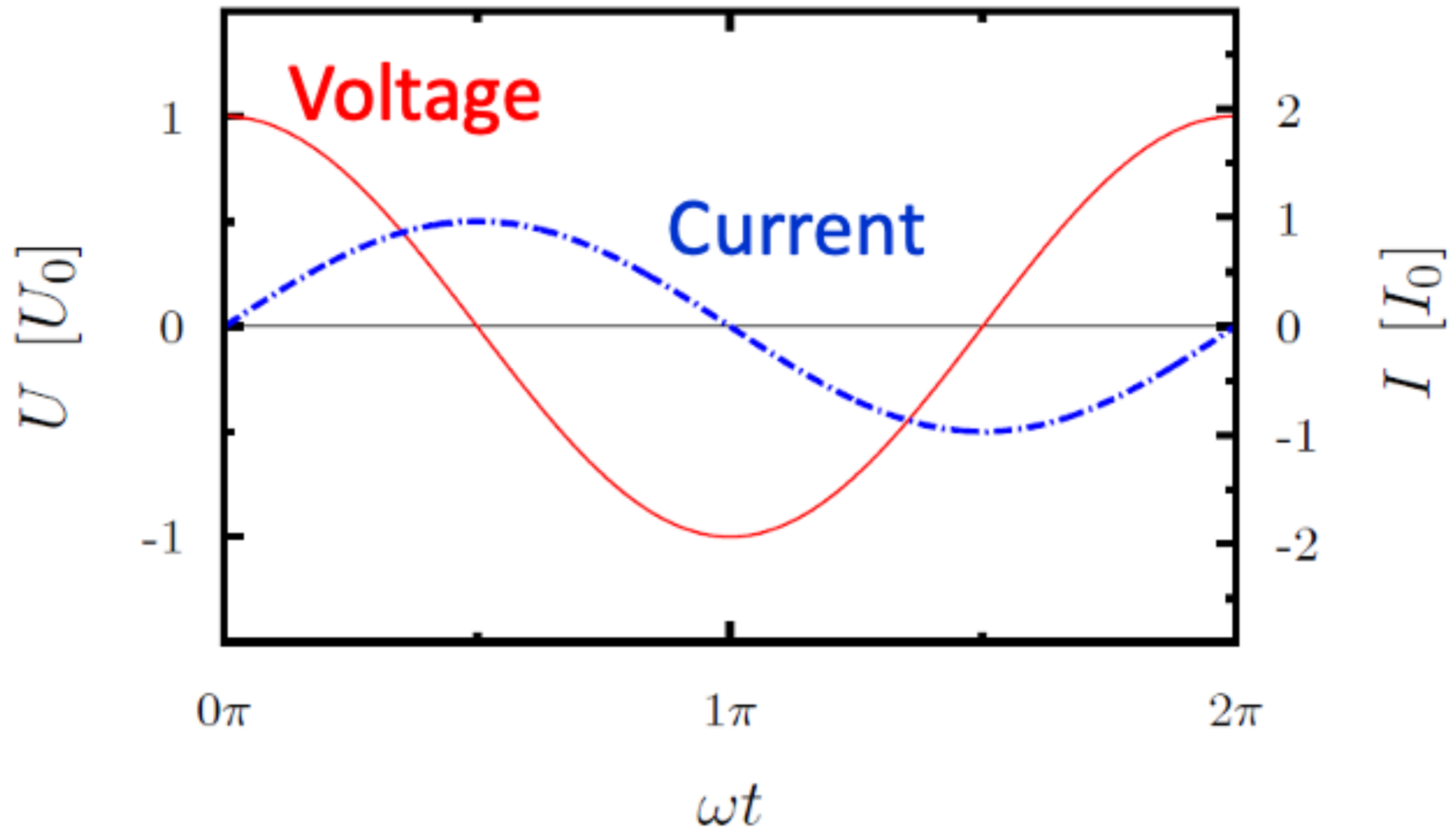
- Harmonic signal: $U(t) = U_0 \sin(\omega t + \varphi)$



$$U = L \frac{dI}{dt}$$

$$I(t) = I_0 \sin(\omega t)$$

=> $U(t) = I_0 \omega L \cos(\omega t)$




90° phase shift:
Current follows voltage.

*Chapter 2*

# Complex Currents and Voltages

*In: Circuits with R, C, L*

# Representation via Complex Numbers

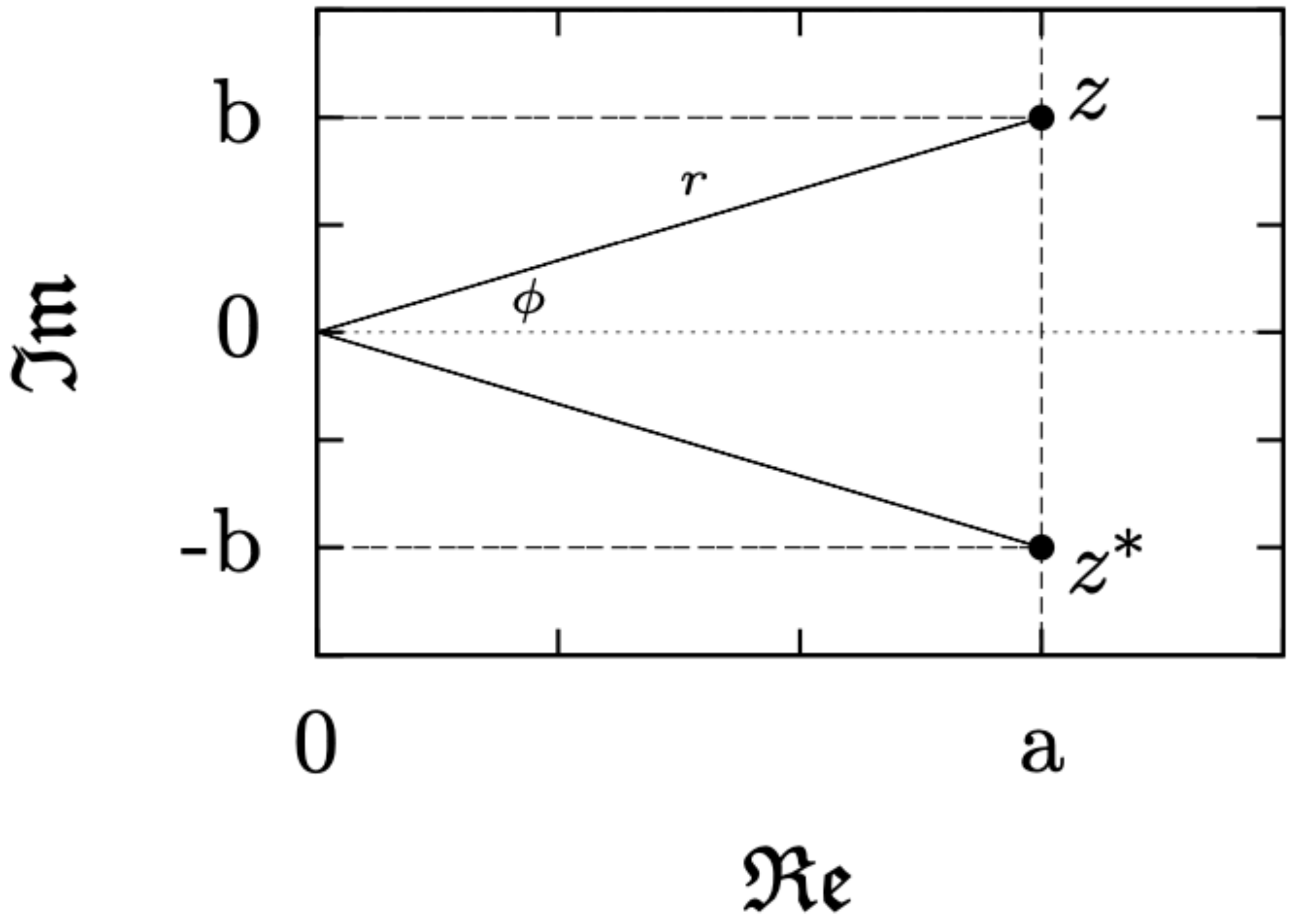
*Time dependence via complex voltages and currents*

- To describe the time dependence it is convenient to use complex numbers:

NB: in EE normally "j" instead of "i" is used for √-1

$$\underline{z} = \mathrm{Re}(\underline{z}) + \mathrm{j}\,\mathrm{Im}(\underline{z}) = a + \mathrm{j}\,b$$

Graphical illustration in complex plane




important relationship for harmonic signals: Euler's formula

$$\mathrm{e}^{\mathrm{j}\,\varphi} = \cos\varphi + \mathrm{j}\,\sin\varphi$$

Complex voltage:

$$\underline{U} = U_0\,\mathrm{e}^{\mathrm{j}\,\omega\,t}$$

# Passive Components in complex Notation

*Summary*

In the time domain (*Zeitbereich*):

Differentiation becomes a multiplication by frequency jω

$$\frac{dF}{dt} \rightarrow j\omega\underline{F}$$

Integration becomes a division by jω

$$\int F dt \rightarrow \frac{1}{j\omega}\underline{F}$$

$$\omega = 2\pi f$$

| | time domain | complex plane |
|---|---|---|
| Resistor | $U = RI$ | $\underline{U} = R\underline{I}$ |
| Capacitor | $I = C\frac{dU}{dt}$ | $\underline{I} = j\omega C\underline{U}$ |
| Inductor | $U = L\frac{dI}{dt}$ | $\underline{U} = j\omega L\underline{I}$ |

# Generalized Ohm's Law

*Verallgemeinertes Ohmsches Gesetz*

- (Complex) impedance Z describing the relationship between complex voltages U and complex currents I

$$\underline{U} = \underline{Z}\,\underline{I}$$

*Impedanz*

Resistor: $\underline{Z}_R = R$

Capacitor: $\underline{Z}_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$

Inductor: $\underline{Z}_L = j\omega L$

Circuits with impedances:

series: $\underline{Z} = \underline{Z}_1 + \underline{Z}_2$

parallel: $\underline{Z} = \left(\frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}\right)^{-1}$

The opposition to alternating currents by capacitors and inductors is also referred to as ***reactance***: No power dissipation (in contrast to resistance)

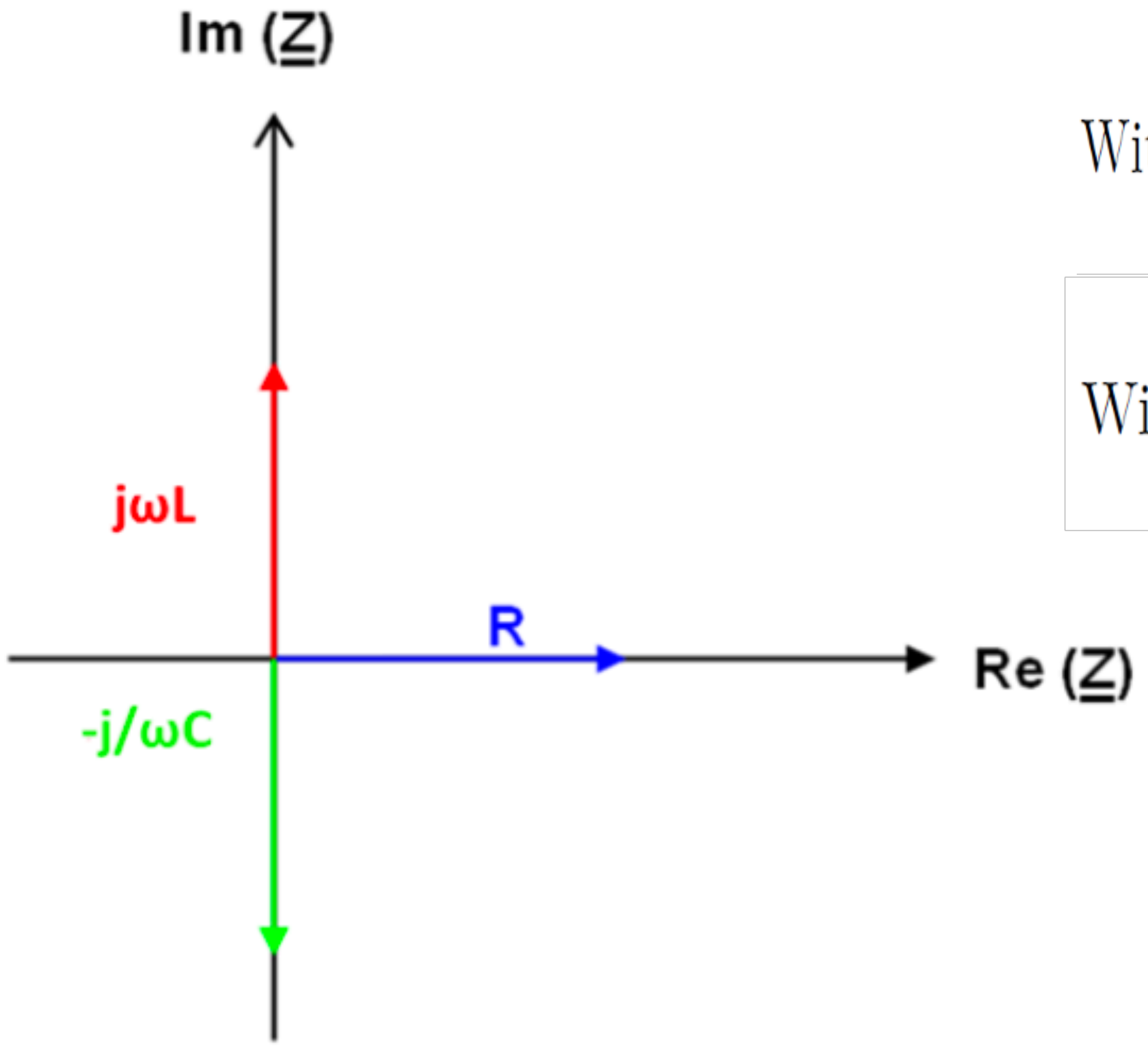
*Blindwiderstand*

# Representation in the Complex Plane

*Darstellung in der komplexen Ebene*

- General remark: The impedance of an ohmic resistor is real, the impedance of capacitors and inductors is imaginary.

Representation of Z in the complex plane:




With $\underline{U} = U_0 \, e^{j\omega t}$ and $\underline{I} = C \frac{d\underline{U}}{dt} \Leftrightarrow \underline{I} = j\omega C U_0 \, e^{j\omega t}$ and $Z_C = \frac{\underline{U}}{\underline{I}} = \frac{1}{j\omega C}$

With $\underline{U} = L \frac{d\underline{I}}{dt} \Leftrightarrow L\underline{I} = \int \underline{U} \, dt = \frac{1}{j\omega} U_0 \, e^{j\omega t}$ and $Z_L = \frac{\underline{U}}{\underline{I}} = j\omega L$

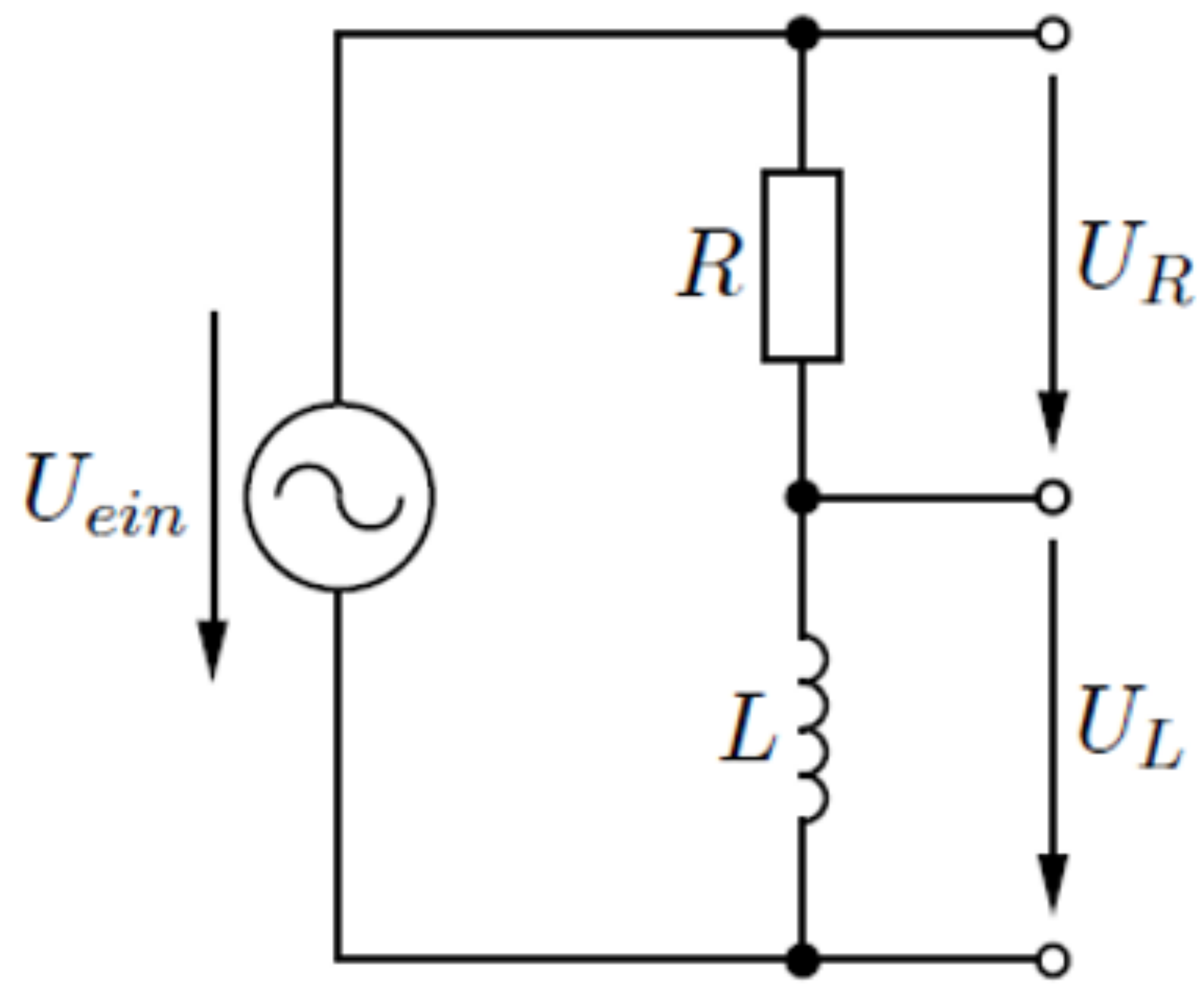
# Frequency Behaviour: Summary

*Frequenzverhalten*

| | $\omega \sim 0$ | $\omega \to \infty$ |
|---|---|---|
| Resistor | $\underline{U} = R\underline{I}$<br>frequency independent | |
| Capacitor<br>$\underline{I} = j\omega C\underline{U}$ | I = 0<br>open circuit | U -> 0<br>short circuit |
| Inductor<br>$\underline{U} = j\omega L\underline{I}$ | U = 0<br>short circuit | I -> 0<br>open circuit |

# Concrete Example: Generalized Ohm's Law

*Resistor and Inductor*




Input voltage

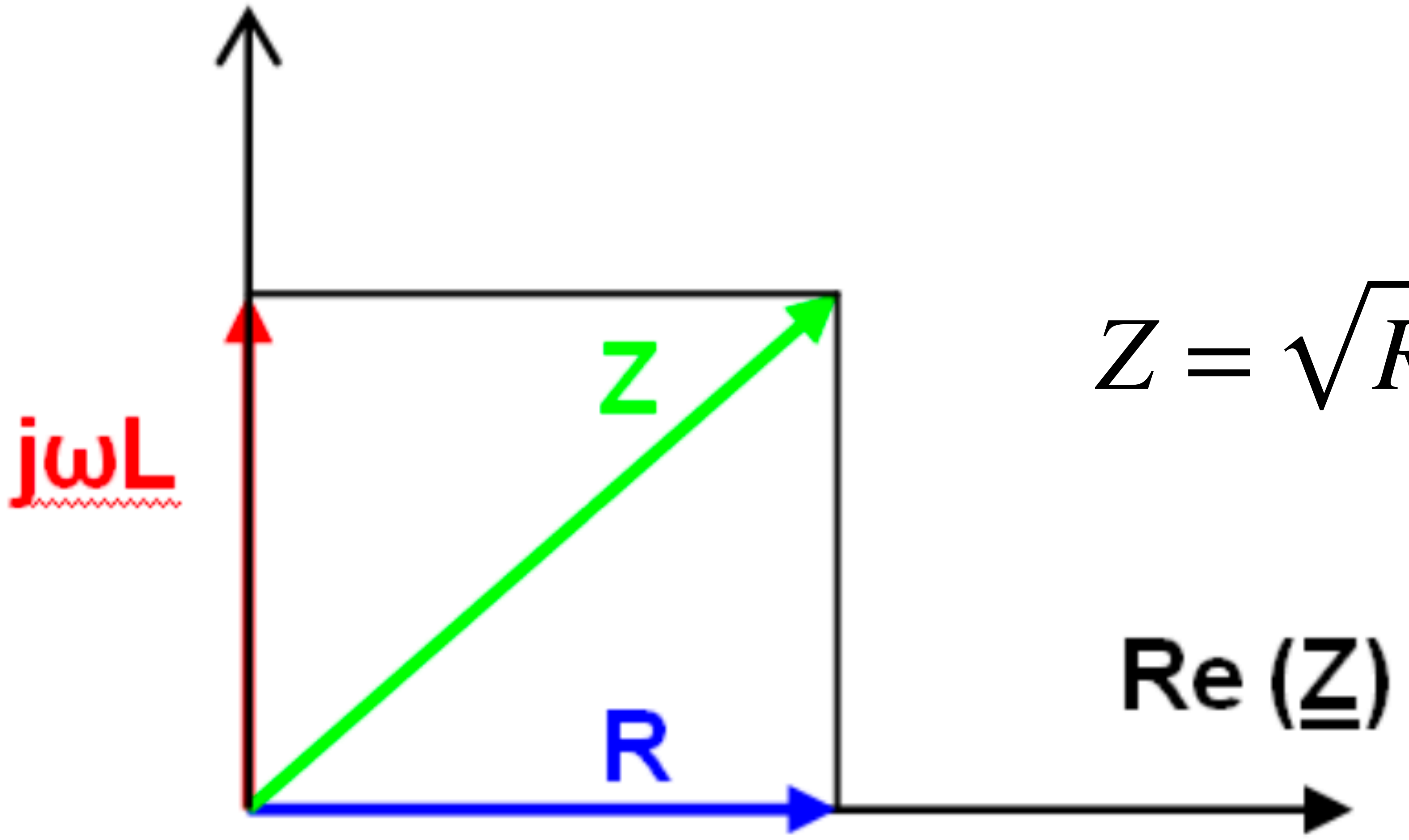
$$\underline{U}_{ein} = U_0 \, e^{j \omega t}$$

The problem:

What are $\underline{U}_R$ and $\underline{U}_L$ ?

and: What are the real voltages?

Ansatz: Voltage divider




$$Z = \sqrt{R^2 + \omega^2 L^2}$$

# Concrete Example: Generalized Ohm's Law

*Solution*

$$\underline{U}_L = \underline{U}_{ein} \frac{\underline{Z}_L}{\underline{Z}_R + \underline{Z}_L} \qquad \underline{U}_R = \underline{U}_{ein} \frac{\underline{Z}_R}{\underline{Z}_R + \underline{Z}_L}$$

↳ Given by generalized Ohm's law

**Solution:**

$$\underline{Z} \equiv \underline{Z}_R + \underline{Z}_L = R + j\omega L \Rightarrow |\underline{Z}| = Z = \sqrt{R^2 + \omega^2 L^2}$$

Impedance / "Scheinwiderstand"

$$\Rightarrow \varphi = \arctan\left(\frac{\omega L}{R}\right)$$

$$\underline{Z} = \sqrt{R^2 + \omega^2 L^2} \cdot e^{j \arctan\left(\frac{\omega L}{R}\right)}$$

$$\Rightarrow \underline{U}_R = U_0 \cdot e^{j\omega t} \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \cdot e^{-j\varphi}$$

($R \leftarrow \underline{Z}_R$)

$$= U_0 \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \cdot e^{j(\omega t - \varphi)}$$

**Info complex numbers**

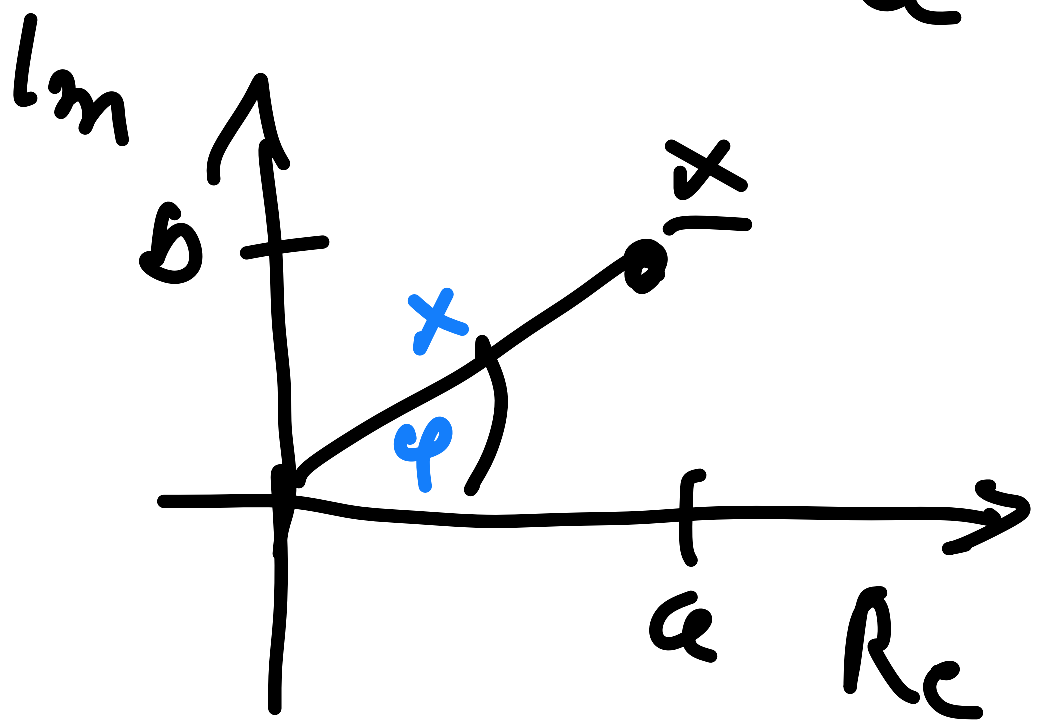
$$\underline{x} = a + jb$$

$$\underline{x} = x \cdot e^{j\varphi}$$

↑ real amplitude

$$x = \sqrt{a^2 + b^2}$$

$$\tan\varphi = \frac{b}{a}$$




**Euler's formula**

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

# Concrete Example: Generalized Ohm's Law

*Solution*

Real voltage given by real part of $\underline{U}_R$

$$\underline{U}_R = U_0 \cdot \frac{R}{\sqrt{R^2+\omega^2 L^2}} \cdot e^{j(\omega t - \varphi)} \Rightarrow U_R = U_0 \cdot \frac{R}{\sqrt{R^2+\omega^2 L^2}} \cdot \cos(\omega t - \varphi)$$

$$= \quad '' \quad \cdot \cos\left(\omega t - \arctan\left(\frac{\omega L}{R}\right)\right)$$

Same for $U_L$:

$$\underline{U}_L = U_0 \cdot e^{j\omega t} \cdot \frac{j\omega L}{\sqrt{R^2+\omega^2 L^2}} \cdot e^{-j\varphi}$$

NB: Multiplication with $j \hat{=}$ Phase shift by $\pi/2$ in complex plane

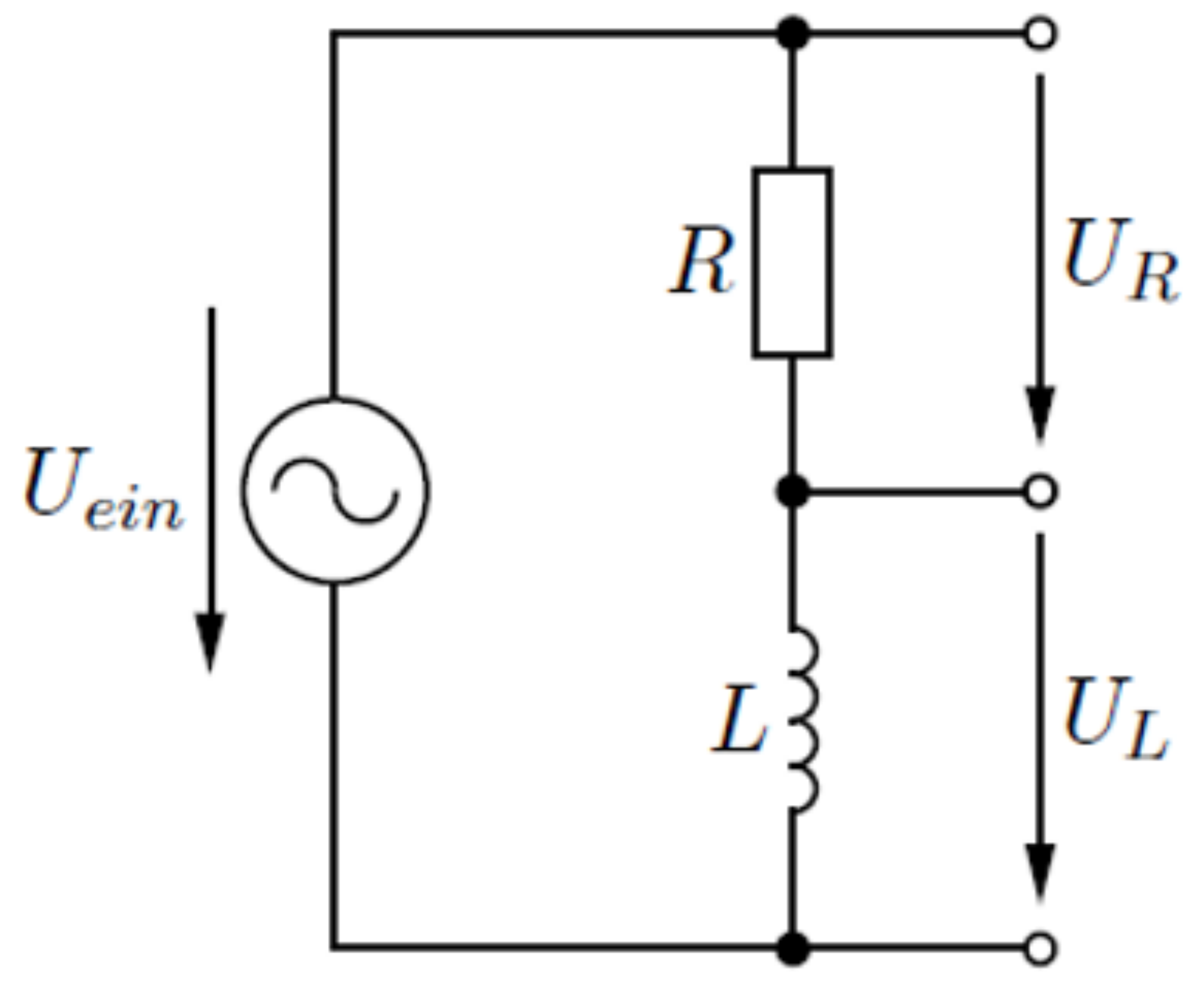
$$= U_0 \cdot e^{j\omega t} \cdot \frac{\omega L}{\sqrt{R^2+\omega^2 L^2}} \cdot e^{-j\varphi + \frac{\pi}{2}} = U_0 \cdot \frac{\omega L}{\sqrt{R^2+\omega^2 L^2}} \cdot e^{j(\omega t - \varphi + \frac{\pi}{2})}$$

results in: $$U_L = U_0 \cdot \frac{\omega L}{\sqrt{R^2+\omega^2 L^2}} \cdot \cos\left(\omega t - \arctan\left(\frac{\omega L}{R}\right) + \frac{\pi}{2}\right)$$

$\Rightarrow U_L$ 90° ahead of $U_R$ !

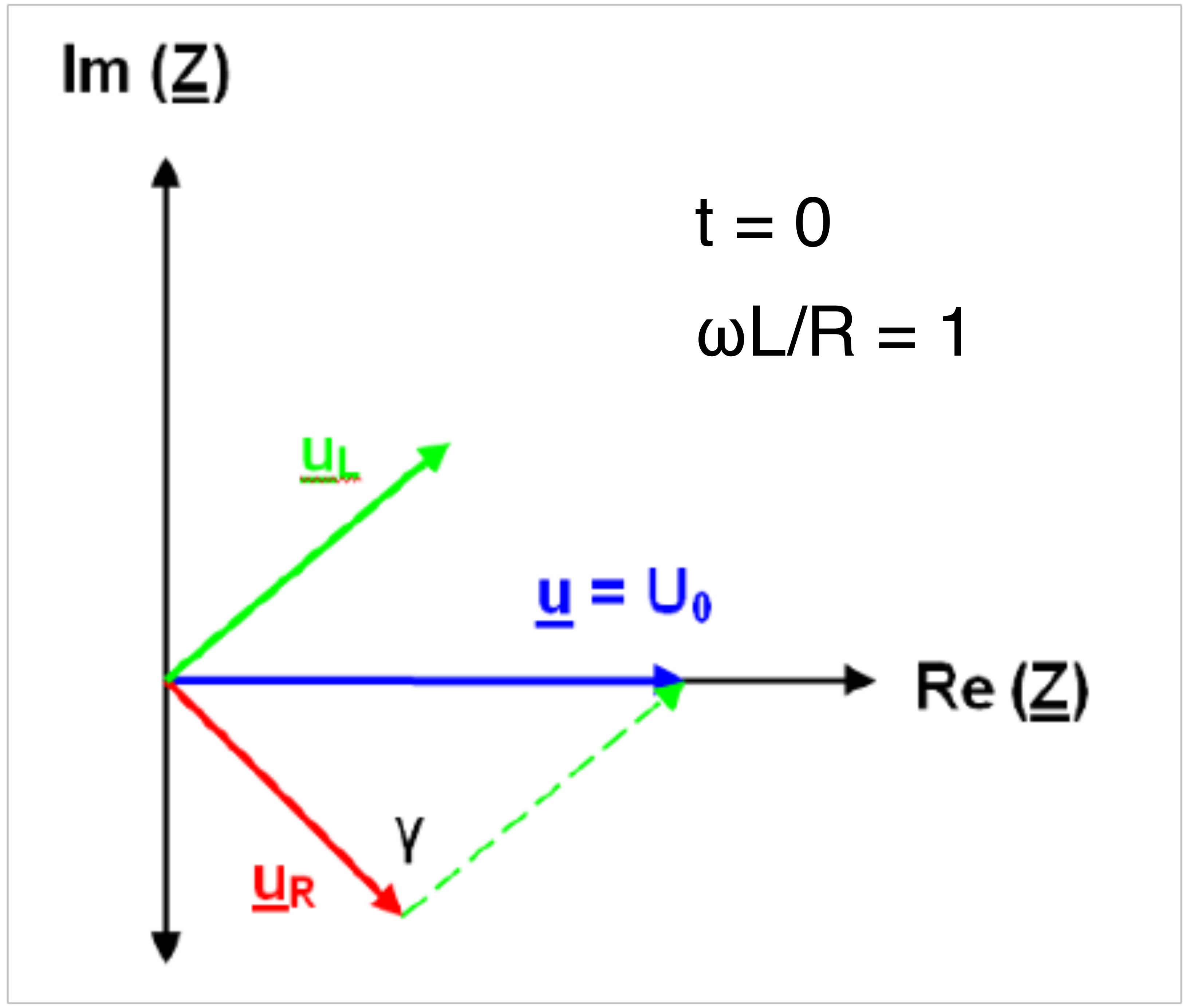
# Concrete Example: Generalized Ohm's Law

*Resistor and Inductor: Result*




$$U_R = U_0 \frac{R}{Z} \cos\left[\omega t - \arctan\left(\frac{\omega L}{R}\right)\right]$$

$$U_L = U_0\, \omega \frac{L}{Z} \cos\left[\omega t - \arctan\left(\frac{\omega L}{R}\right) + \frac{\pi}{2}\right]$$

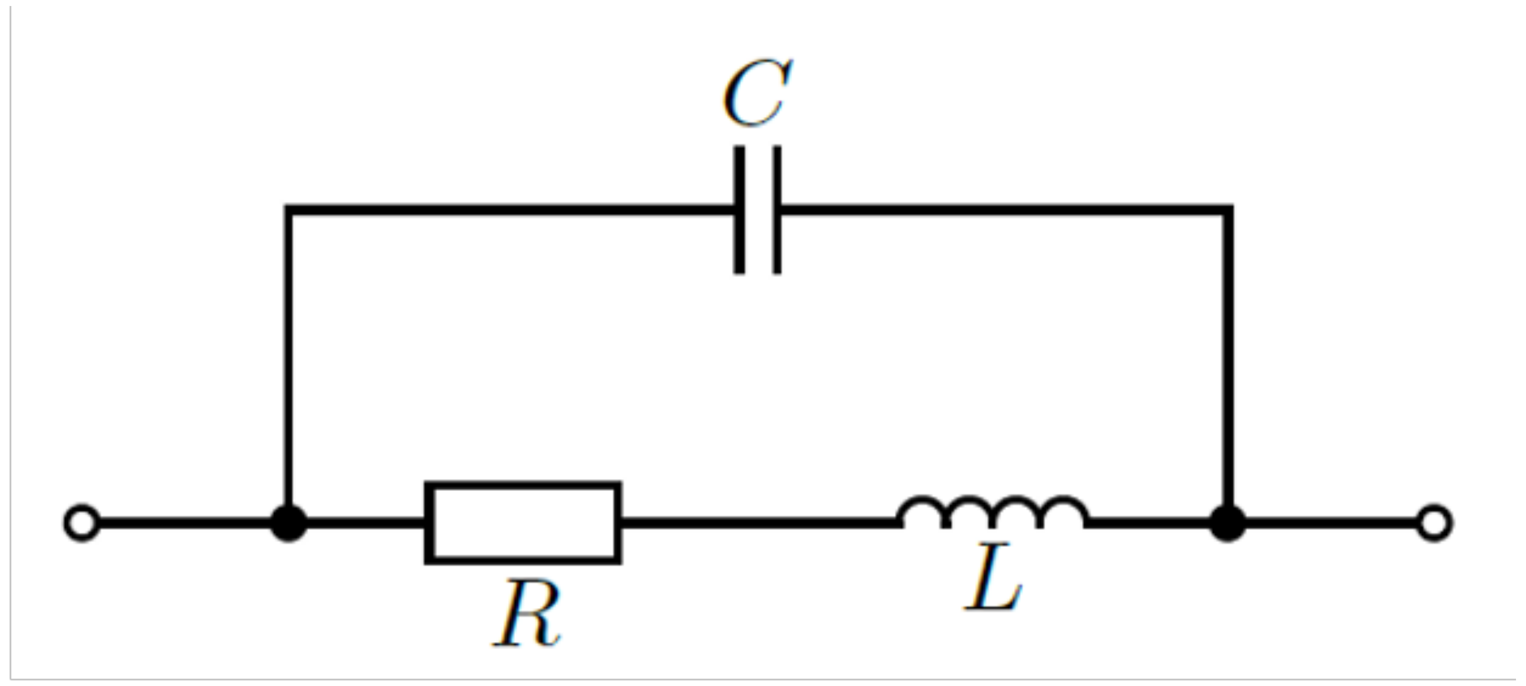


# Why this matters: Real Components

*Short overview*

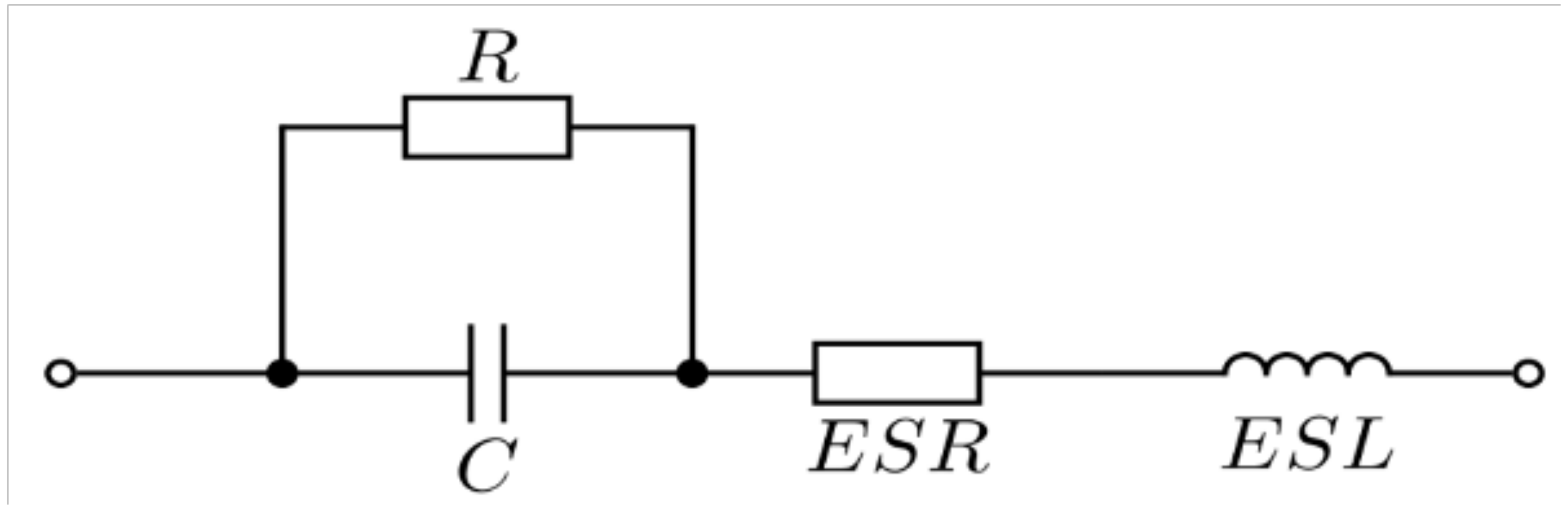
- Real components never behave like ideal elements:

Example resistor:




equivalent circuit
(*Ersatzschaltbild*)

Example capacitor:




ESR: equivalent series resistance
ESL: equivalent series inductance

Consequence: frequency dependent behavior of real electronics components beyond naive expectation.

# Exampe: Compensated Voltage Divider

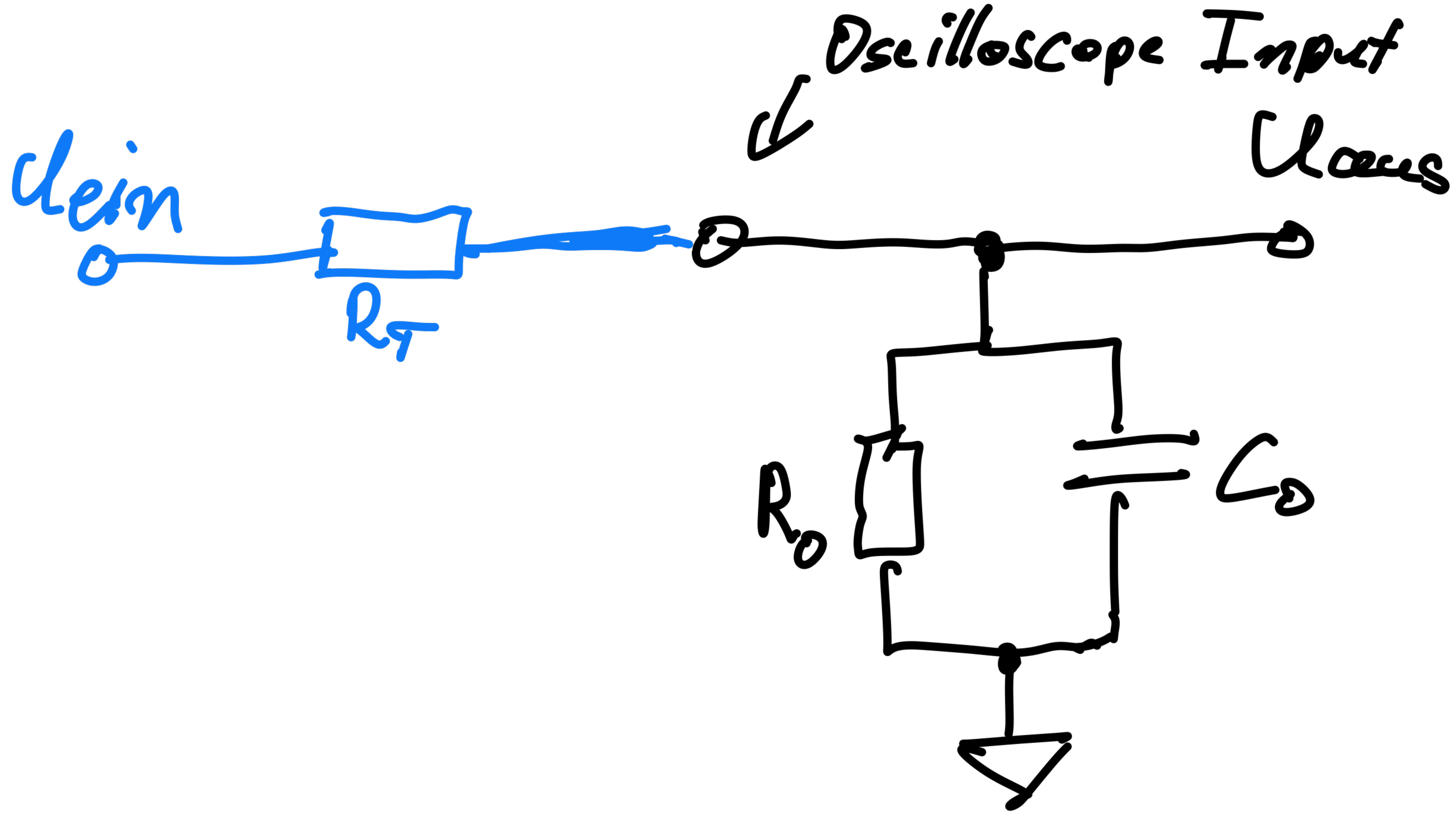
*Oscilloscope Probe*



- Typical application: Oscilloscope to extend the measurement range

Practical Course!

Naive approach: 10:1 Voltage divider with two resistors: x10 in dynamic range. (One of the resistors is the input resistor of the oscilloscope, typ. 1 MΩ or 50 Ω, depending on use case.)




But: Capacitance (typ. ~ 100 pF) results in a frequency dependence, which is modified by the additional resistor: Distortion of the signal!

# Exampe: Compensated Voltage Divider

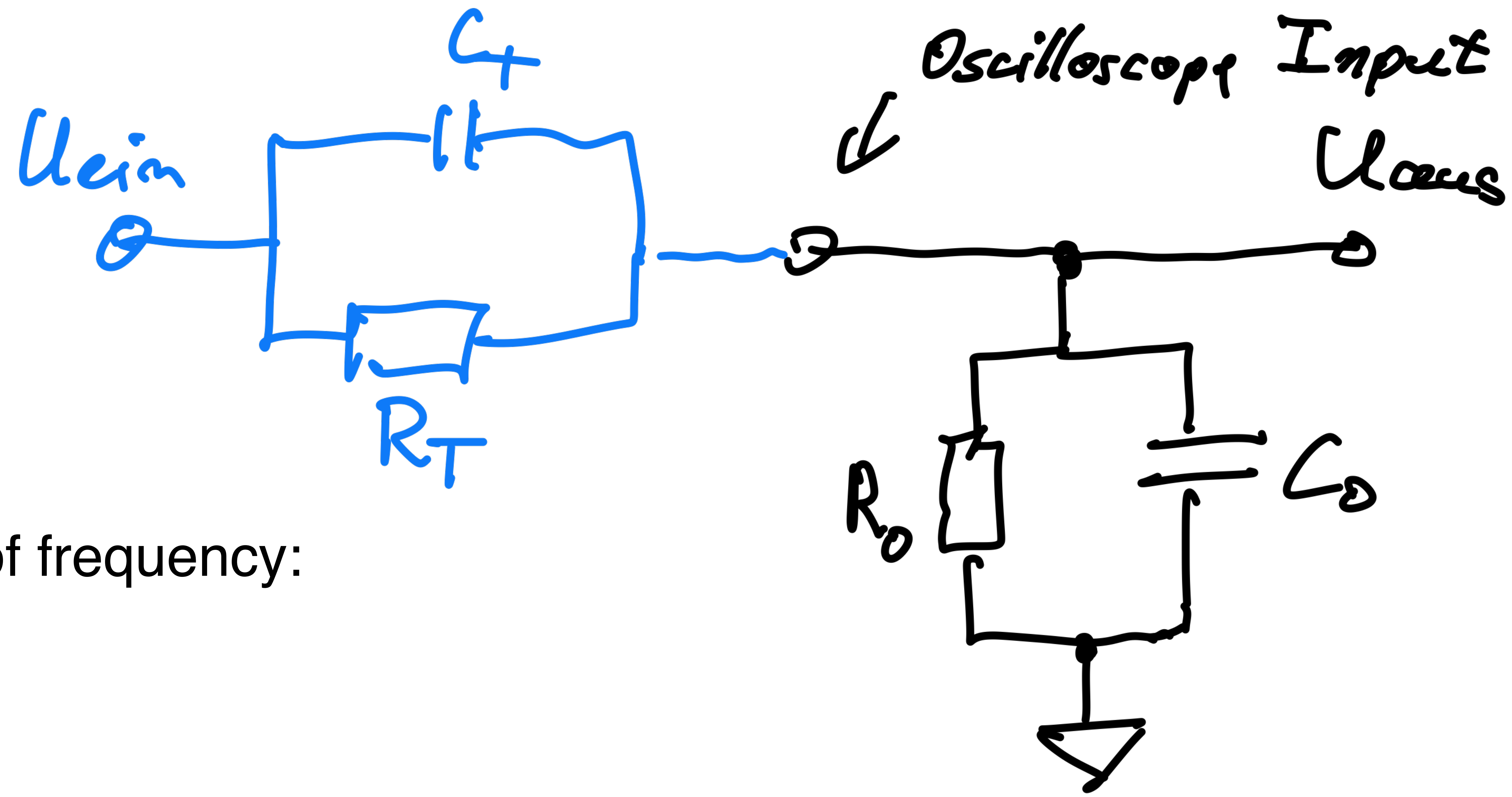
*Oscilloscope Probe*

Karlsruhe Institute of Technology

- Typical application: Oscilloscope to extend the measurement range

Practical Course!

Die solution: Probe with a parallel circuit of resistor and capacitance, which is chosen based on the input impedance of the oscilloscope.




$C_T$ has to be chosen such that $U_{ein}/U_{aus}$ is frequency independent.

Result:

$R_O C_O = R_T C_T$, and with that $C_T = 1/9 * C_O$

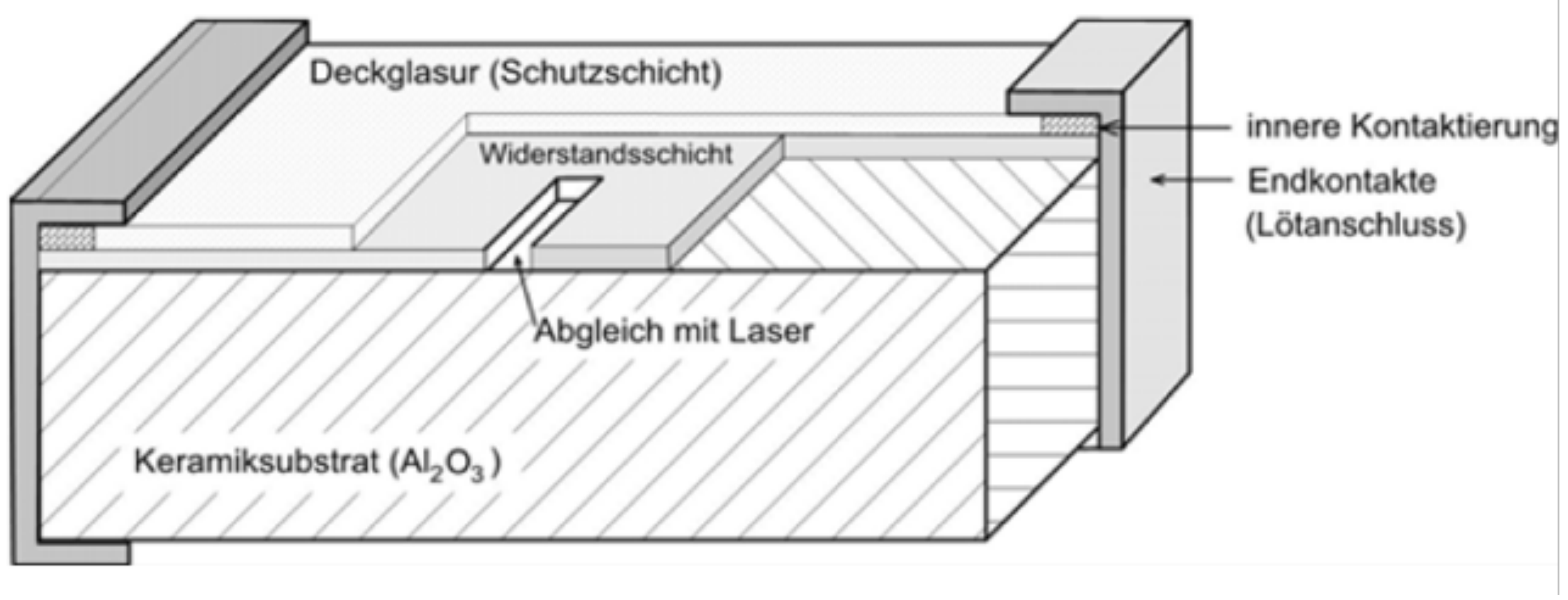
Provides 10:1 voltage division independent of frequency:

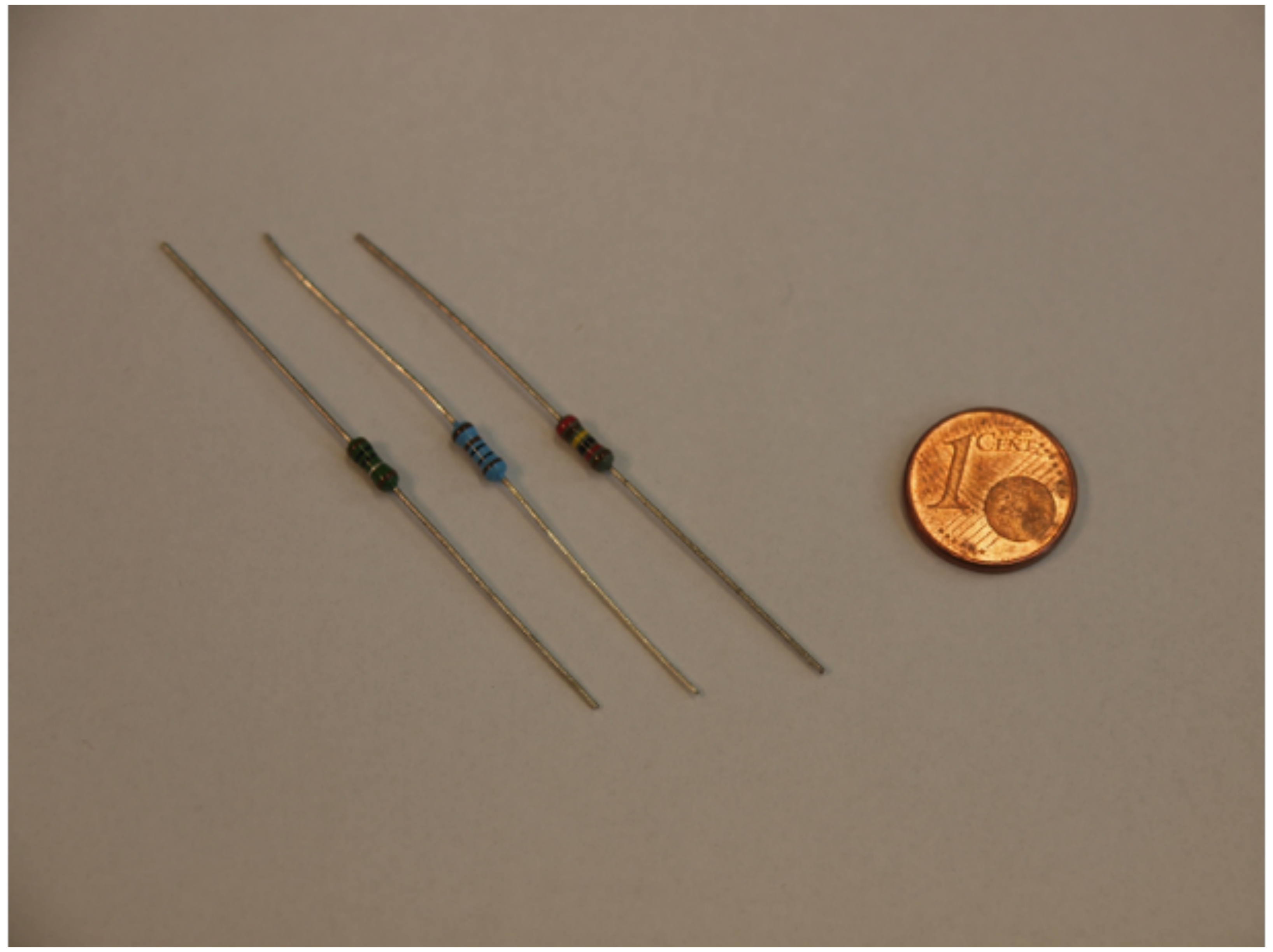
$$\frac{U_{ein}}{U_{aus}} = \frac{U_{ein}}{U_{aus}} = 1 + \frac{R_T}{R_O}$$

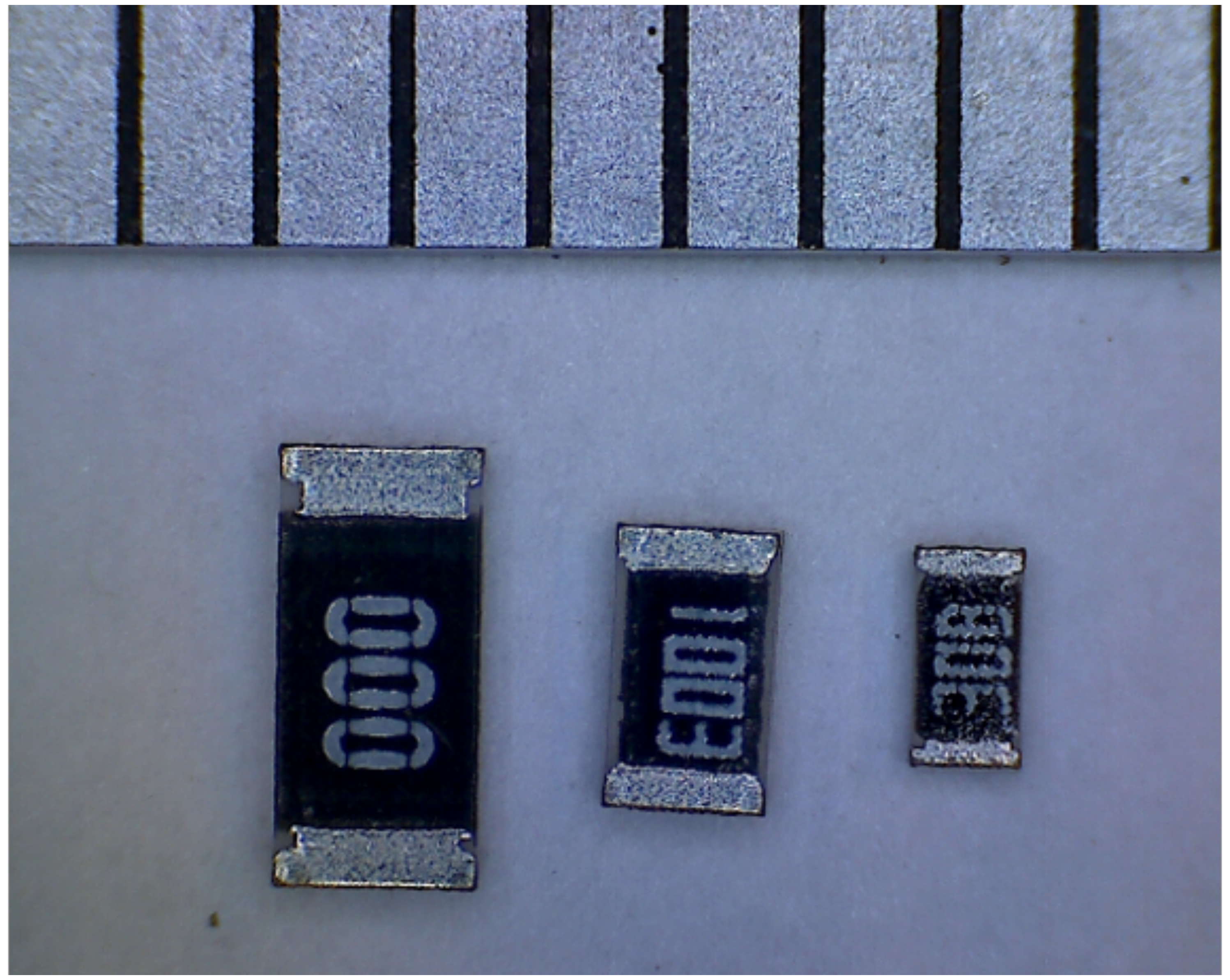
# Real Components: Resistors

*Some more details*

- Typical parameters to consider:
  - Maximum ratings
  - Precision
  - Temperature stability
  - Types and sizes
  - HF properties
  - Costs



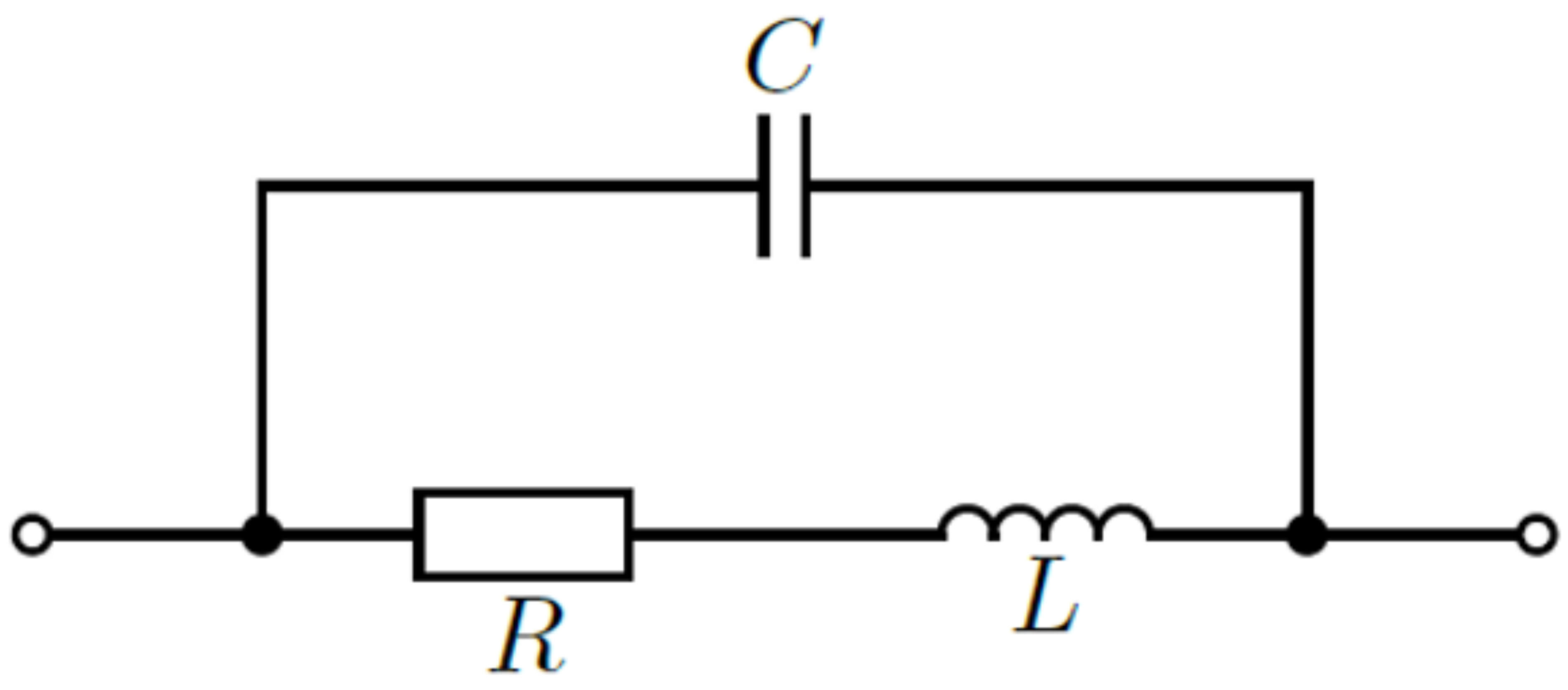


SMD components

"**s**urface **m**ounted **d**evice"

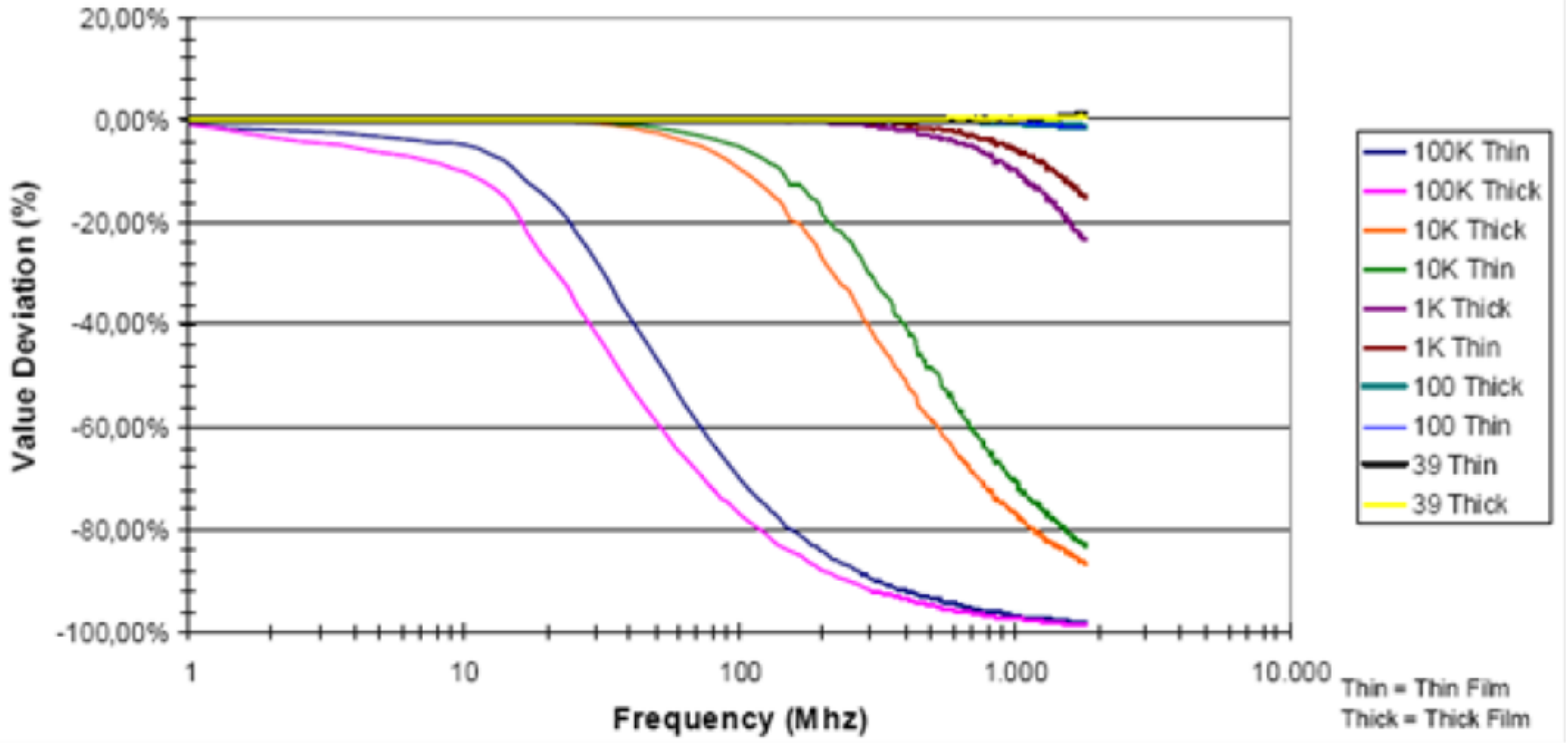
# Real Components: Resistors

*Some more details*




frequency behavior -

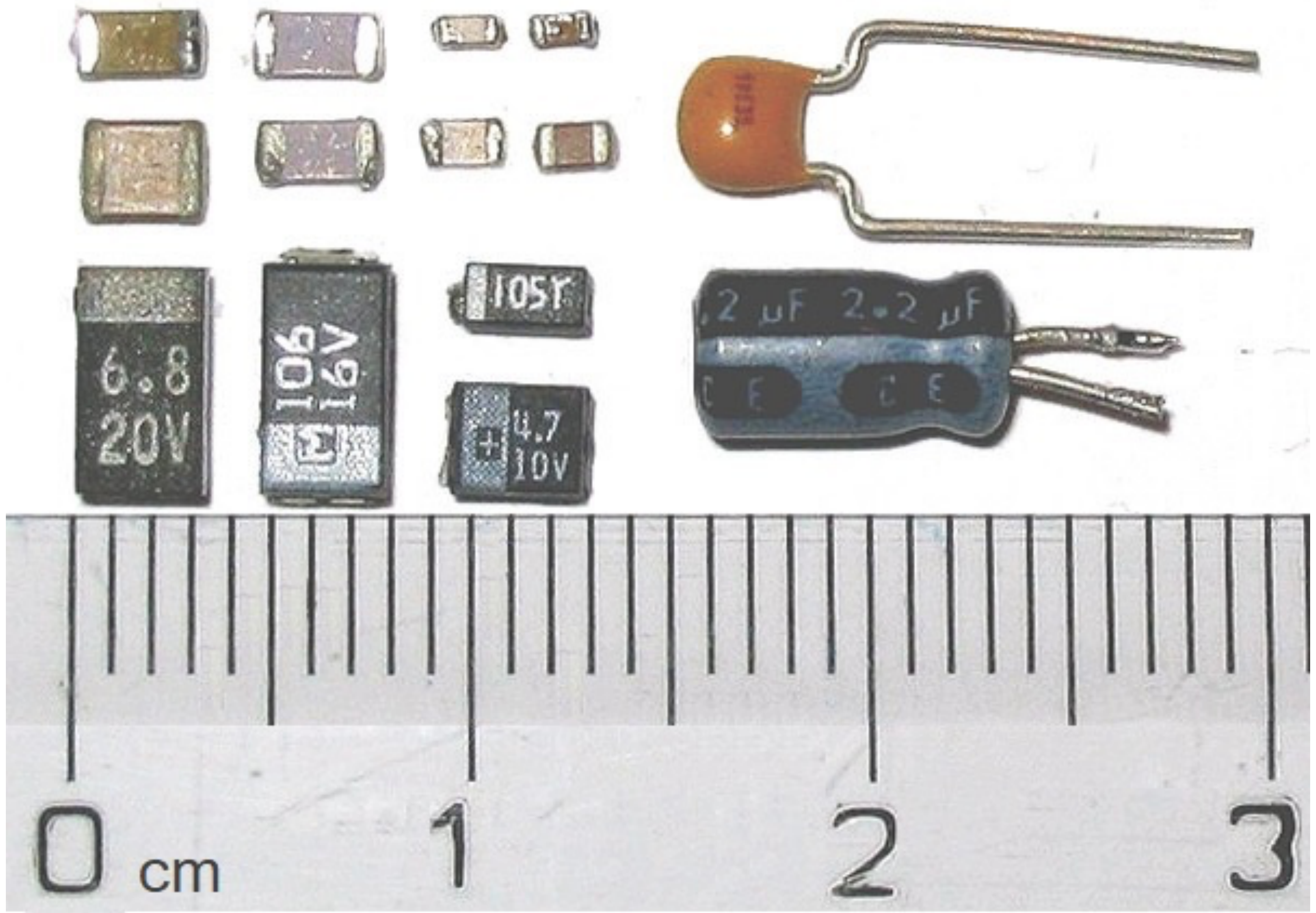
some random examples



# Real Components: Capacitors

*Some more details*

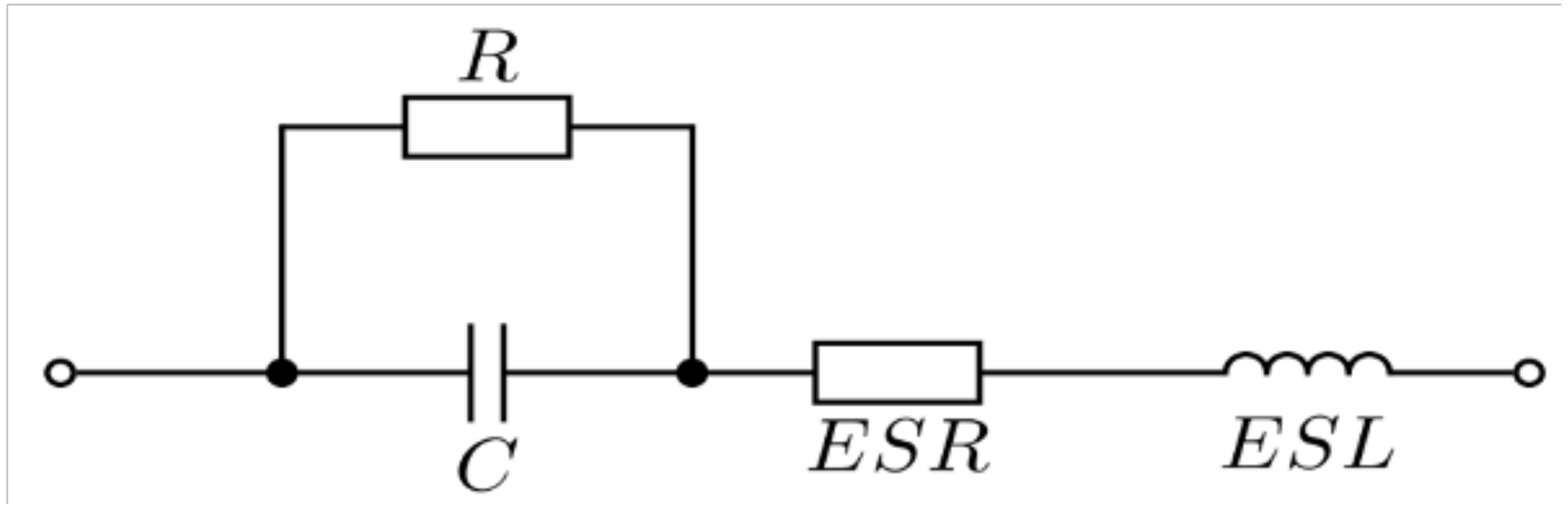
- Different types and form factors
  - Different dielectrics in use - ceramics, liquid,… with dielectric constants ranging typically from a few 10 to a few 100

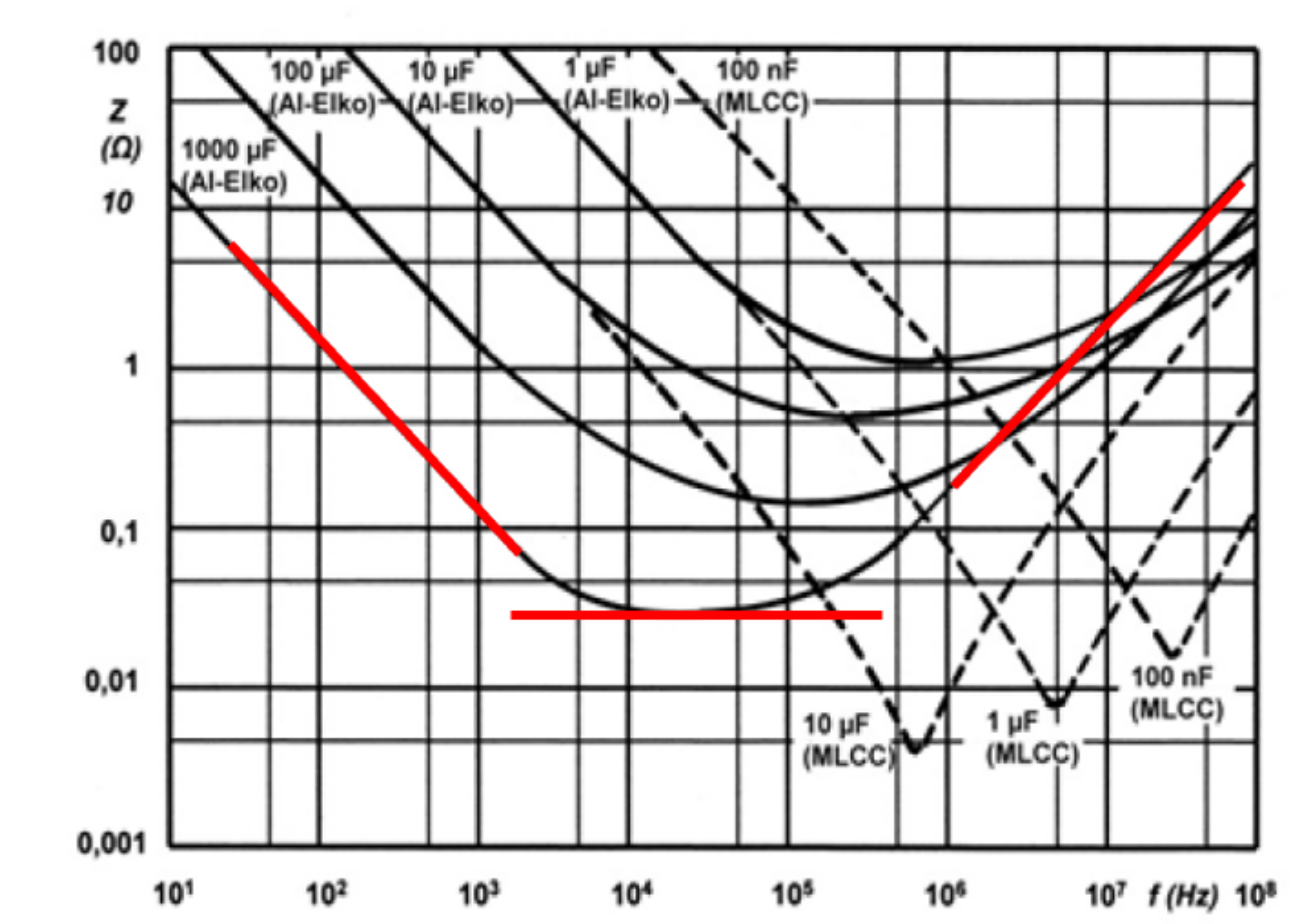


# Real Components: Capacitors

*Some more details*

Frequency behavior - can be understood via equivalent circuit




- For small ω, the capacitive term 1/ωC dominates.
- For large ω, the term ωL dominates.
- Around the natural frequency behavior primarily resistive ($\omega_0 = 1 / \mathrm{Sqrt}(\, ESL\ C\,)$ ).
- The frequency response of a capacitor is often more important than the exact value of its capacitance.

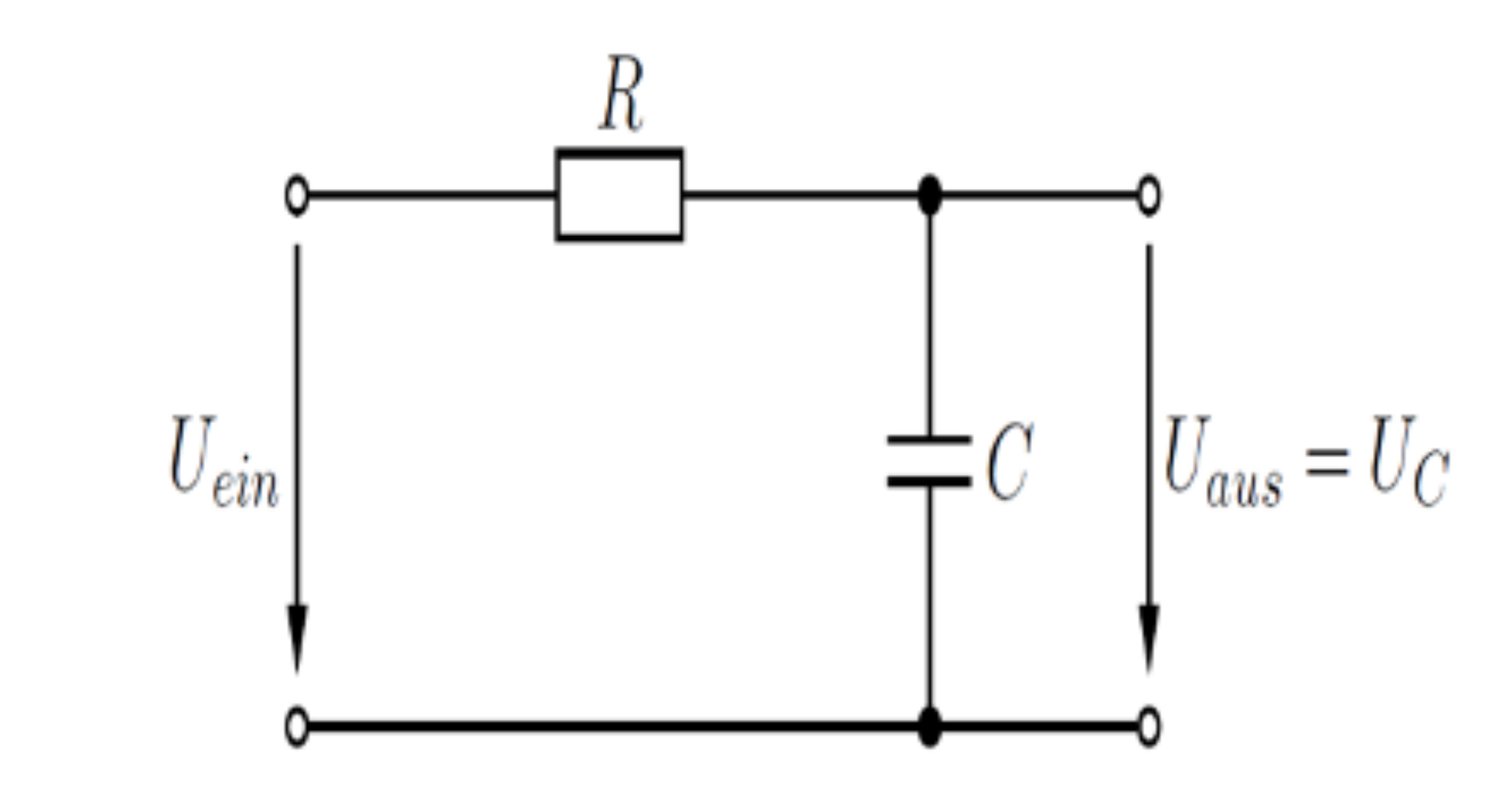
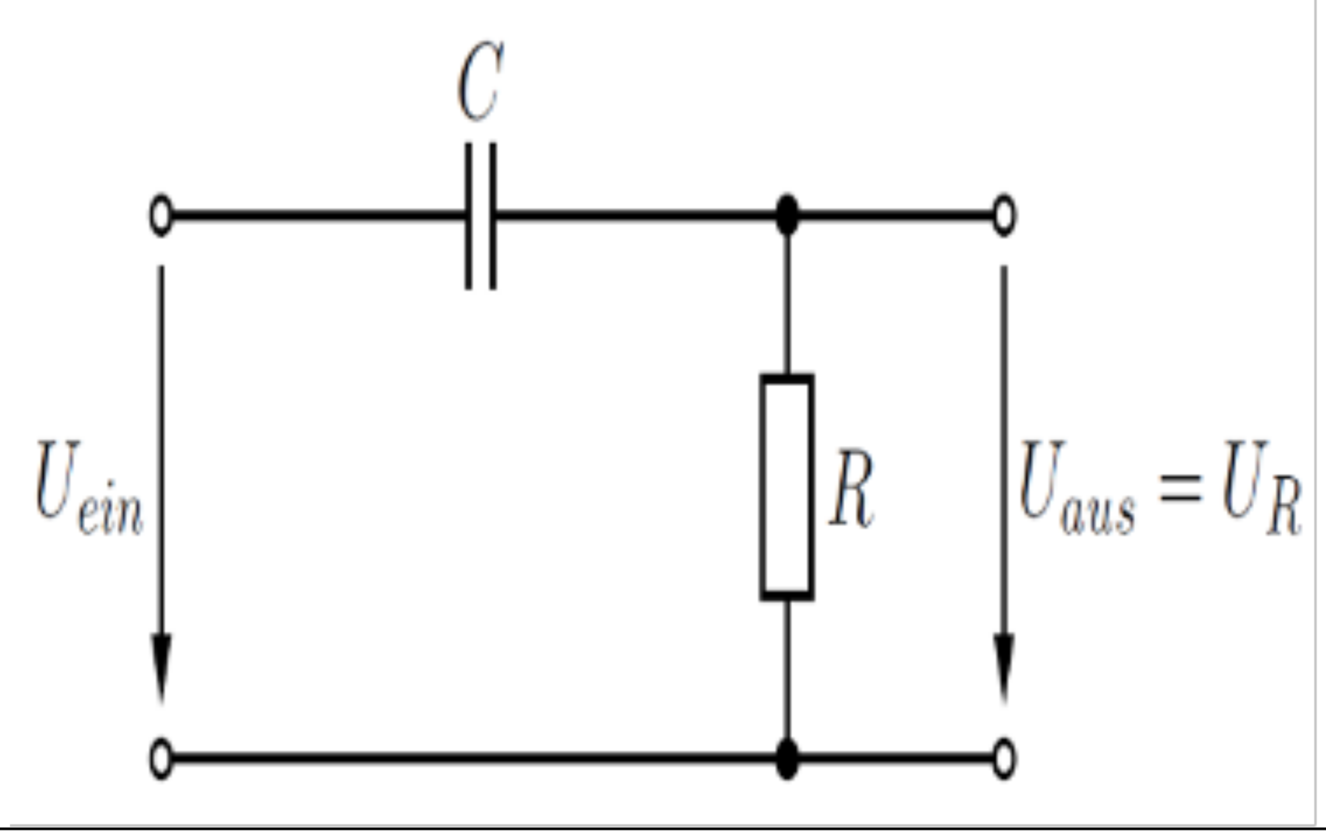
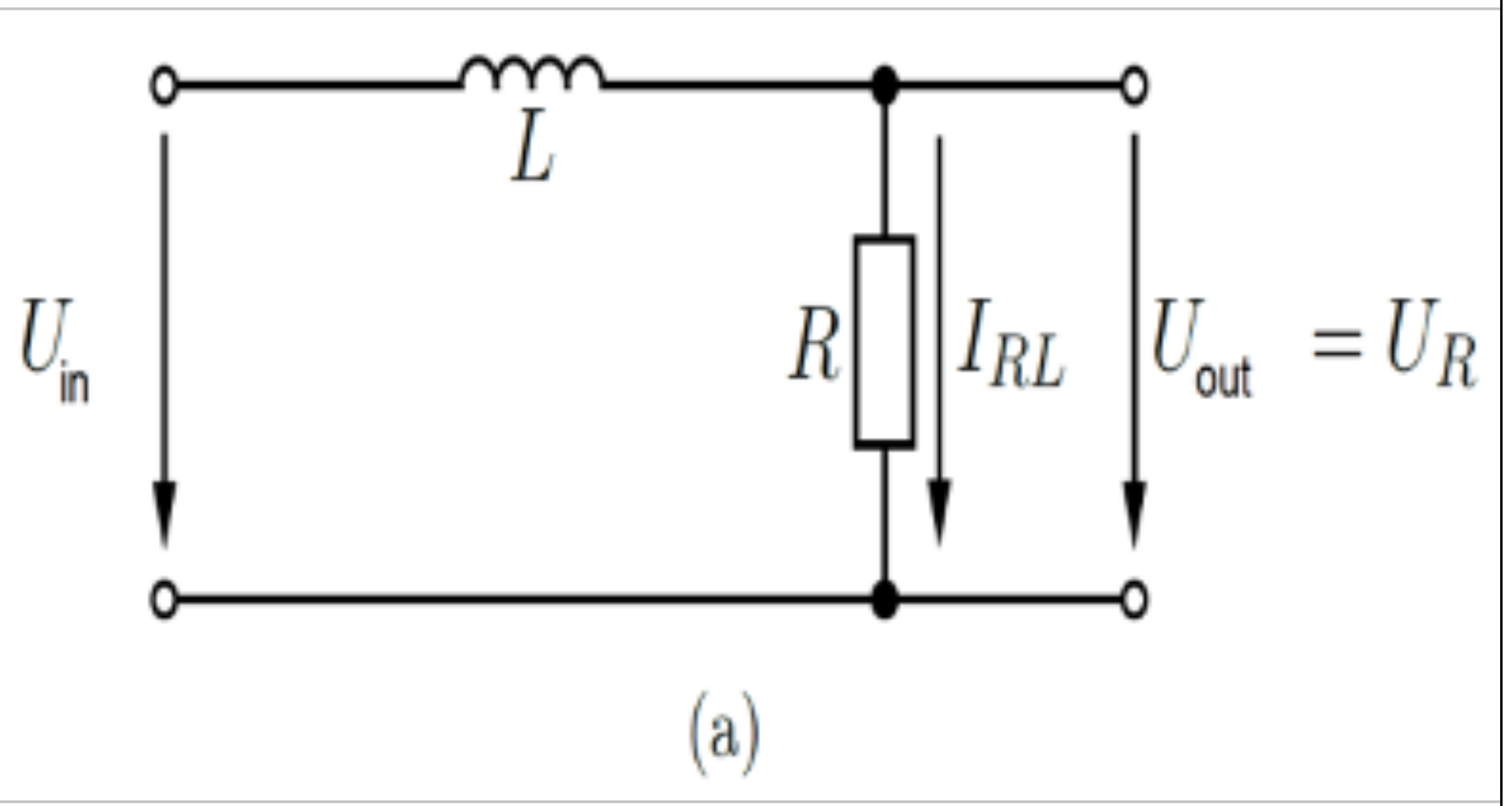
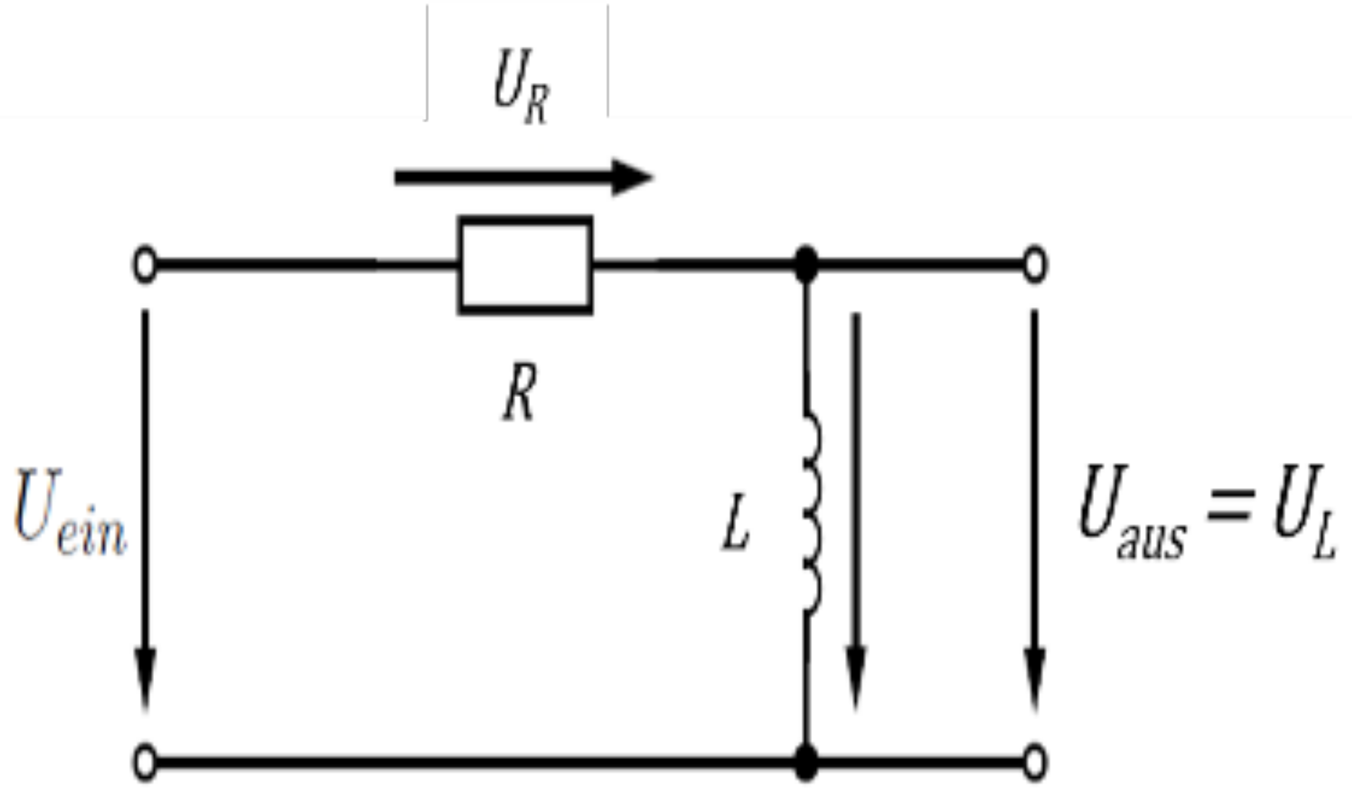
*Chapter 2*

# Filters

*In: Circuits with R, C, L*

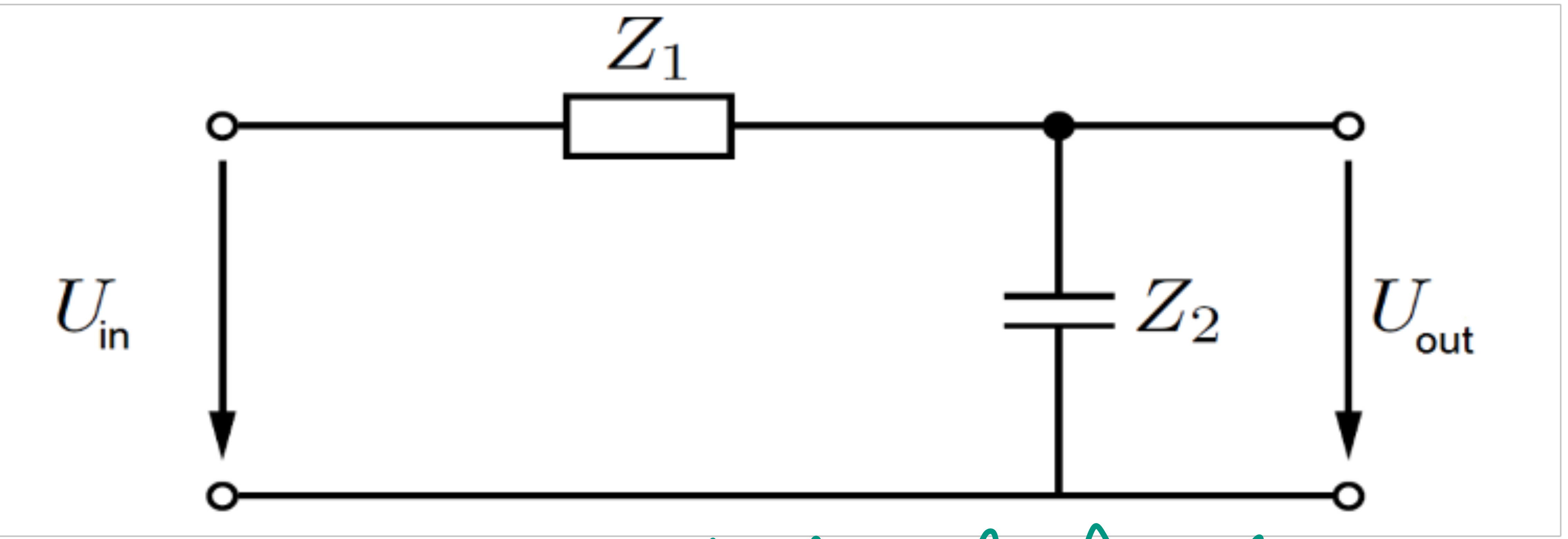
# High- and Low Pass Filters

*Reminder / Summary*

| Type | low pass | high pass |
|---|---|---|
| RC |  |  |
| RL |  |  |

# Low Pass

*Tiefpass*




Integrator / low pass filter

$$\underline{U}_{out} = \underline{U}_{in} \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$$ (voltage divider!)

complex transfer function

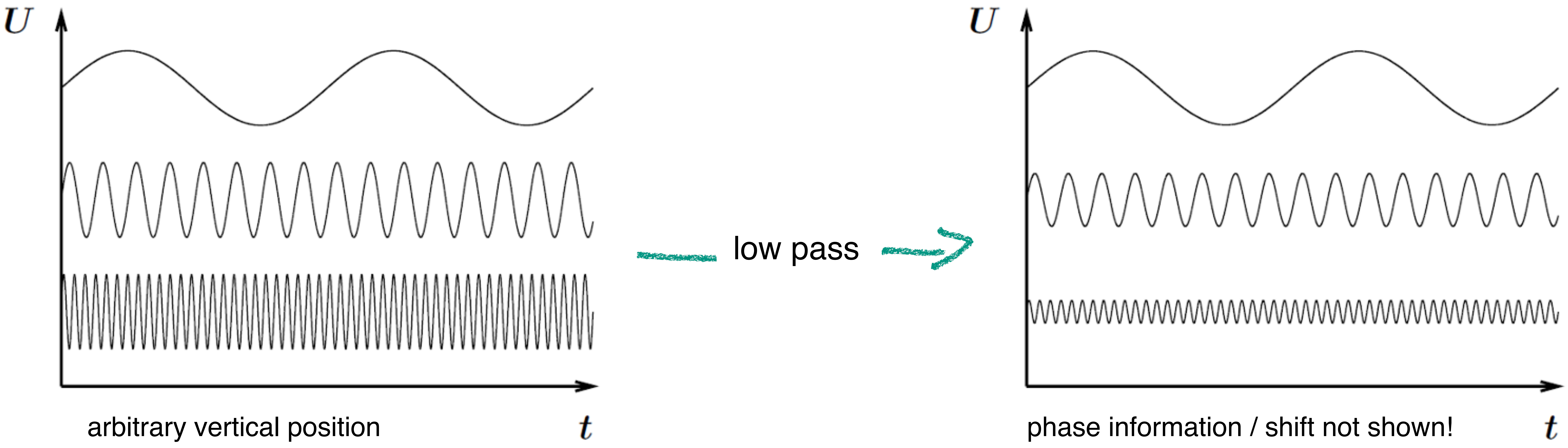
$$\frac{\underline{U}_{out}}{\underline{U}_{in}} = \underline{Z} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC} = \frac{1 - j\omega RC}{1 + (\omega RC)^2}$$

with $\underline{Z} = Z e^{j\varphi}$ : $Z = \frac{1}{\sqrt{1 + (\omega RC)^2}}$ and $\tan\varphi = \frac{\mathrm{Im}(\underline{Z})}{\mathrm{Re}(\underline{Z})} = -\omega RC$

$Z \approx 1$ for $\omega \approx 0$ and $Z \approx 0$ for $\omega >> 1/(RC)$

# Low Pass

*Impact on Signals*

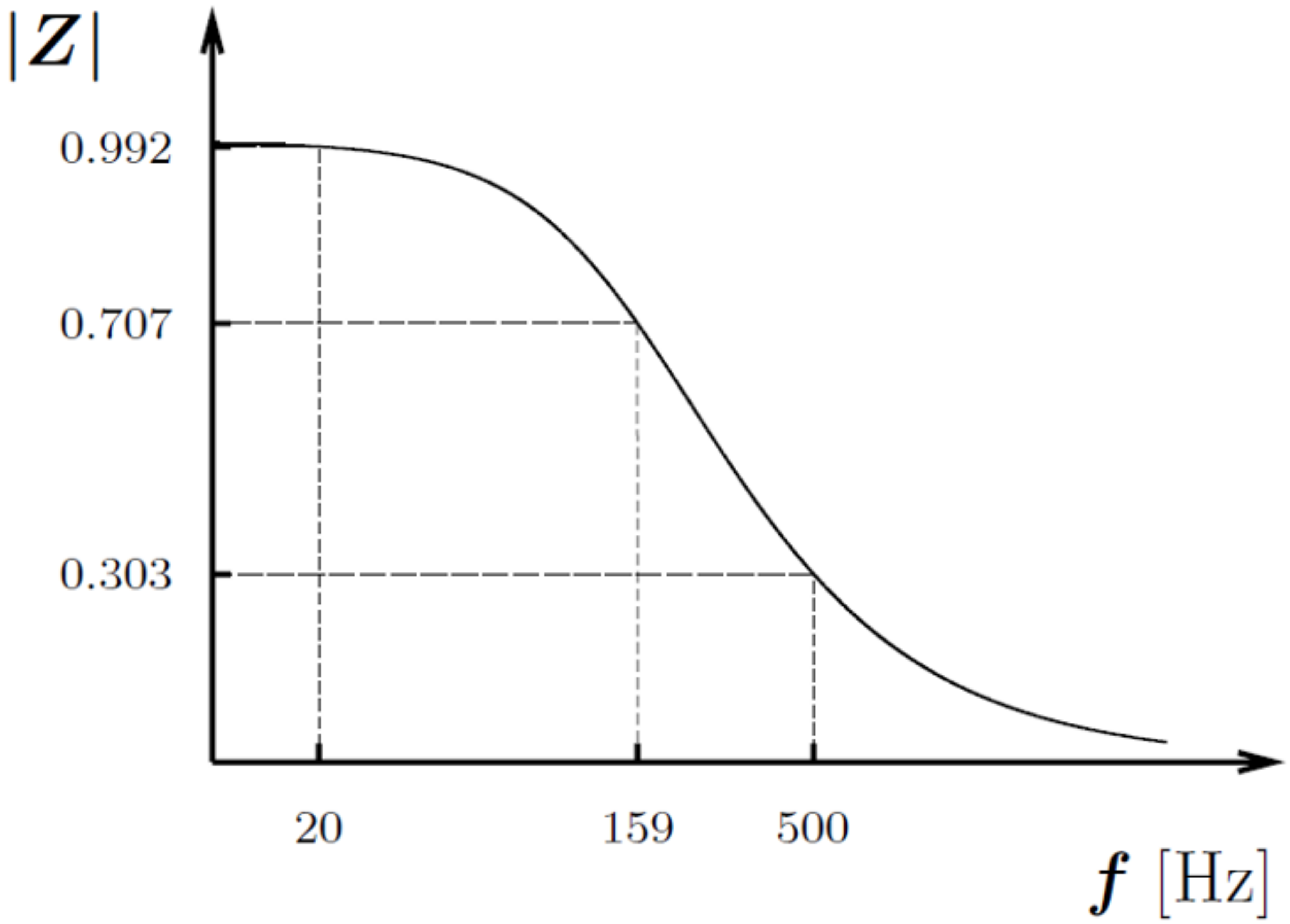



- Low frequency signals pass unchanged
- High frequency signals are damped
- For high frequencies the output signal is delayed in time with respect to the input signal

# Low Pass

*Frequence Dependence*

- Amplitude of output signal depends on frequency.
- Phase depends on frequency.
- The frequency of input and output signal is identical.




$$Z = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

# Brief Excursion: Decibel

*Characterizing Filter Properties*

Definitions: $10\ log \frac{P_{in}}{P_{out}}$ for power $20\ log \frac{U_{in}}{U_{out}}$ or $20\ log \frac{I_{in}}{I_{out}}$ for current / voltage

| dB | voltage ratio $20 \log \frac{U_{aus}}{U_{ein}}$ | power ratio $10 \log \frac{P_{aus}}{P_{ein}}$ |
|---|---|---|
| $100$ | $10^5$ | $10^{10}$ |
| $40$ | $100$ | $10^4$ |
| $20$ | $10$ | $100$ |
| $6,02$ | $2$ | $4$ |
| $3,01$ | $\sqrt{2}$ | $2$ |
| $0$ | $1$ | $1$ |
| $-3,01$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ |
| $-6,02$ | $\frac{1}{2}$ | $\frac{1}{4}$ |
| $-20$ | $0,1$ | $10^{-2}$ |
| $-40$ | $0,01$ | $10^{-4}$ |
| $-100$ | $10^{-5}$ | $10^{-10}$ |

NB: log = base 10 logarithm

Reminder:

log(ab) = log(a) + log(b):

dB values can be added!

Additional terms:

dBm: dB relative to 1 mW: $10\ log \frac{P[\mathrm{W}]}{1\,\mathrm{mW}}$

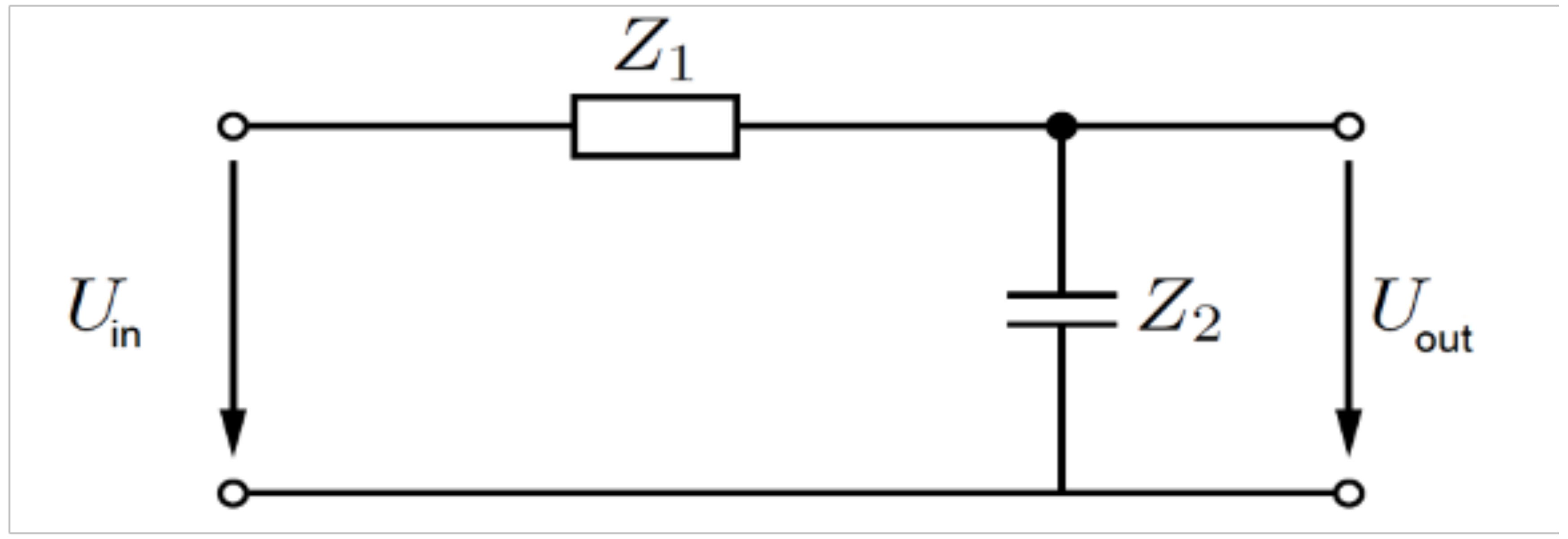
dBmA: dB relative to 1 mA: $20\ log \frac{I[\mathrm{A}]}{1\,\mathrm{mA}}$

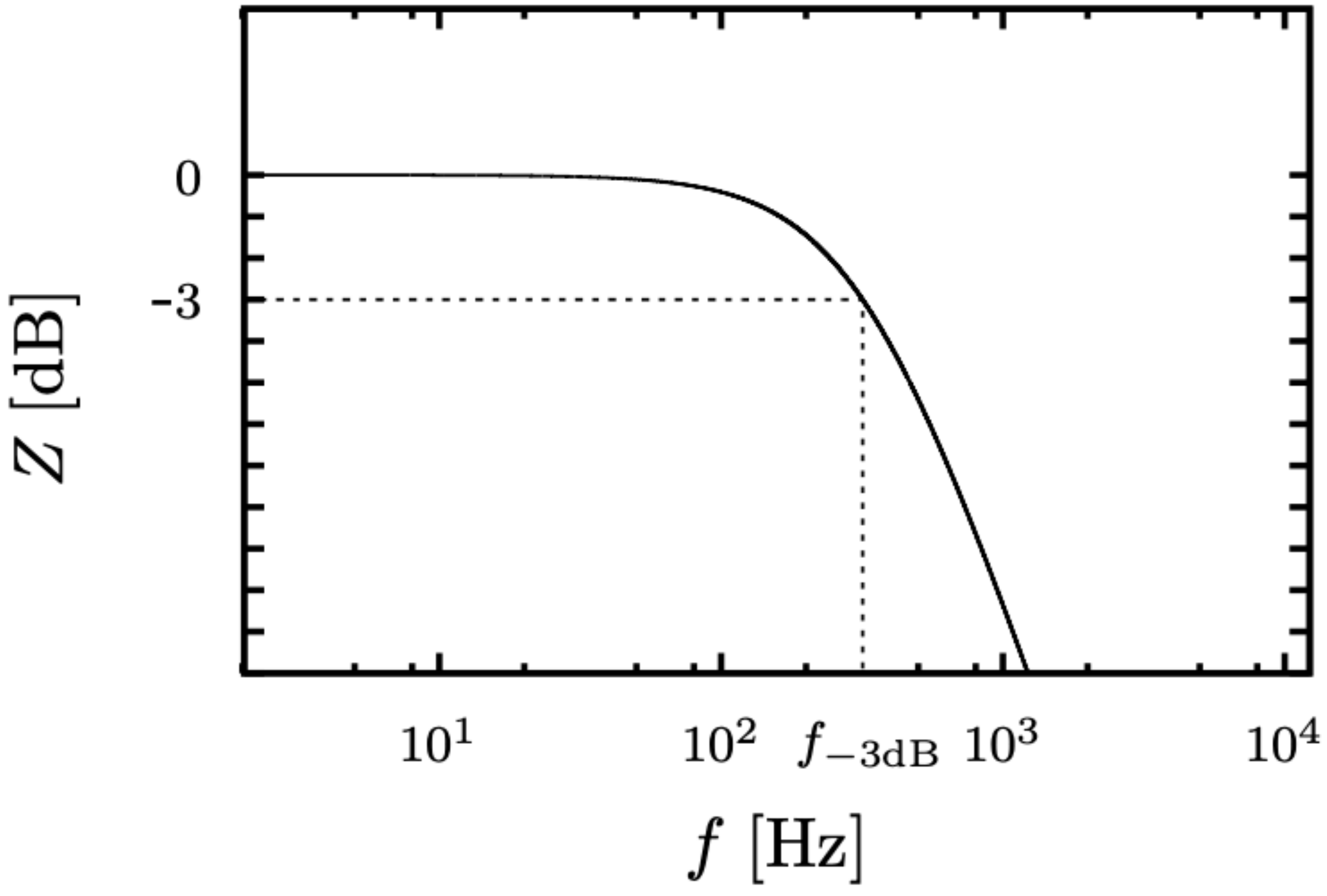
# Bode Plot: Introduction, Low Pass

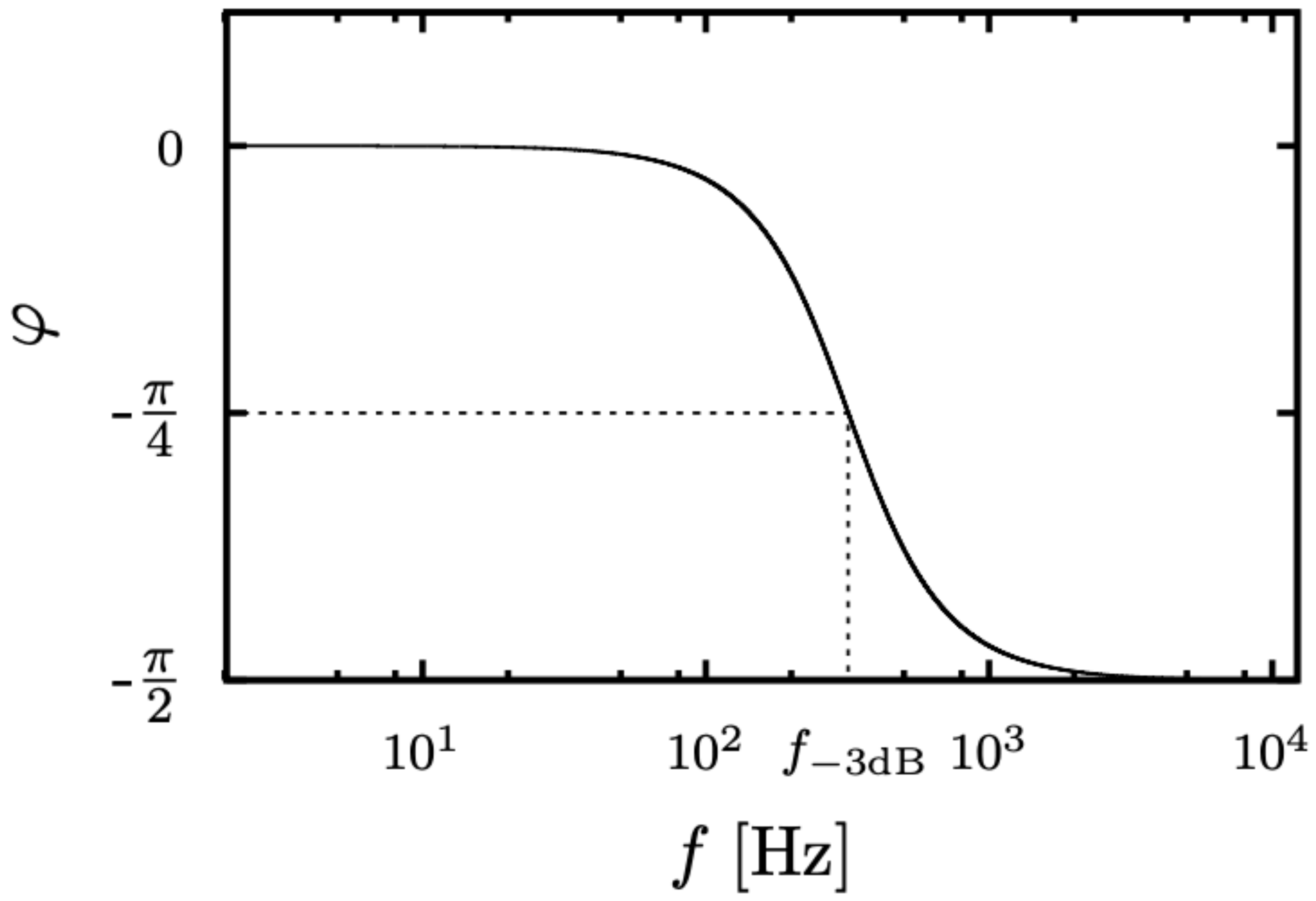
*Bodediagramm*

- A commonly used representation of the transfer function (*komplexe Übertragungsfunktion*) $Z e^{j\varphi}$
- Two plots:
  - Amplitude of transfer function ***Z*** in dB vs log(f)
  - Phase **ϕ** vs log(f)

$$f = \frac{\omega}{2\pi}$$



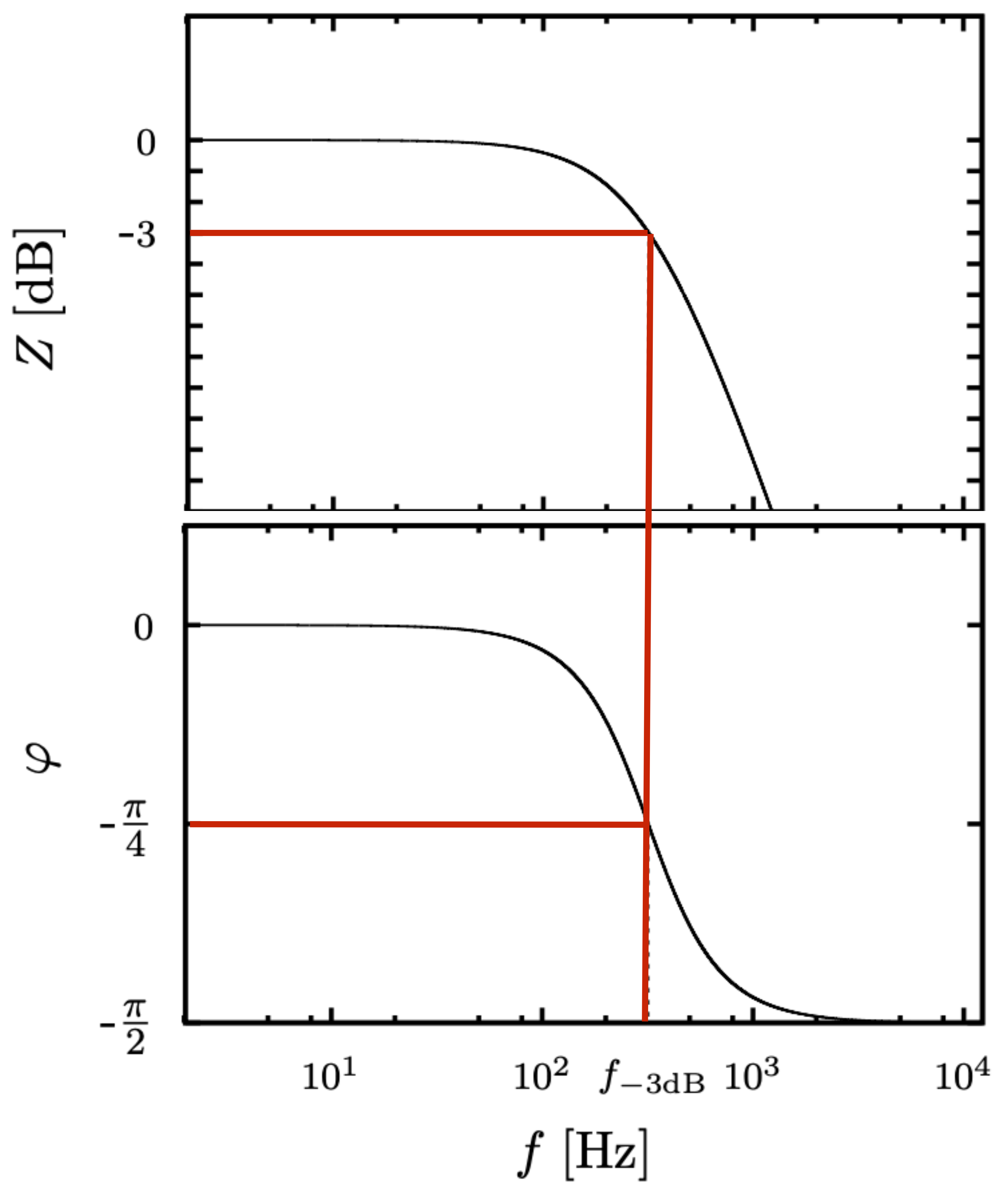

$$Z = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\tan \varphi = -\omega RC$$

here:

R = 10 kΩ

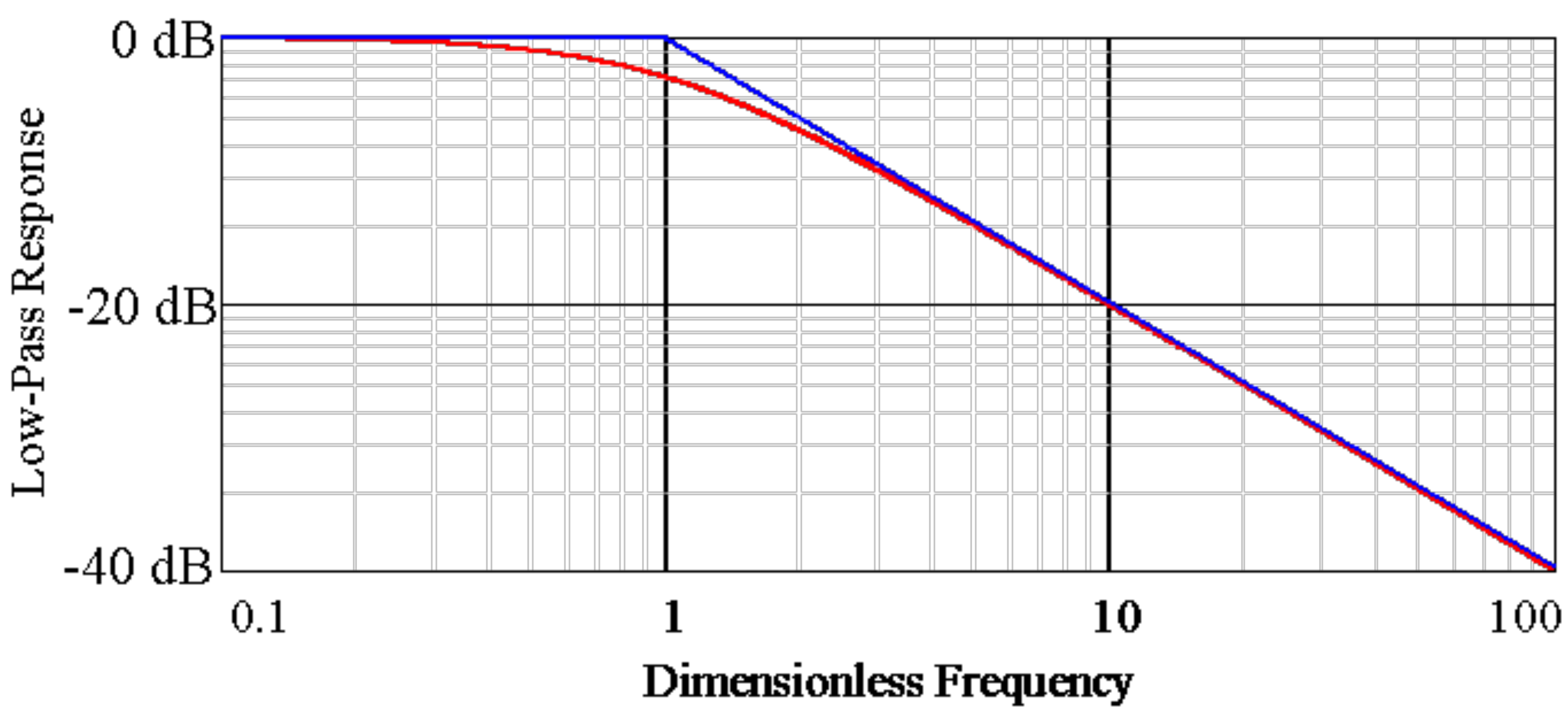
C = 100 nF




Cutoff frequency: Z has dropped by 1/√2 (-3 dB)

$$f_{-3\,\mathrm{dB}} = \frac{1}{2\pi RC} \qquad \omega_{-3\,\mathrm{dB}} = \frac{1}{RC}$$

Since $\tan\varphi = -\omega RC$ : for critical frequency: -1; -> ϕ = - π/4




At high frequencies Z drops by 20 dB per frequency decade.

Corresponds to 6 dB / Octave (frequency doubling / halving)

**Next Lectures:**

**Digital - Thursday, November 9**

**Analog 05 - Chapters 02, 03 - Tuesday, November 14**

# Time Plan for Next Lectures

*A few Changes coming up!*

| Calender Week | Tuesday | Thursday |
|---|---|---|
| 45 | 07.11. **Analog** | 09.11. **Digital** |
| 46 | 14.11. **Analog** | 09.11. **Digital** |
| 47 | 21.11. **Digital** | 23.11. **Analog** |
| 48 | 28.11. **Digital** | 30.11. **Digital** |
| 49 | 05.12. **Digital** | 07.12. **Analog** |
| 50 | 12.12. **Digital** | 14.12. **Analog** |
| 51 | 19.12. **Analog** | 21.12. **Digital** |
| 2 | 09.01. **Analog** | 11.01. **Analog** |
| 3 | 16.01. **Digital** | 18.01. **Digital** |
| 4 | 23.01. **Analog** | 25.01. **Digital** |
| 5 | 30.01. **Analog** | 01.02. **Digital** |
| 6 | 06.02. **Analog** | 08.02. **Digital** |
| 7 | 13.02. **Analog** | 15.02. **Digital** |

# Electronics for Physicists

## *Analog Electronics*

*Chapter 2; Lecture 04*

**Frank Simon**
**Institute for Data Processing and Electronics**

07.11.2023

Karlsruhe Institute of Technology

***KIT, Winter 2023/24***

Institute for Data Processing and Electronics