Electronics for Physicists

Analog Electronics

Chapter 4; Lecture 07

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Karlsruhe Institute of Technology

KIT, Winter 2023/24

Institute for Data Processing and Electronics

Chapter 4

Operational Amplifiers

- OpAmp Basics
- Simple Circuits: Feedback etc.
- OpAmp Circuits I
- Realistic OpAmps
- OpAmp Circuits II

Overview

Simple Circuits: Feedback etc.

In: Chapter 4: Operational Amplifiers

Feedback

Rückkopplung

- Principle: A fraction of the output signal of a circuit is fed back to its input
  - Positive feedback
  - Negative feedback (the dominant application!)

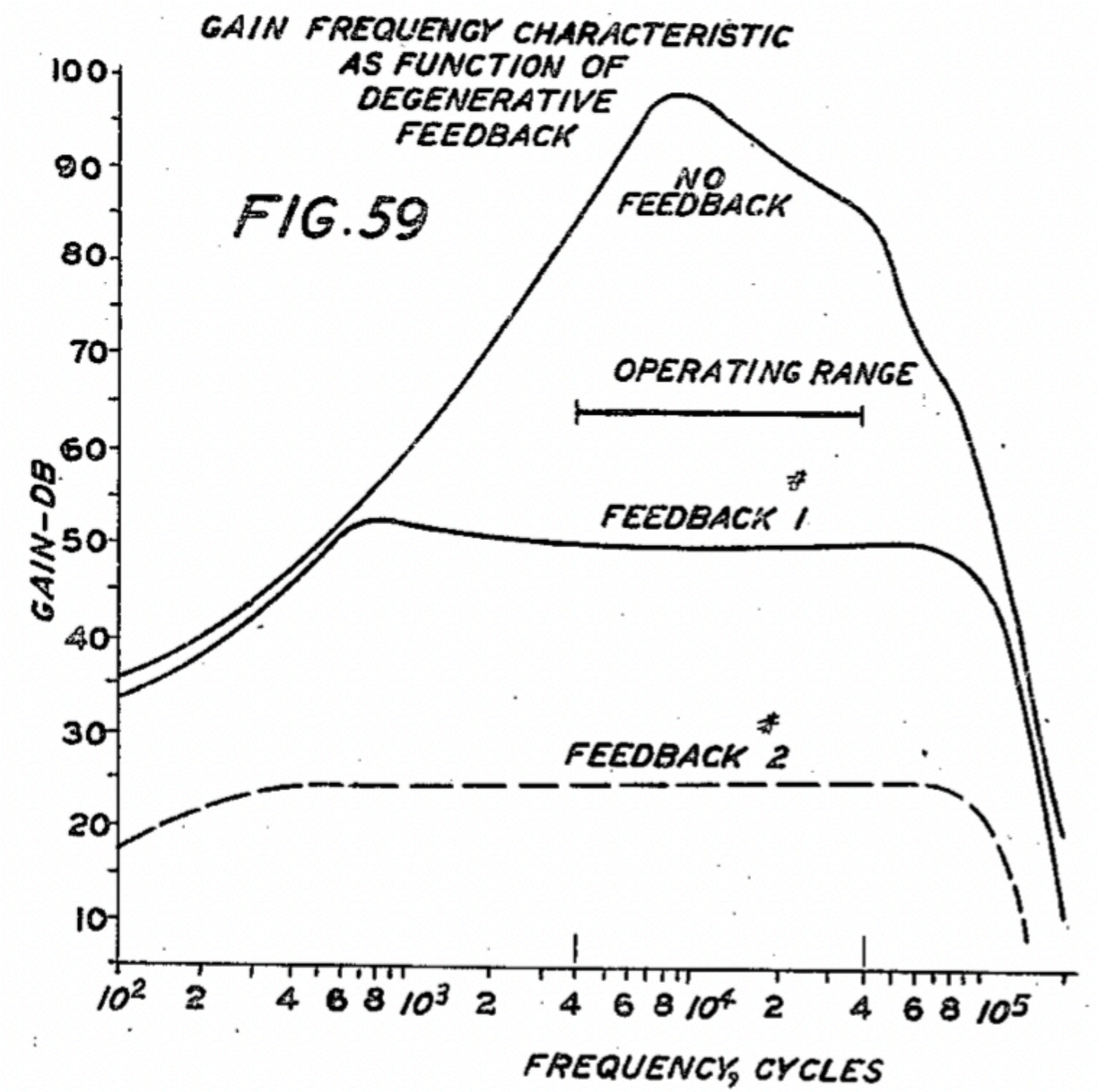
Invented 1927 by Harold Black (Bell Labs)

US Pat. 2,102,671 (1937) “Wave Translation System”

Dec. 21, 1937 H. S. BLACK 2,102,671

WAVE TRANSLATION SYSTEM

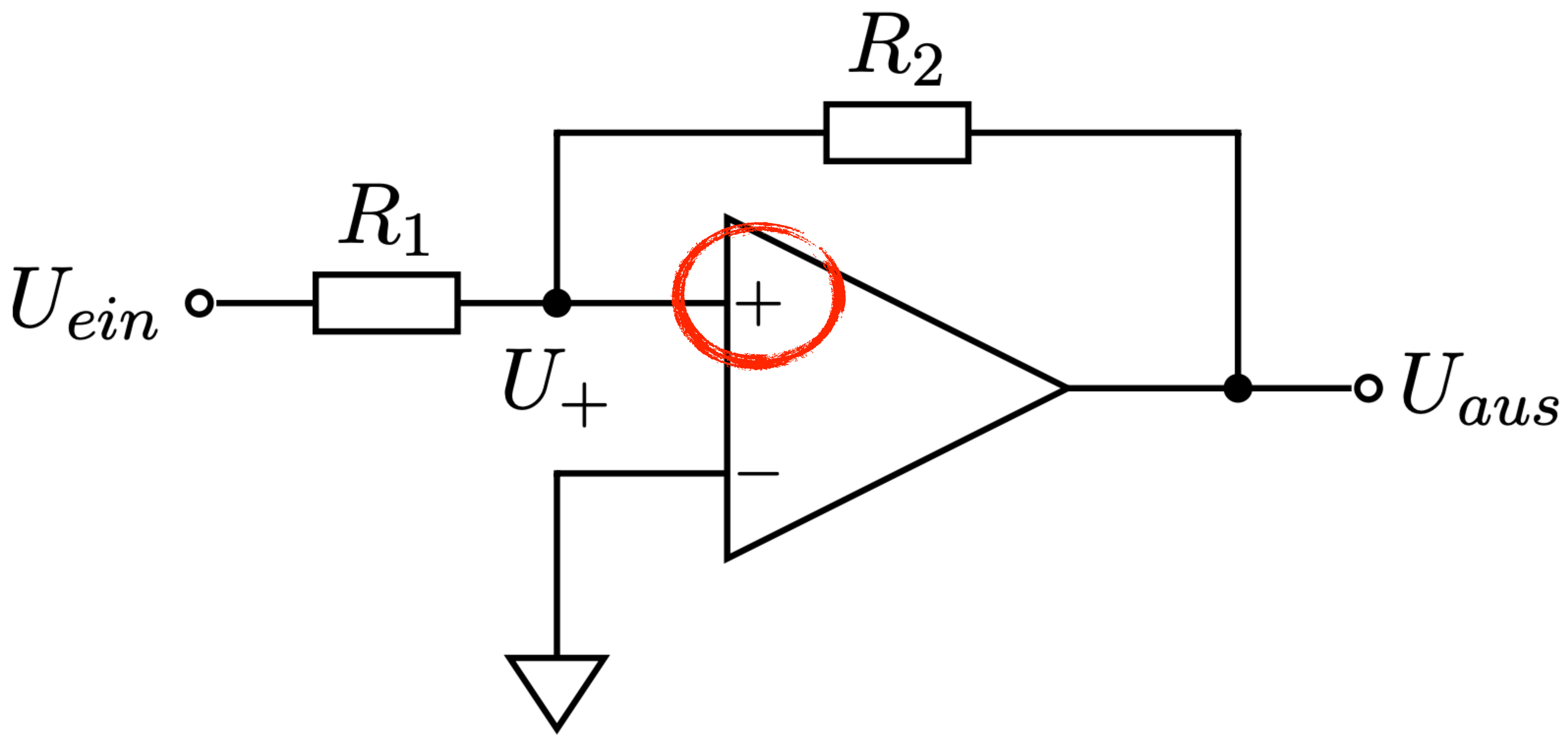
Filed April 22, 1932 35 Sheets-Sheet 27



Feedback with Op Amps

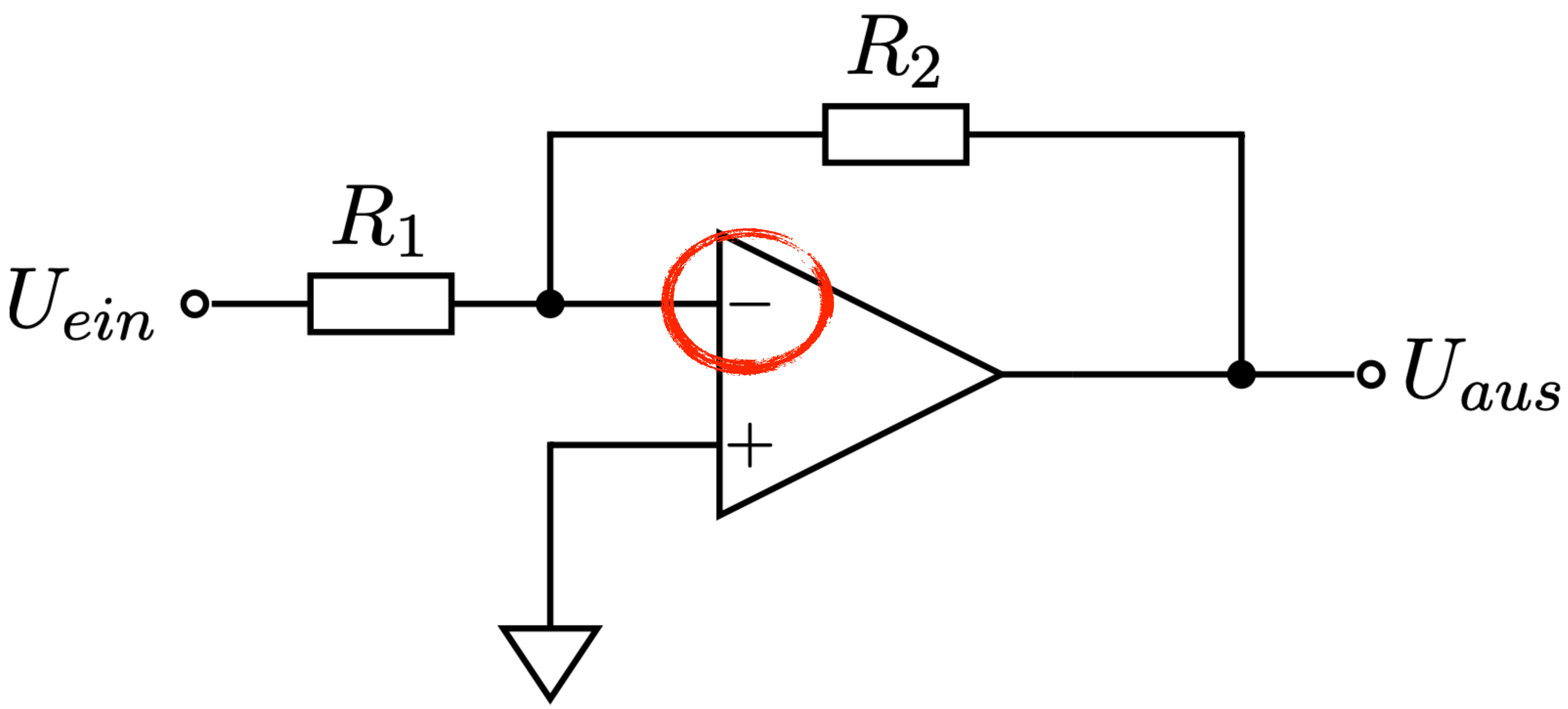
Positive, negative

- Positive feedback: Output signal coupled into the non-inverting input



Schmitt trigger

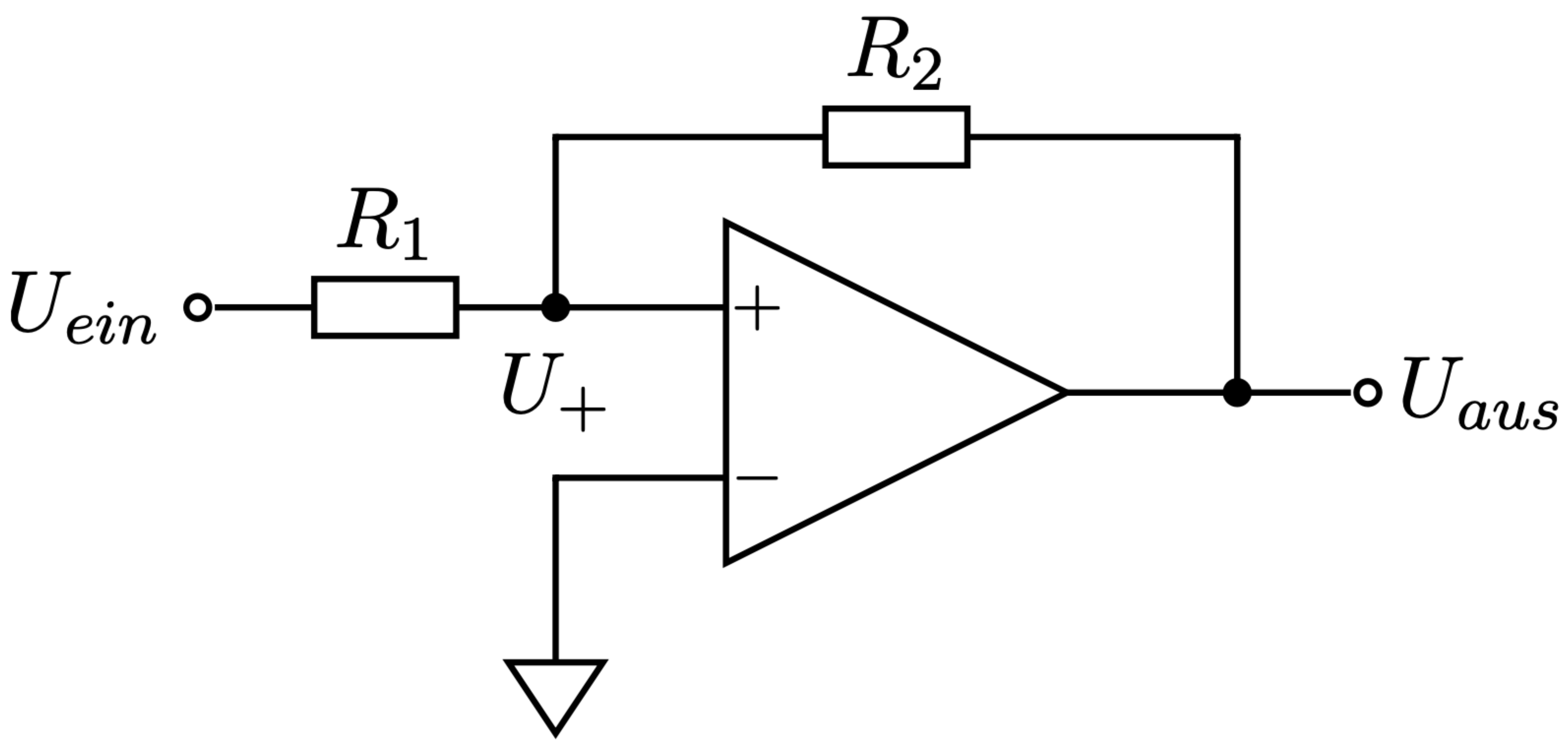
- Negative feedback: Output signal coupled into the inverting input



inverting amplifier

The Schmitt Trigger

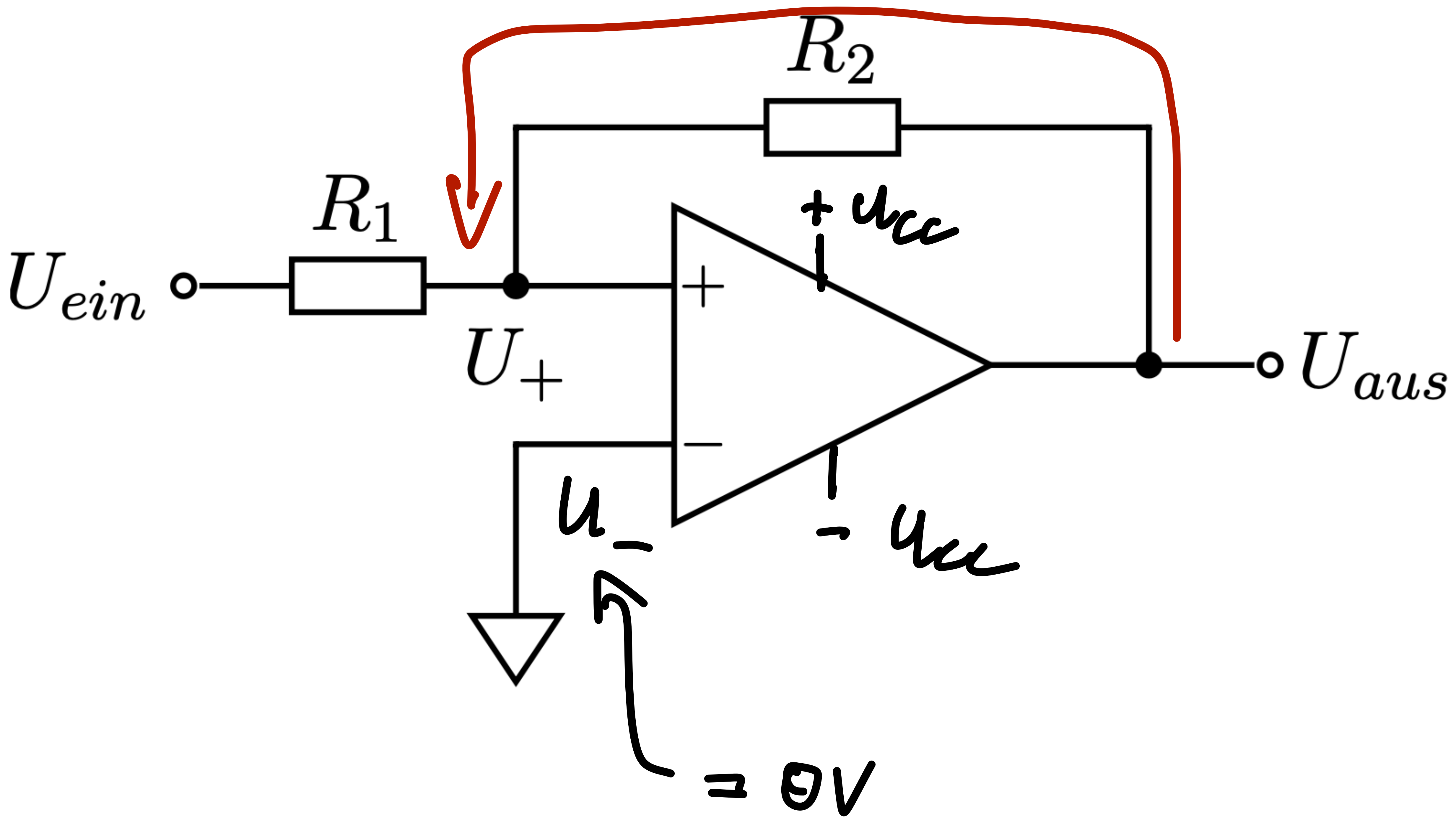
Positive feedback



How does this circuit work?

The Schmitt Trigger

Positive Feedback



How does this circuit work?

For $U_{ein} > 0V$

$U_+ > U_- \rightarrow U_{aus} \gg 0V$

large amplification

Feedback: U_+ is further increased $\rightarrow$ U_{aus} increases even further.

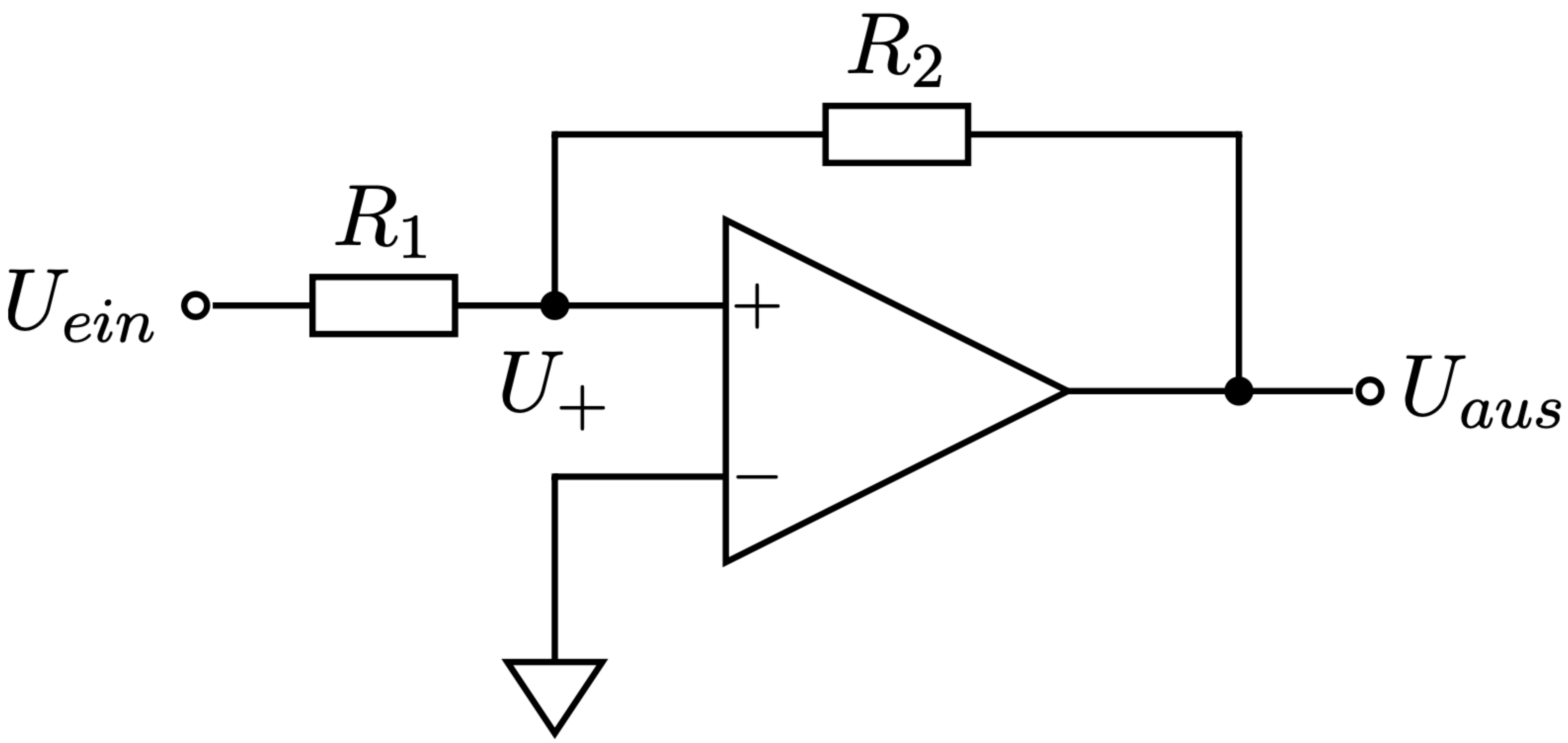
$\Rightarrow$ $U_{aus} = +U_{cc}$ (or the highest possible output voltage of the OpAmp)

For $U_{ein} < 0V$: same principle, $U_{aus} = -U_{cc}$

$\Rightarrow$ "Trigger function": Two possible states

The Schmitt Trigger

Switching between states



- Assumption for initial state:
  $U_{aus} = U_{max}$

To change the state:
$U_+ < 0$ required

KCL: $I_1 = I_2$
(large input resistance of OpAmps, input current ~ 0)

$$\frac{U_{ein} - U_+}{R_1} = \frac{U_+ - U_{aus}}{R_2}$$

Obtaining $U_+ \sim 0$: for

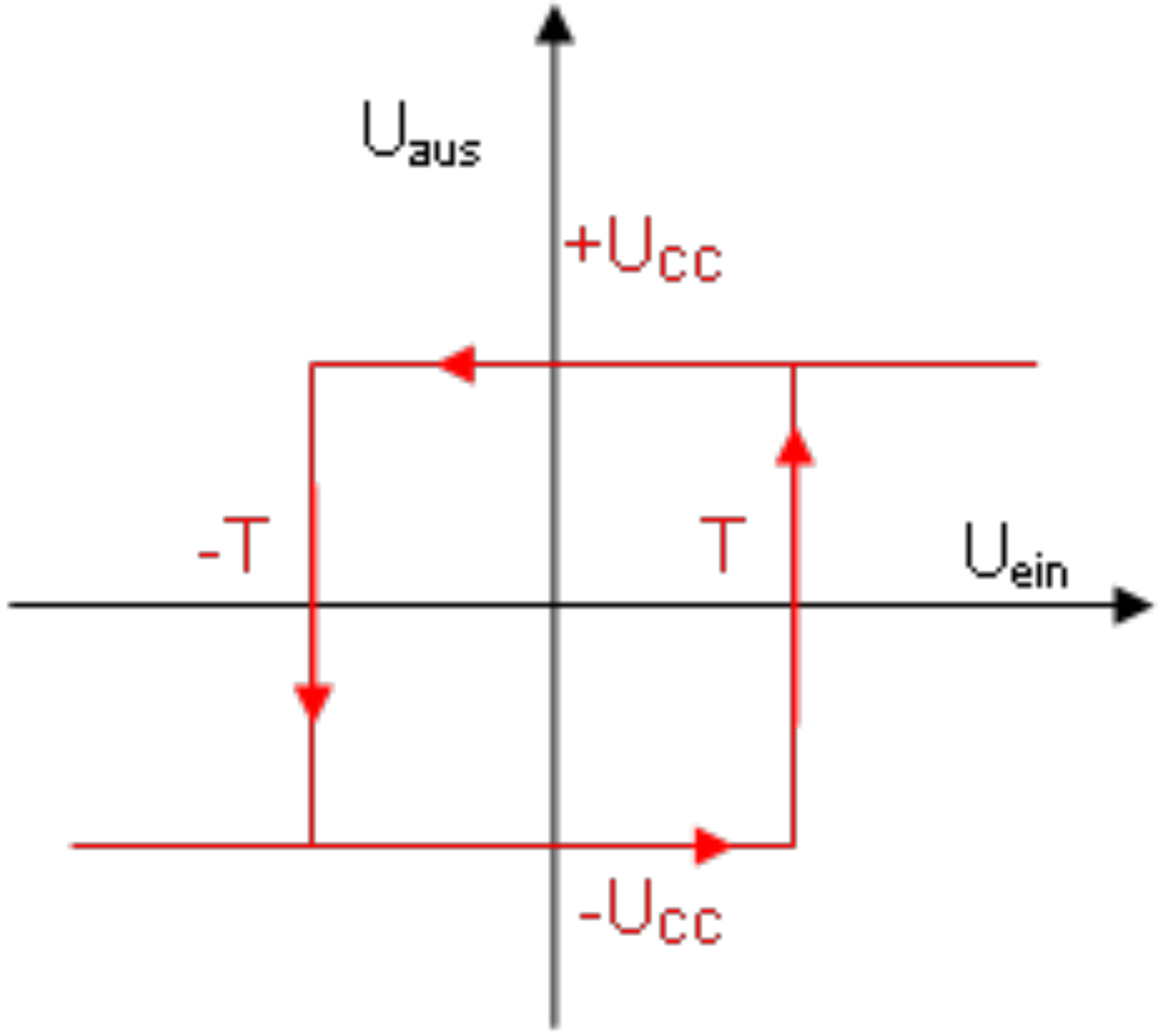
$$U_{ein} < -U_{aus}\frac{R_1}{R_2}$$

changes to $U_{aus} = U_{min}$

Schmitt Trigger

Hysteresis

- The input signal has to "overcome: the positive feedback to switch states



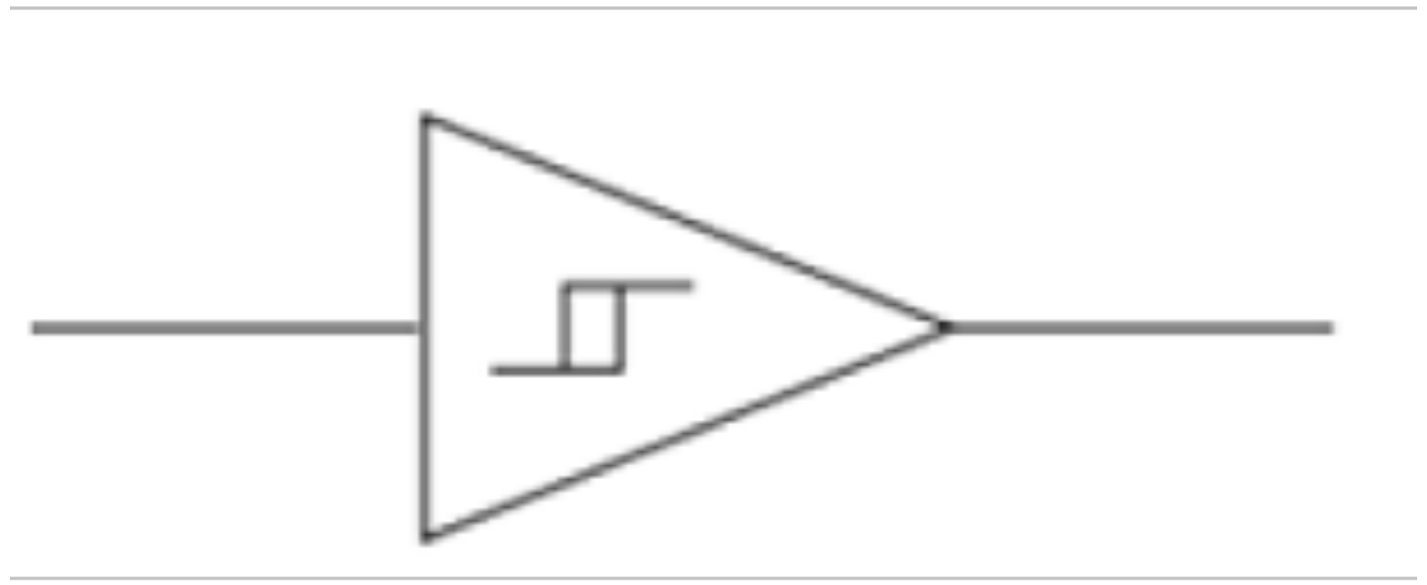
Idealized assumption: $U_{max/min}$ = +/- U_{CC}

switching for

$$U_{\text{ein}} > U_{\text{cc}} \frac{R_1}{R_2}$$

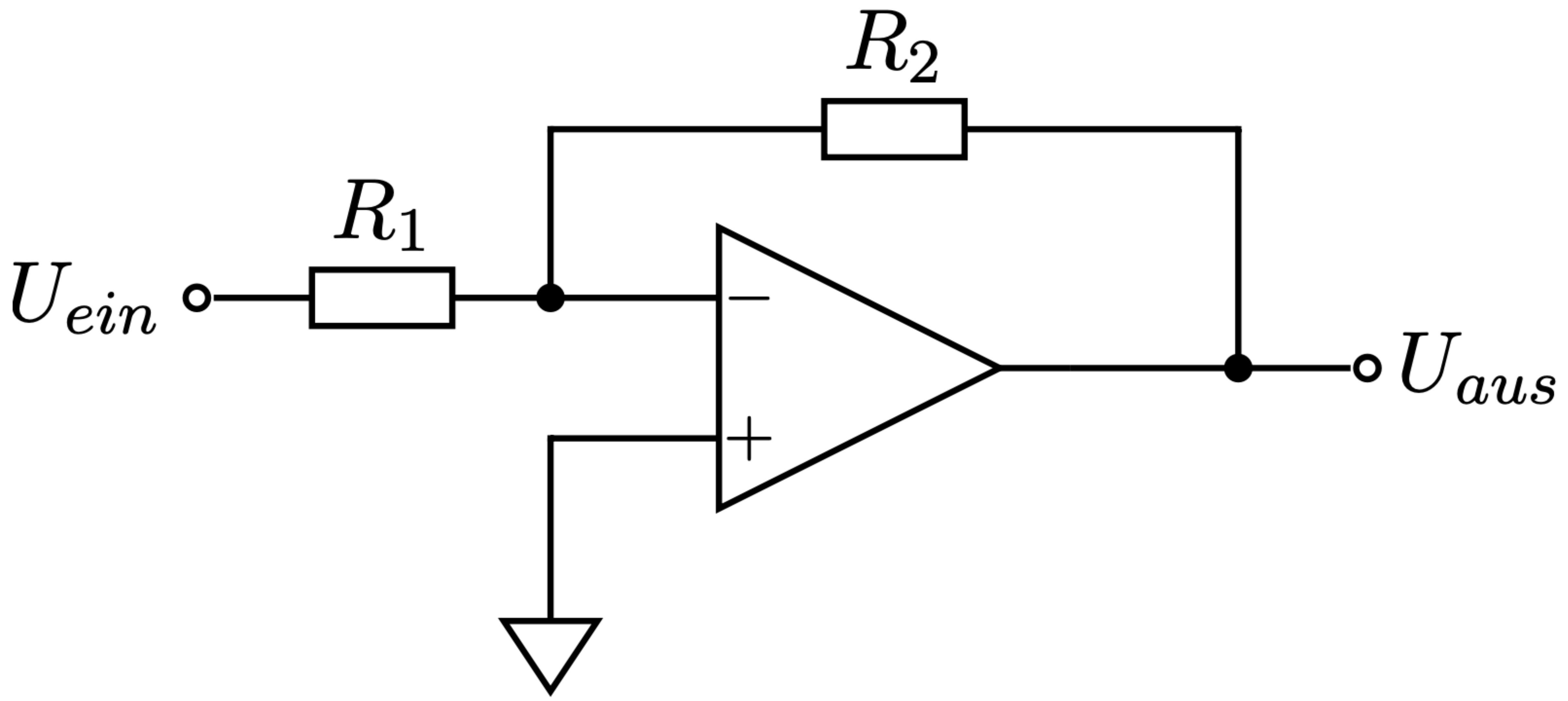
$$U_{\text{ein}} < -U_{\text{cc}} \frac{R_1}{R_2}$$

Symbol for the Schmitt Trigger

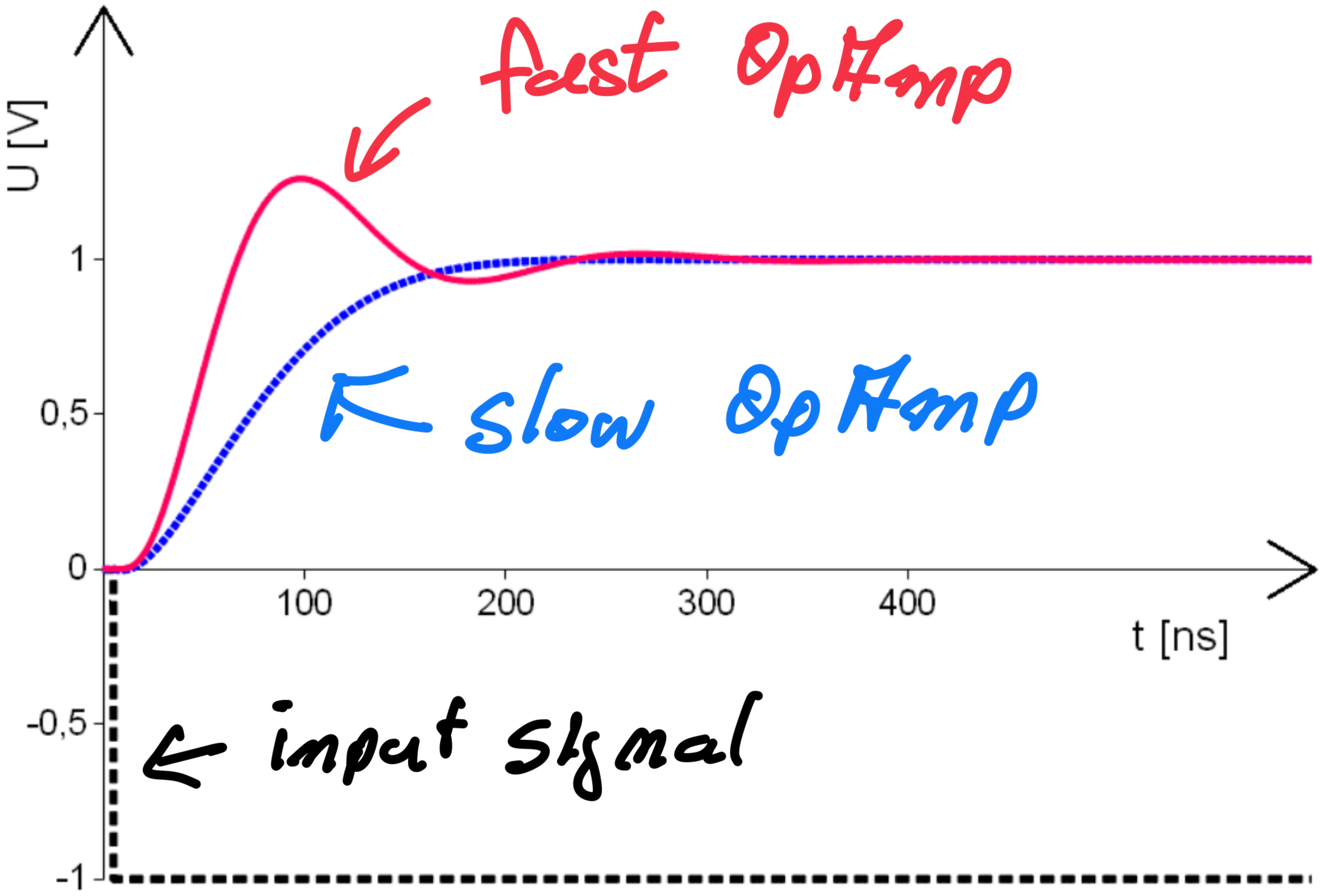


Negative Feedback

Inverting Amplifier



How does this circuit behave?



Negative Feedback

Inverting Amplifier

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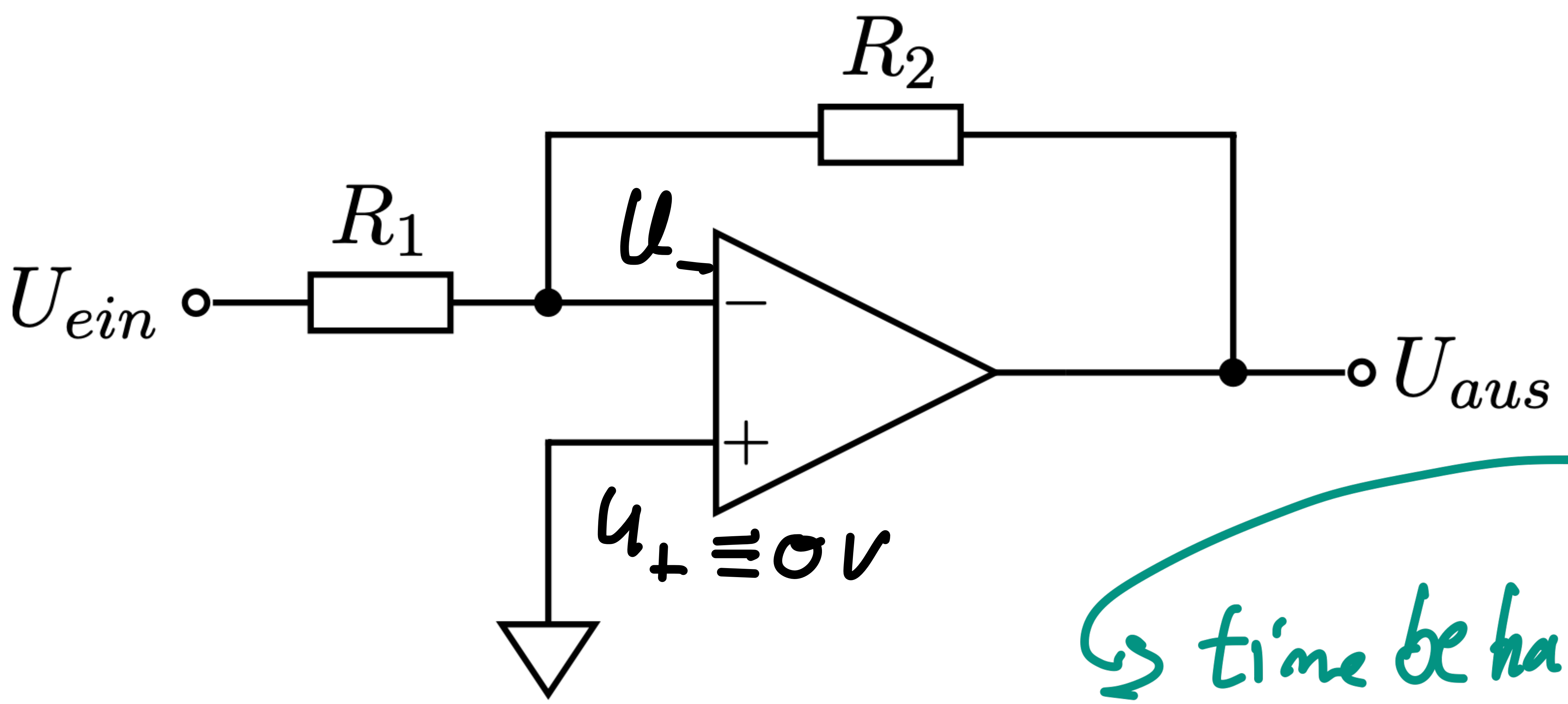
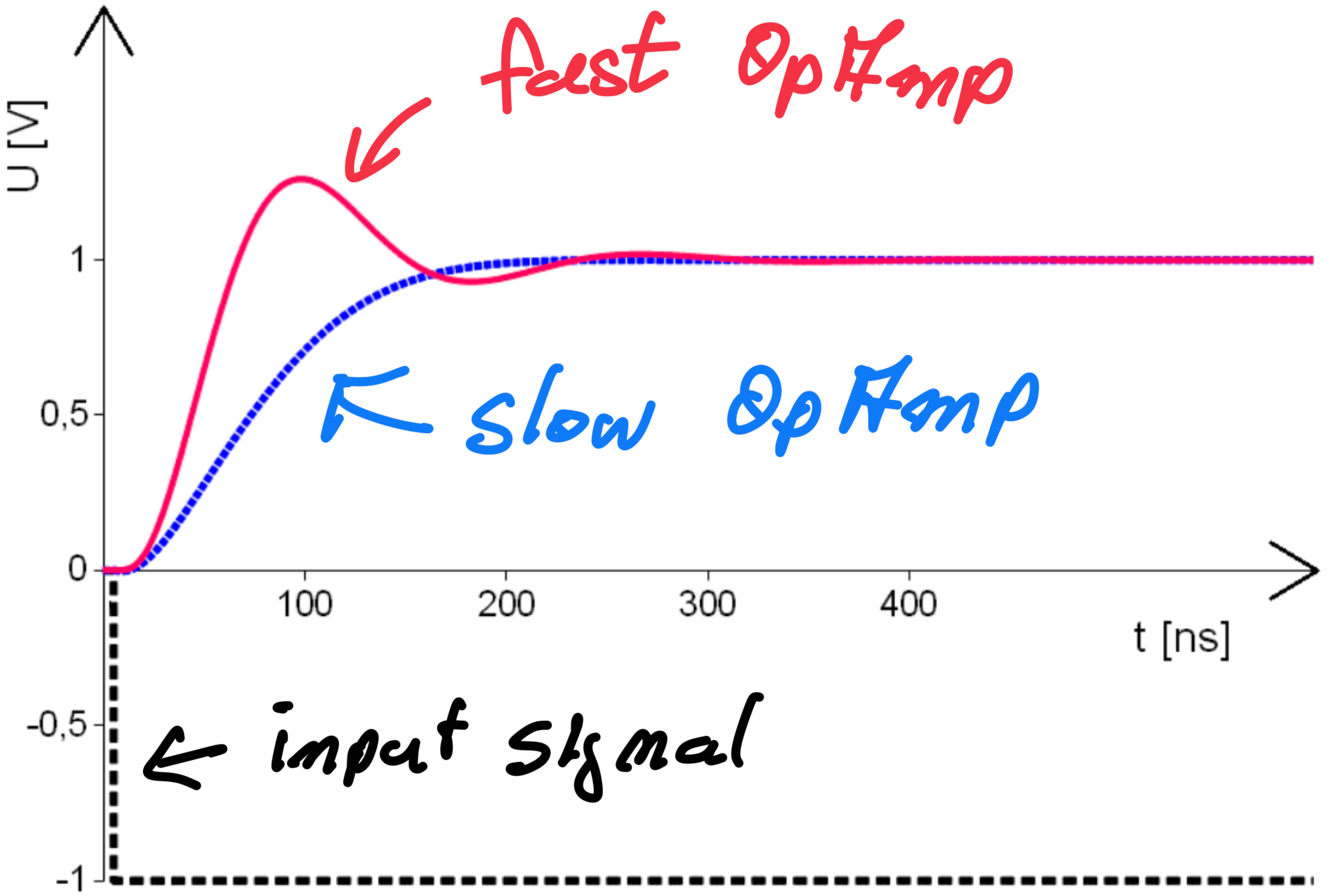


For $U_{ein} < 0V \Rightarrow U_- < 0V$

$\Rightarrow U_{aus} >> 0V$

$\Rightarrow U_-$ becomes > U_{ein}, lowering $U_{aus} \Rightarrow$ equilibrium

$\Rightarrow$ time behavior depends on OpAmp details.



How does this circuit behave?

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The Golden Rules

OpAmps with negativer Feedback

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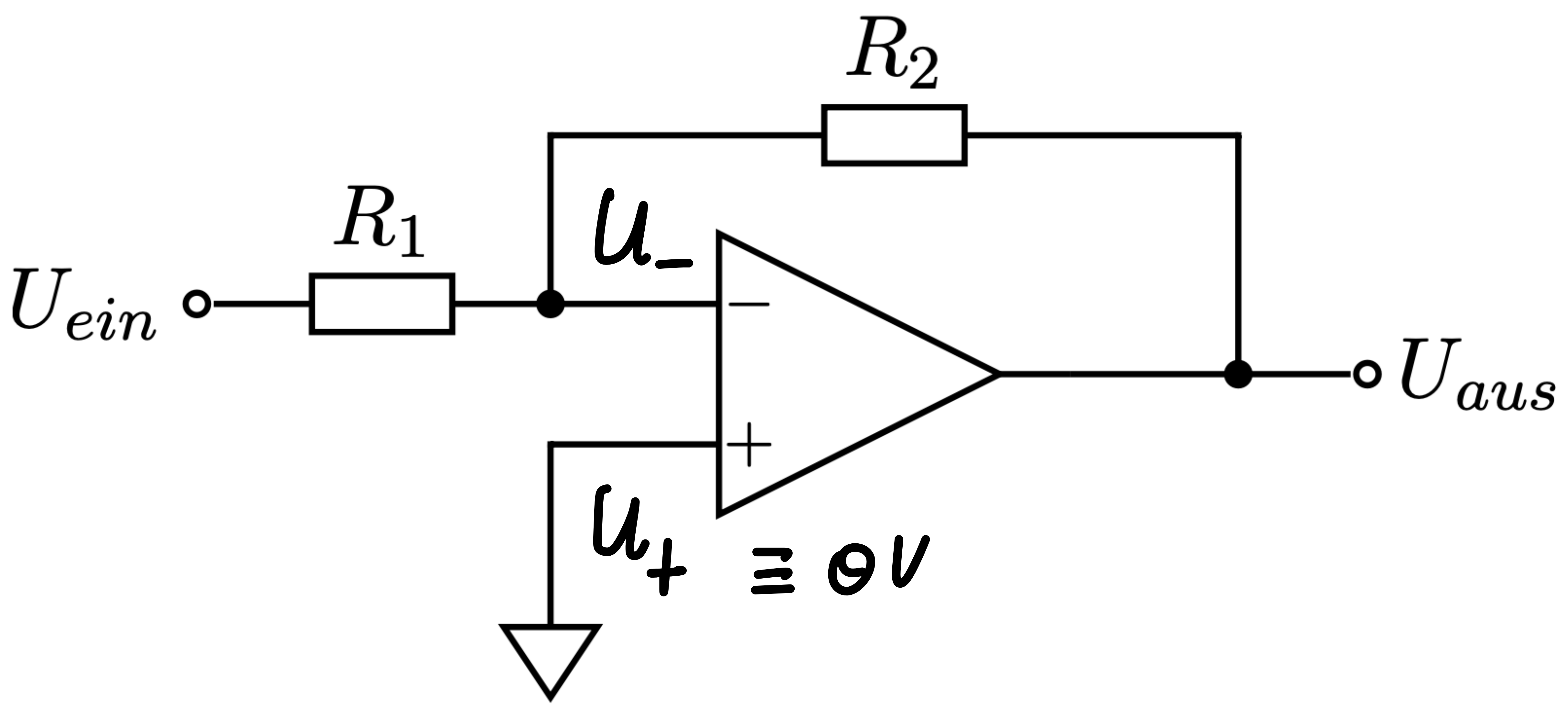
- The analysis of the behavior of ((idealized) OpAmp circuits with negative feedback results in the ***Golden Rules:***
  1. **The OpAmp does whatever is necessary to achieve $U_+ = U_-$.**
  2. **The input draws no current.** (input impedance ∞)

  This is sufficient to describe and understand the behavior of negative feedback OpAmp circuits in most practical cases.
  In addition we can define a 3rd rule:

  3. **The output impedance is very small.** (Output voltage does not depend on load or current.)

The inverting Amplifier

Applying the Golden Rules



"virtual ground"

results in input impedance $Z_i = R_1$

(normally relatively small …)

> Interesting observation:
> The open loop gain $\boldsymbol{A_D}$ of the OpAmp does not appear in the gain of the circuit!

1st Rule: $U_- = U_+ = 0V$

0V - U_{aus} over R_2 ab, U_{ein} - 0V over R_1:

$$I_1 = \frac{U_{ein}}{R_1} \qquad I_2 = \frac{-U_{aus}}{R_2}$$

2nd Rule + KCL: $I_1 = I_2$

with that $$\frac{U_{ein}}{R_1} = \frac{-U_{aus}}{R_2}$$

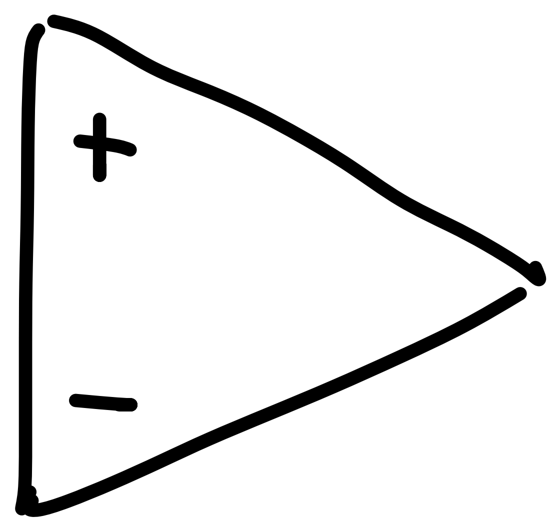
Gain:

$$A = \frac{U_{aus}}{U_{ein}} = -\frac{R_2}{R_1}$$

Non-inverting Amplifier

with negative feedback

- How can we do this?

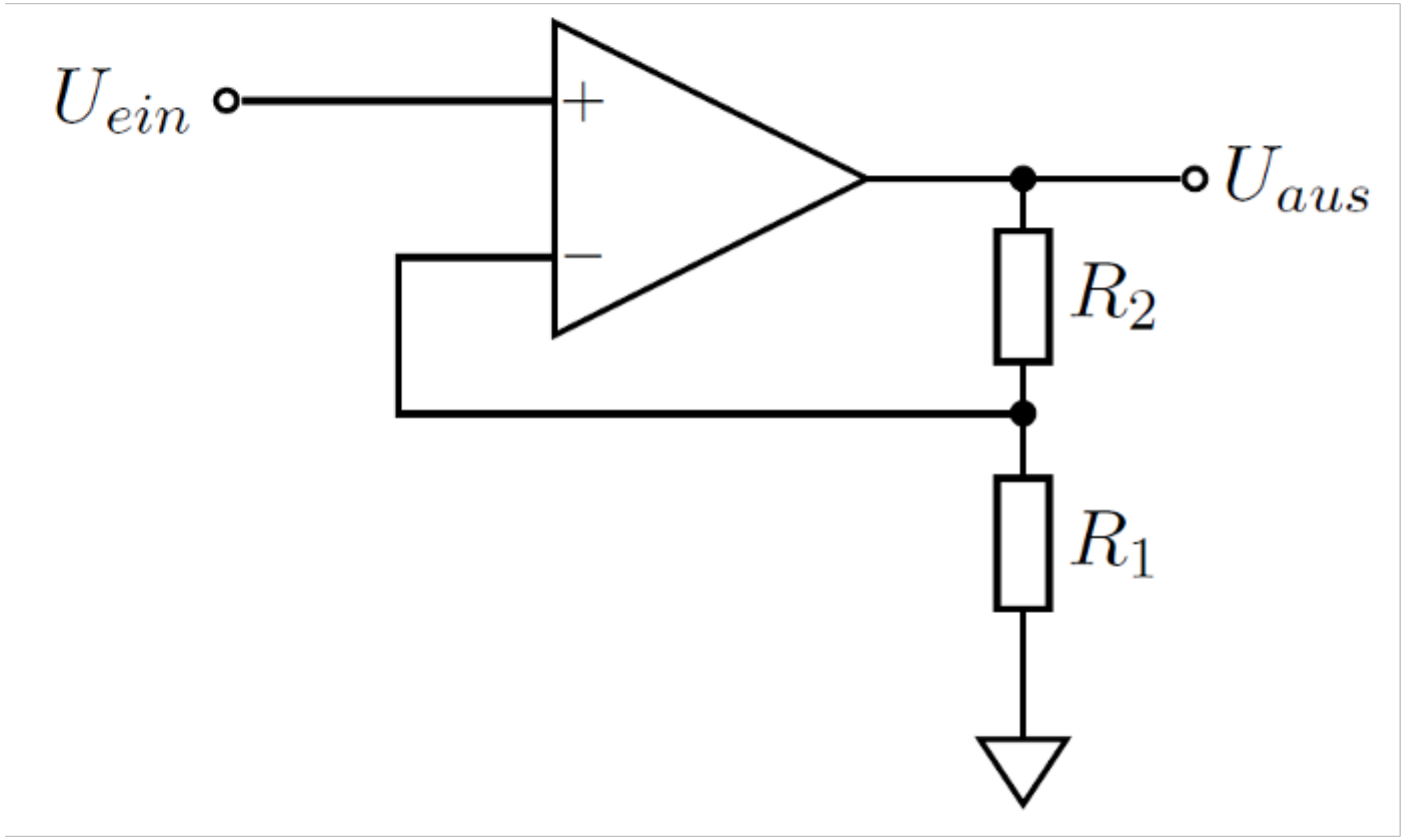


Non-inverting Amplifier

With negative Feedback

- How can we do this?

Input signal into the non-inverting input



Negative feedback via voltage divider on output.

Golden Rules:

$$U_{ein} = U_+ = U_- = U_{aus} \frac{R_1}{R_1 + R_2}$$

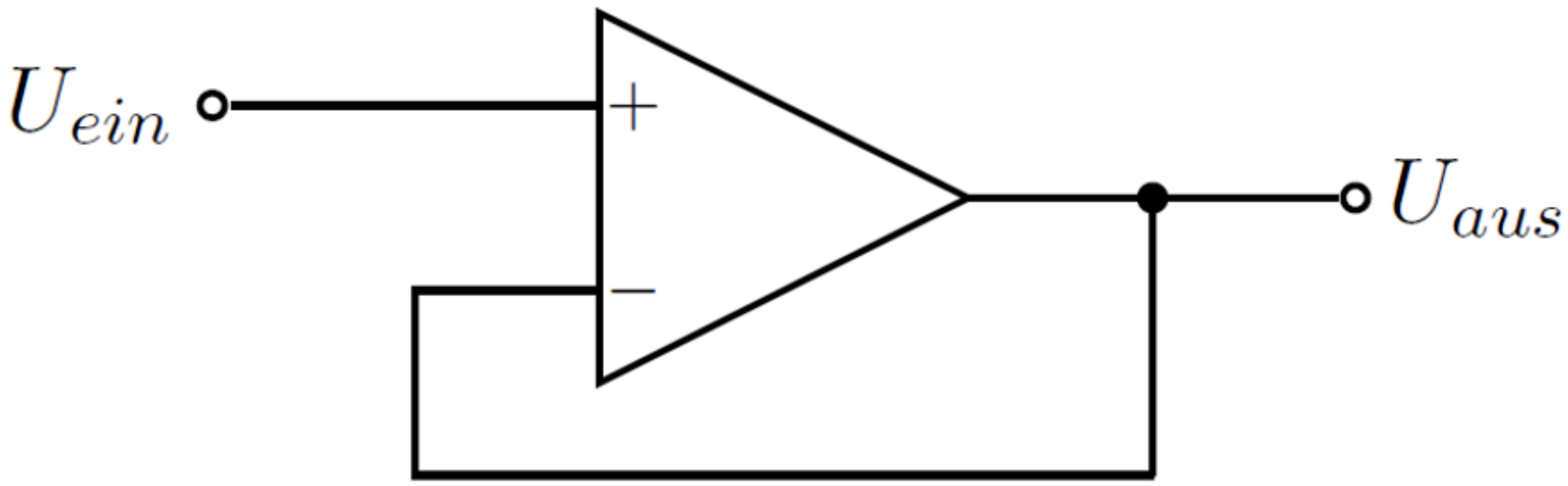
Amplification:

$$\frac{U_{aus}}{U_{ein}} = 1 + \frac{R_2}{R_1}$$

Input impedance: very high!

Voltage Buffer

Impedanzwandler / Spannungsfolger

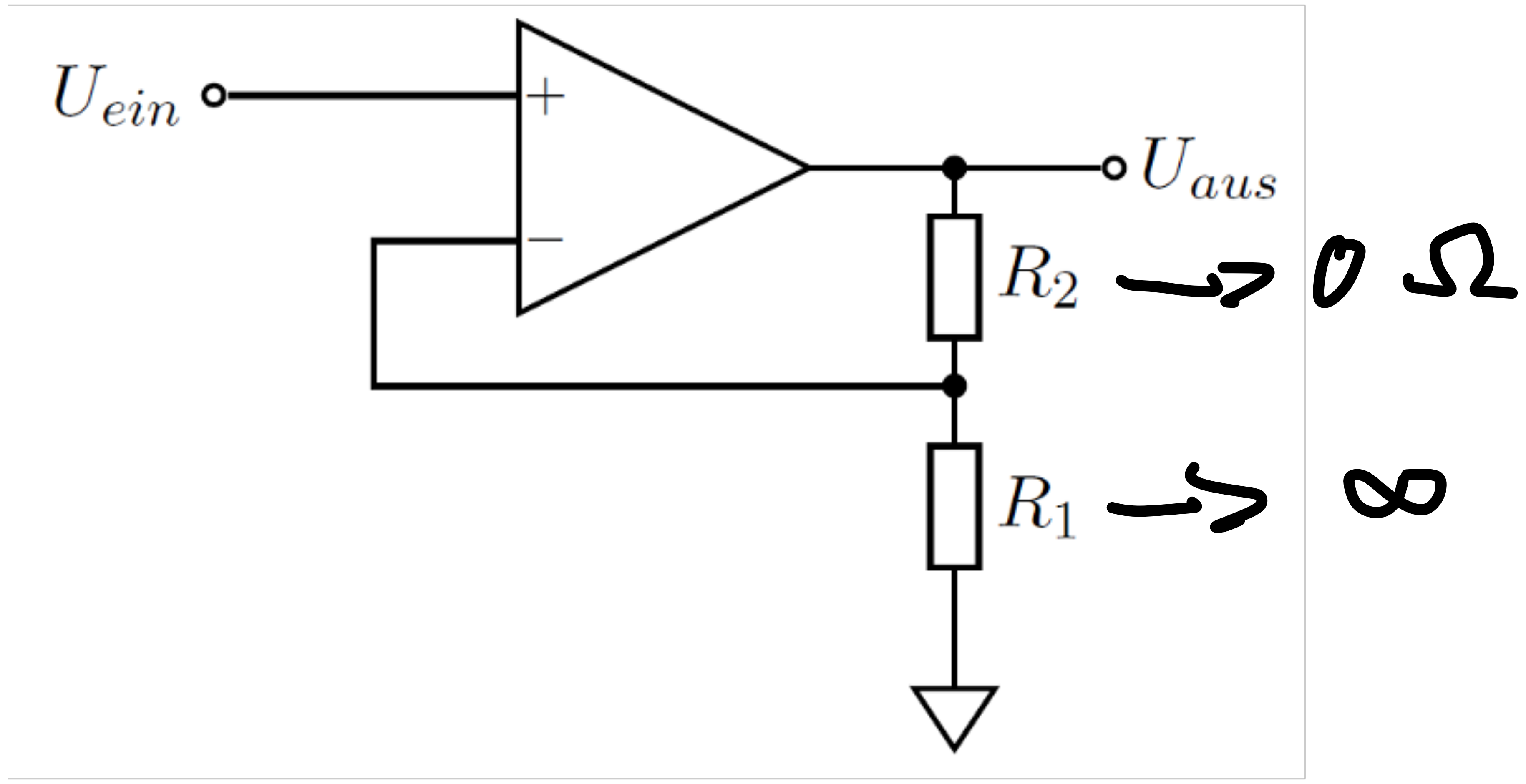


What's the point?

- "Extreme case" of non-inverting amplifier..

Amplification:

$$\frac{U_{aus}}{U_{ein}} = 1 + \frac{R_2}{R_1} \approx 1$$



But: Output current can be high (small output impedance of OpAmp) without putting a load on the input source.
NB: In realistic cases one would use a (not too large) resistor for the feedback loop.

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A brief Break

Lecture Survey

Lecture Survey

Please fill it!

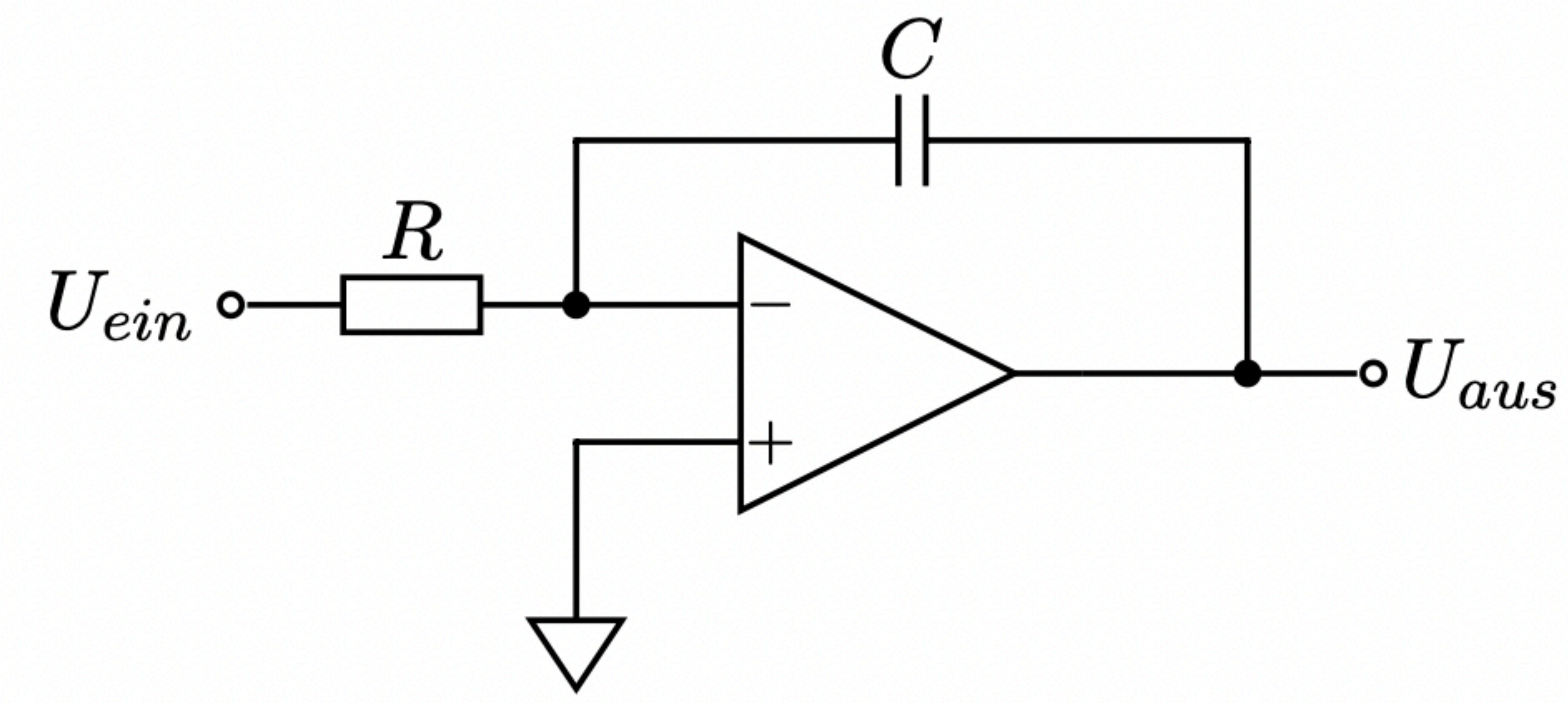
https://onlineumfrage.kit.edu/evasys/online.php?p=HVC7V

OpAmp Circuits - Part I

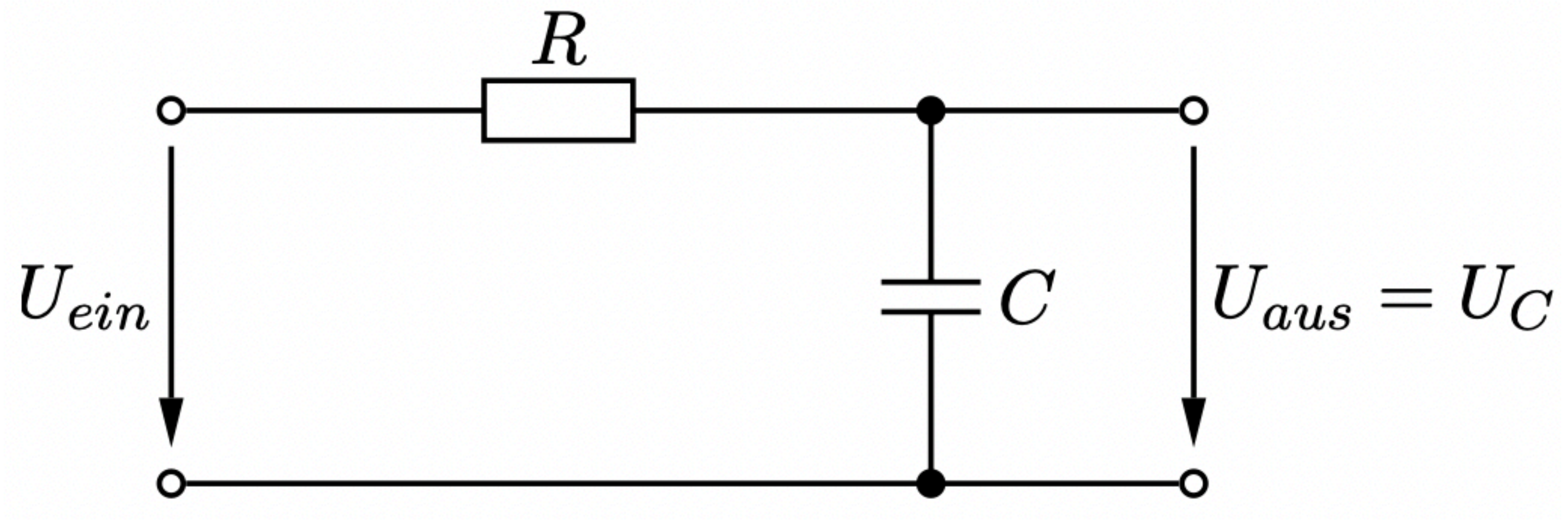
In: Chapter 4: Operational Amplifiers

Integrator / Low Pass

Active vs passive Circuit



for reference: passive integrator



Inverting amplifier, with C instead of R

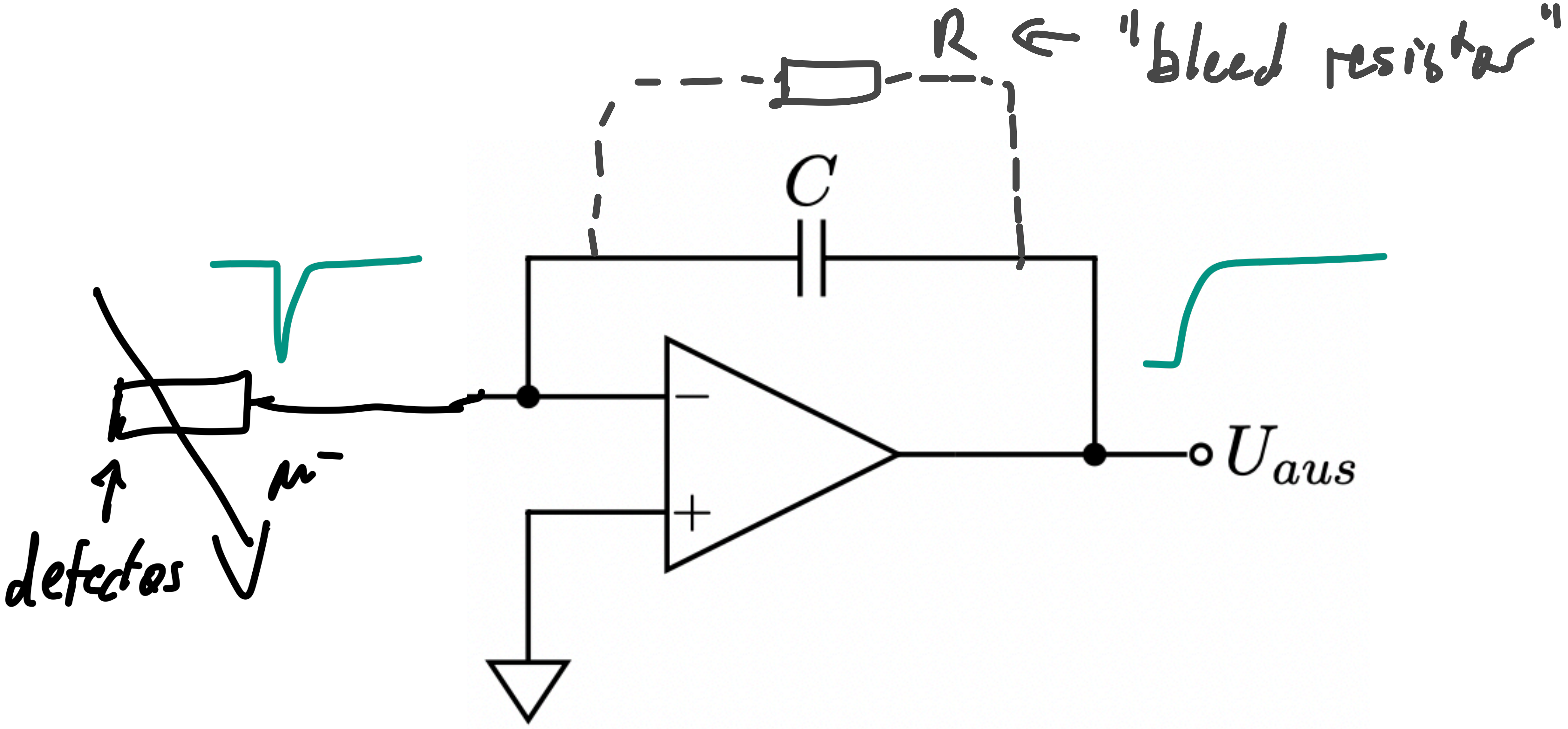
Using the Golden Rules: $$\frac{U_{ein}}{R_1} = -C\frac{dU_{aus}}{dt}$$ (I = 0 on the input)

$$U_{aus} = -\frac{1}{RC}\int U_{ein}dt$$

In contrast to the passive integrator: no restriction of the integration time to t << RC

Integrator: Charge-sensitive Amplifier

Charge-integrating Amplifier (CIA / CSA)



Important application: charge-sensitive pre-amplifier for particle detectors.

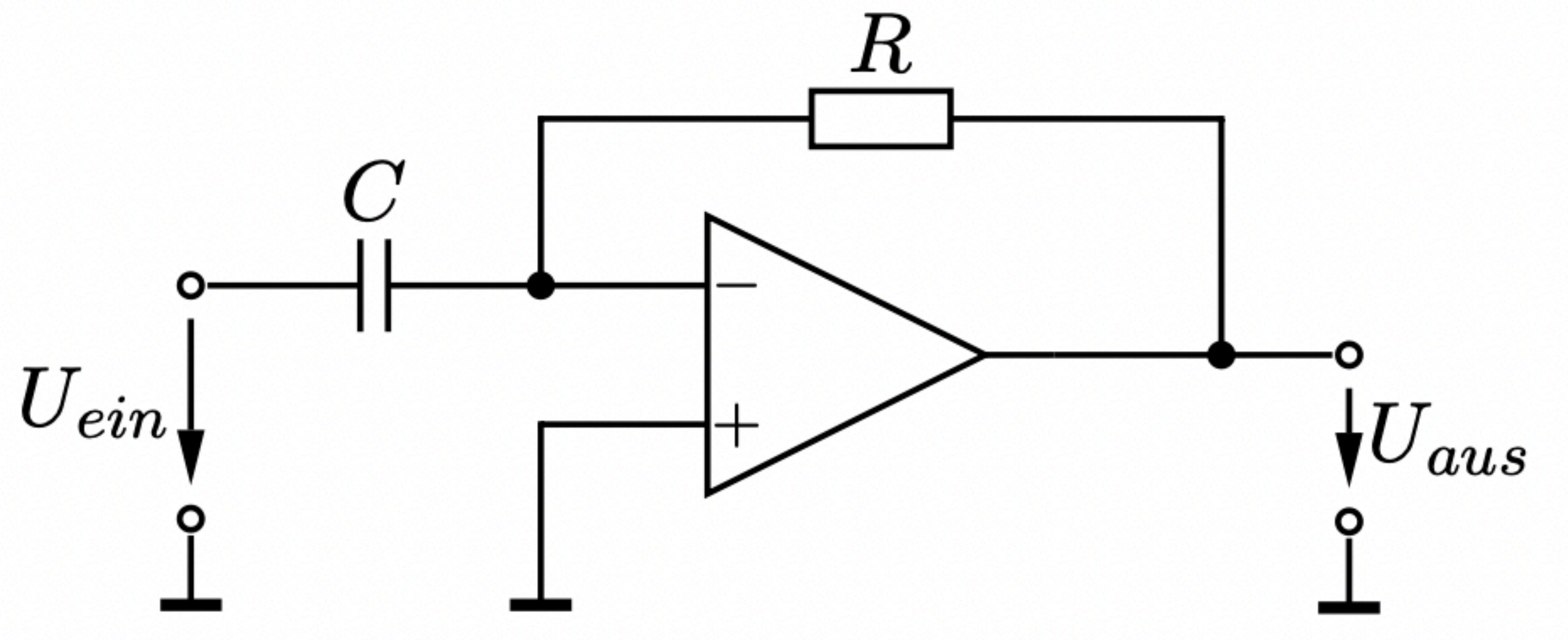
Current source (charge source) as input (instead of U_{ein} and R):

$$U_{aus} = -\frac{1}{C}\int I_{ein} dt = -\frac{Q}{C}$$

Typical gains for detector applications: mV/fC. What does this say about the scale of C?

Differentiator

and the comparision with the passive high pass

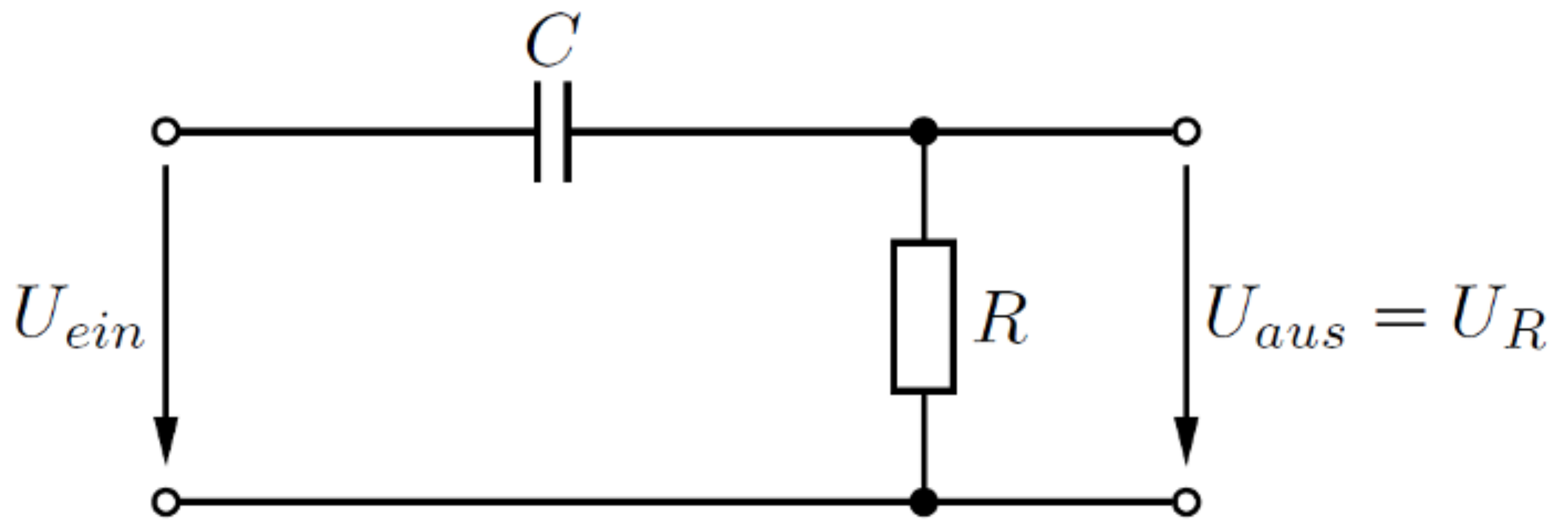


Applying the Golden Rules: $U_{aus} = -RC\frac{dU_{ein}}{dt}$

(I = 0 on input)

Superior to the passive high pass:

Good linearity!



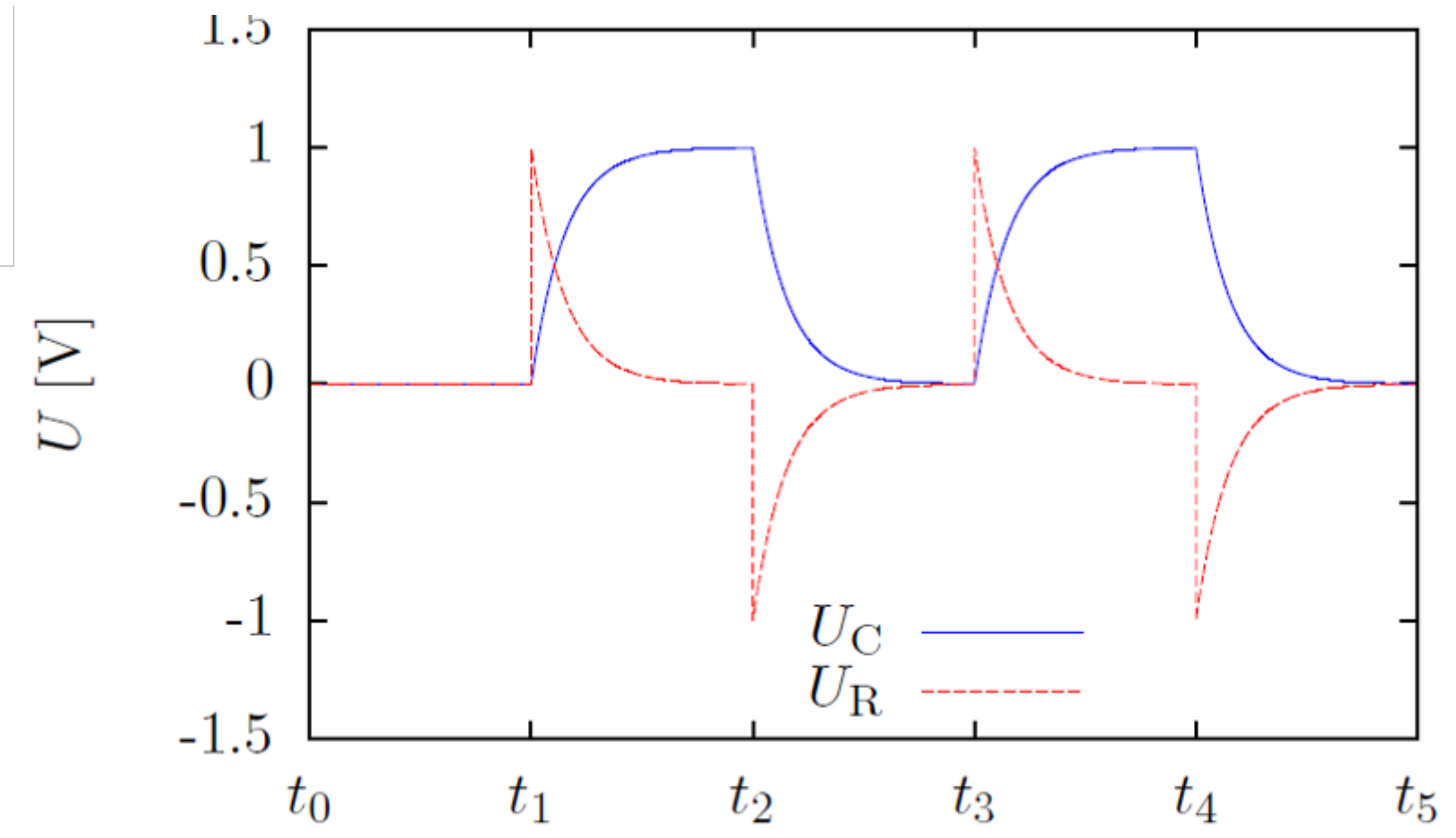
see Chapter 2

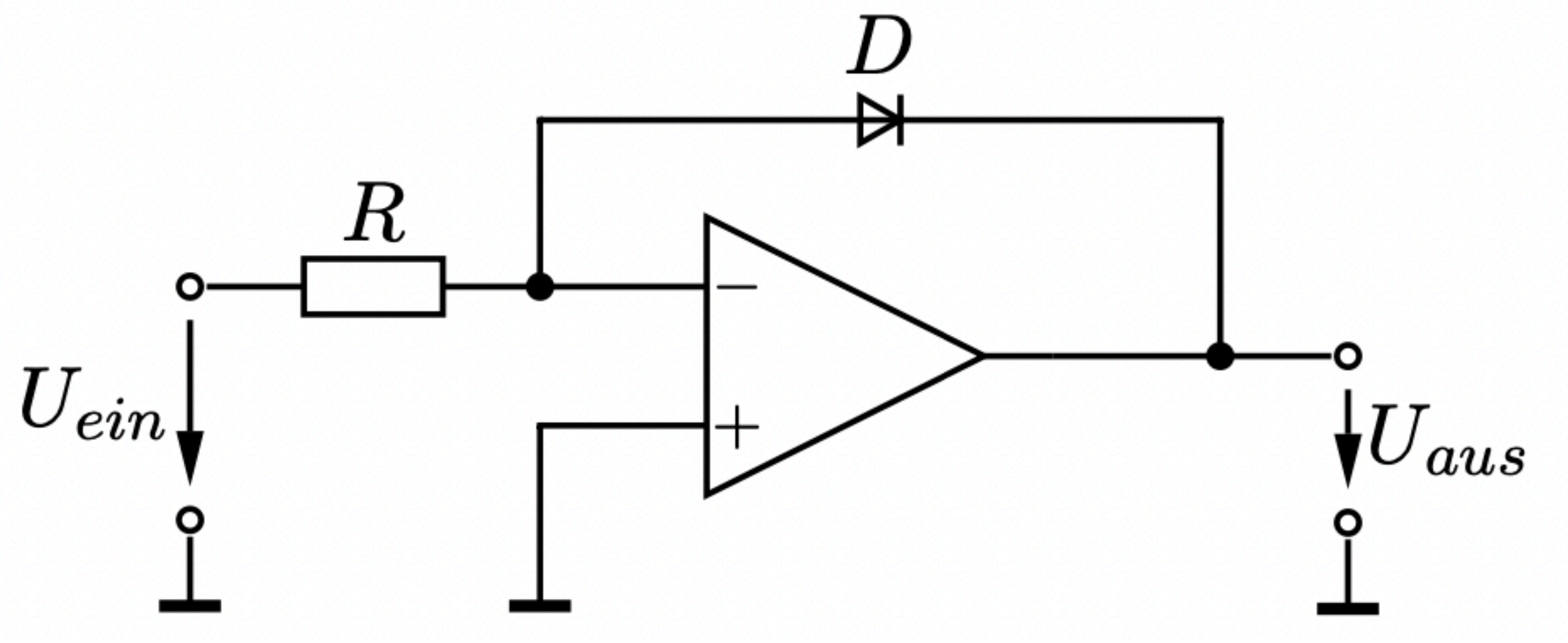
$$U_{ein} = U_C + RC\frac{dU_C}{dt}$$

$$U_{ein} - U_C = RC\frac{d(U_{ein} - U_{aus})}{dt} = U_{aus}$$

For t << RC or small R:

$$U_{aus} = RC\frac{dU_{ein}}{dt}$$





Using the exponential IV curve of a forward-biased diode:

$$U_{ein} = RI_{ein}$$

IV curve:

$$I = I_S \left[e^{\frac{q_e U}{k T}} - 1 \right] = I_S \left[e^{\frac{U}{U_T}} - 1 \right]$$

See Chapter 03: Shockley Equation

Results in:

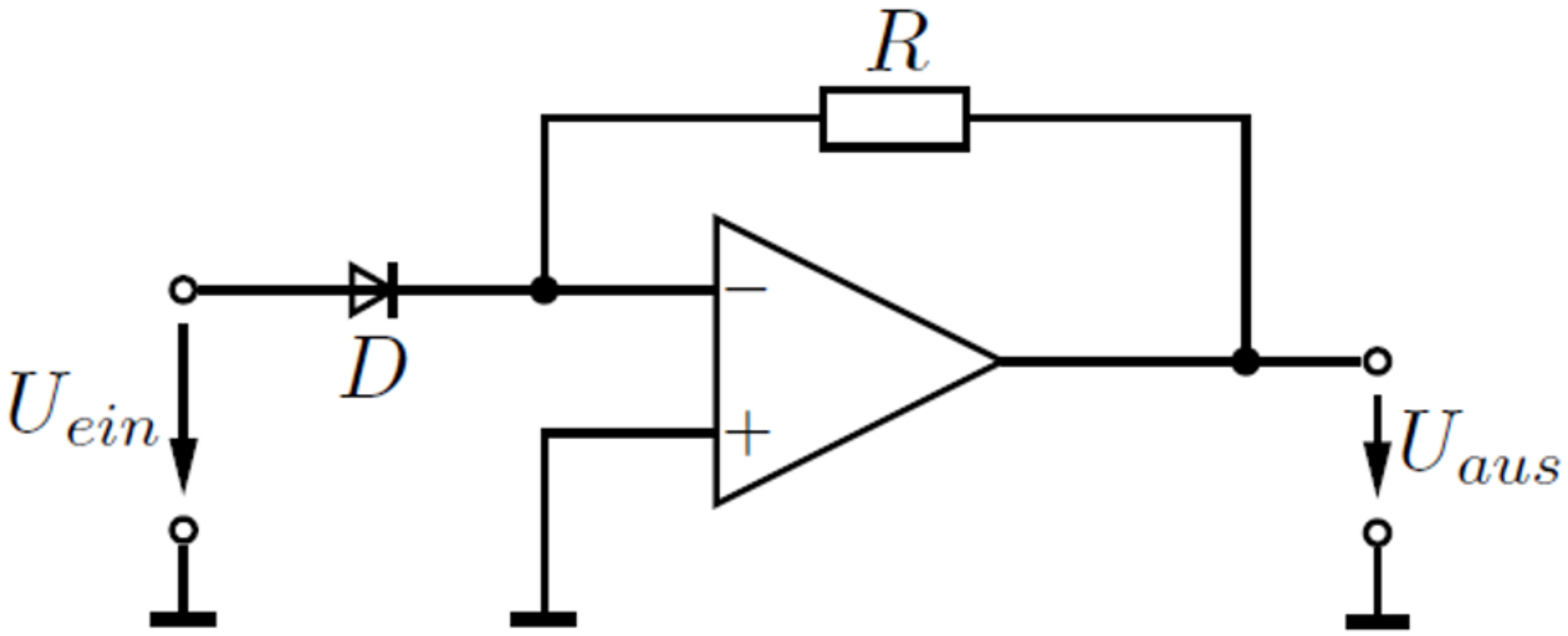
$$\frac{U_{ein}}{RI_S} + 1 = e^{-\frac{q_e U_{aus}}{kT}}$$

$$U_{aus} = -\frac{kT}{q_e} ln(\frac{U_{ein}}{RI_S} + 1) \approx -\frac{kT}{q_e} ln(\frac{U_{ein}}{RI_S})$$

Output signal depends on logarithm of input signal!

Exponential Amplifier

Combining OpAmp and Diode



- Same principle, but putting the diode on the input:

$$I_{ein} = I_D = I_S(e^{-\frac{q_e U_{ein}}{kT}} - 1)$$

Golden Rules:

$$I_R = I_{ein} = -\frac{U_{aus}}{R}$$

=> $U_{aus} \approx -RI_S\, e^{-\frac{q_e U_{ein}}{kT}}$ With that: Output signal depends exponentially on input signal

Taking all this together: Why is this relevant?

Analog Computers

Using OpAmps et al.

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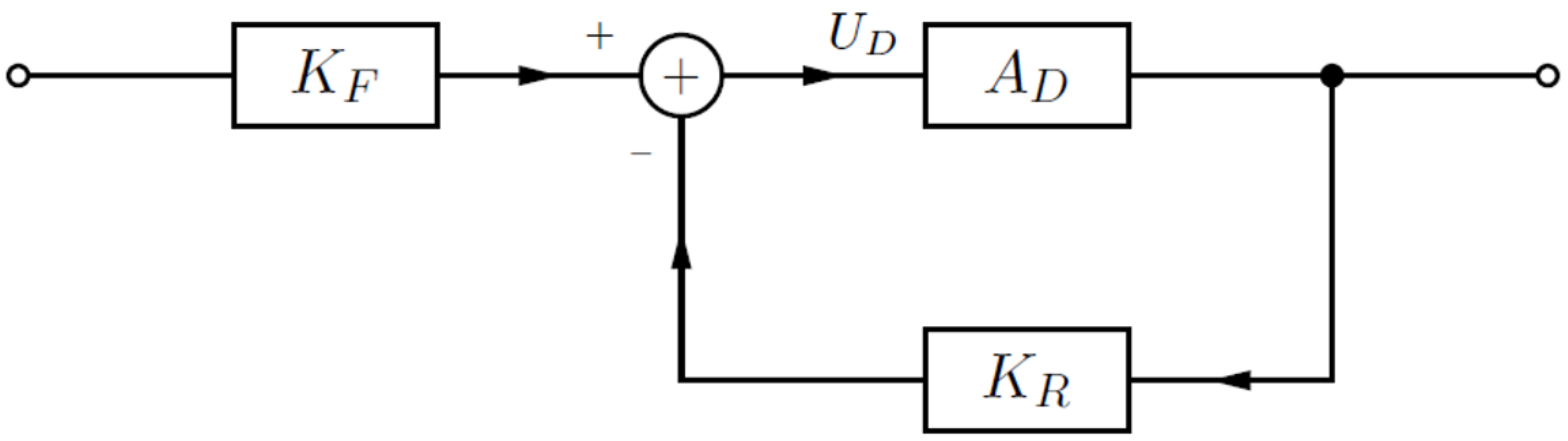
- In the 50ies, 60ies und into the 70ies electrical analog computers were used.
- OpAmps that were interconnected to add, subtract, multiply, integrate,…

Real OpAmps, Replacement Circuits

In: Chapter 4: Operational Amplifiers

OpAmp as a Control Loop

Regelkreis



An OpAmp with negative feedback can be considered as a *control loop.*

K_F: Attenuation of the input voltage (for simplicity often assume =1)

K_R: Attenuation of the output voltage in the feedback loop.

U_D: Differential voltage

A_D: Open loop gain of OpAmp

$$U_D = K_F U_{ein} - K_R U_{aus}$$

$$U_{aus} = A_D U_D$$

$$U_{ein} = U_D \frac{1 + K_R A_D}{K_F}$$

Amplification of full circuit (with feedback, *closed-loop gain*)

$$A = \frac{U_{aus}}{U_{ein}} = \frac{K_F A_D}{1 + K_R A_D} \approx \frac{K_F}{K_R}$$

Summary of Definitions

Parameters of OpAmp Circuits - Non-inverting Amplifier as Example

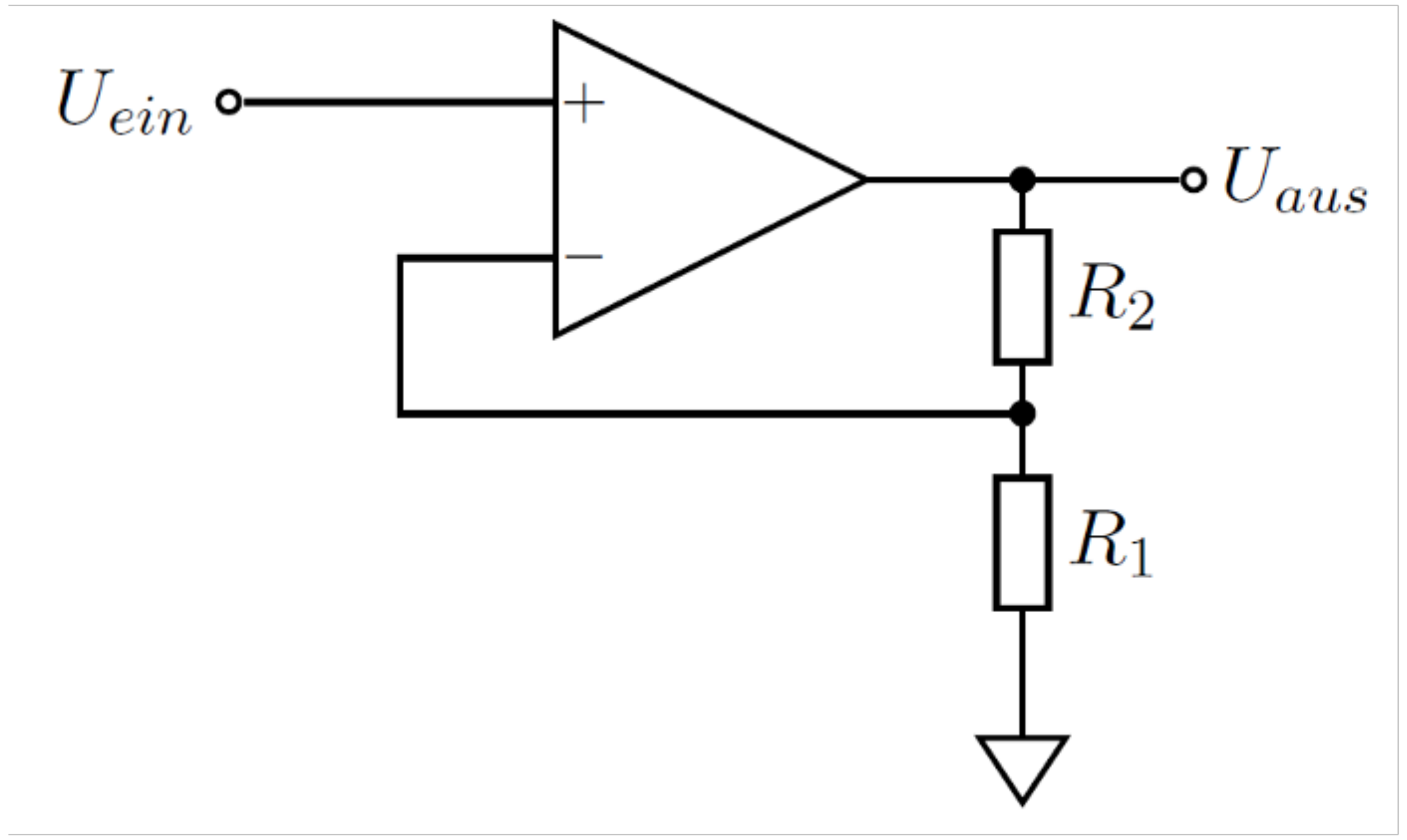
A_D: Open Loop gain of OpAmps (*Leerlaufverstärkung*)

A: Closed loop gain (*Verstärkung mit Rückkopplung*)

$$A = \frac{U_{aus}}{U_{ein}} = \frac{K_F\, A_D}{1 + K_R\, A_D} \approx \frac{K_F}{K_R}$$

$K_R A_D$: Loop gain (*Schleifenverstärkung*)

Example: non-inverting amplifier



$$K_F = 1$$

$$K_R = \frac{R_1}{R_1 + R_2}$$

$$A = \frac{K_F}{K_R} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_1}{R_2}$$

Numerical example

$R_1 = 9\ R_2$

=> $K_R = 1/10$

=> $A = 10$

$A_D = 10^5$

with that $K_R A_D = 10^4$

Summary of Definitions

Parameters of OpAmp Circuits - Inverting Amplifier as Example

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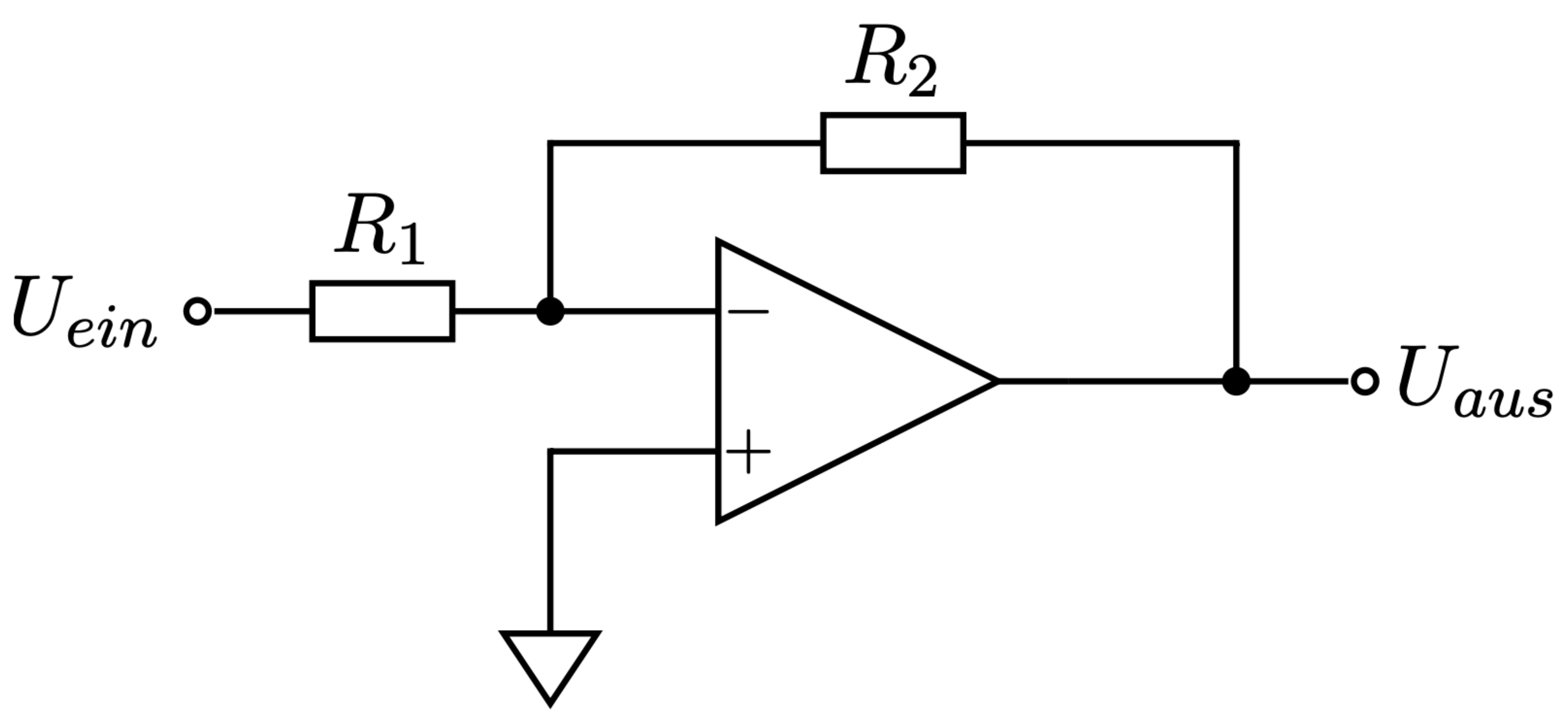
A_D: Open Loop gain of OpAmps (*Leerlaufverstärkung*)

A: Closed loop gain (*Verstärkung mit Rückkopplung*)

$$A = \frac{U_{aus}}{U_{ein}} = \frac{K_F A_D}{1 + K_R A_D} \approx \frac{K_F}{K_R}$$

$K_R A_D$: Loop gain (*Schleifenverstärkung*)

Example: inverting amplifier



$$K_F = -\frac{R_2}{R_1 + R_2}$$

$$K_R = \frac{R_1}{R_1 + R_2}$$

$$A = \frac{K_F}{K_R} = -\frac{R_1}{R_2}$$

Numerical example

$R_1 = 9\ R_2$

=> $K_R = 9/10$

=> $K_F = -1/10$

=> $A = -9$

$A_D = 10^5$

resulting in $K_R A_D = 9 \times 10^4$

Real Operational Amplifiers

Parameters

Größe	Symbol	ideal OPV	real OPV
common-mode impedance	R_G	∞	$10^8 \ldots 10^{12}\,\Omega$
differential input impedance	R_D	∞	$10^5 \ldots 10^7\,\Omega$
output impedance	R_{aus}	0	$0,1 \ldots 100\,\Omega$
differential gain	A_D	∞	$10^4 \ldots 10^7$
common-mode gain	A_G	0	$1 \ldots 10$
input bias current	I_{OG}	0	$1\,\mathrm{pA} \ldots 1\,\mu\mathrm{A}$
offset voltage	U_{OS}	0	$10\,\mu\mathrm{V} \ldots 10\,\mathrm{mV}$

- Parameters w/o feedback

Amplification of $U_+ - U_-$

Amplification of $U_+ + U_-$

$$U_{aus} = A_D\,(U_+ - U_-) + A_G\,(U_+ + U_-)$$

common mode rejection ration (CMRR)" **A_D/A_G** should be as large as possible! *"Gleichtacktunterdrückung"*

idealized: U_{aus} = 0 V for $U_+ = U_-$ = 0 V.
Deviation **U_{OS}** ("offset voltage")
depends on temperature, supply voltage, but also component ageing,…

Real Operational Amplifiers

Parameters - Table in German "for the record"

Größe	Symbol	ideal OPV	real OPV
Gleichtakteingangsimpedanz	R_G	∞	$10^8 \dots 10^{12}\,\Omega$
Differenzeingangsimpedanz	R_D	∞	$10^5 \dots 10^7\,\Omega$
Ausgangsimpedanz	R_{aus}	0	$0,1 \dots 100\,\Omega$
Differenzverstärkung	A_D	∞	$10^4 \dots 10^7$
Gleichtaktverstärkung	A_G	0	$1 \dots 10$
Eingangsruhestrom	I_{OG}	0	$1\,\text{pA} \dots 1\,\mu\text{A}$
Offsetspannung	U_{OS}	0	$10\,\mu\text{V} \dots 10\,\text{mV}$

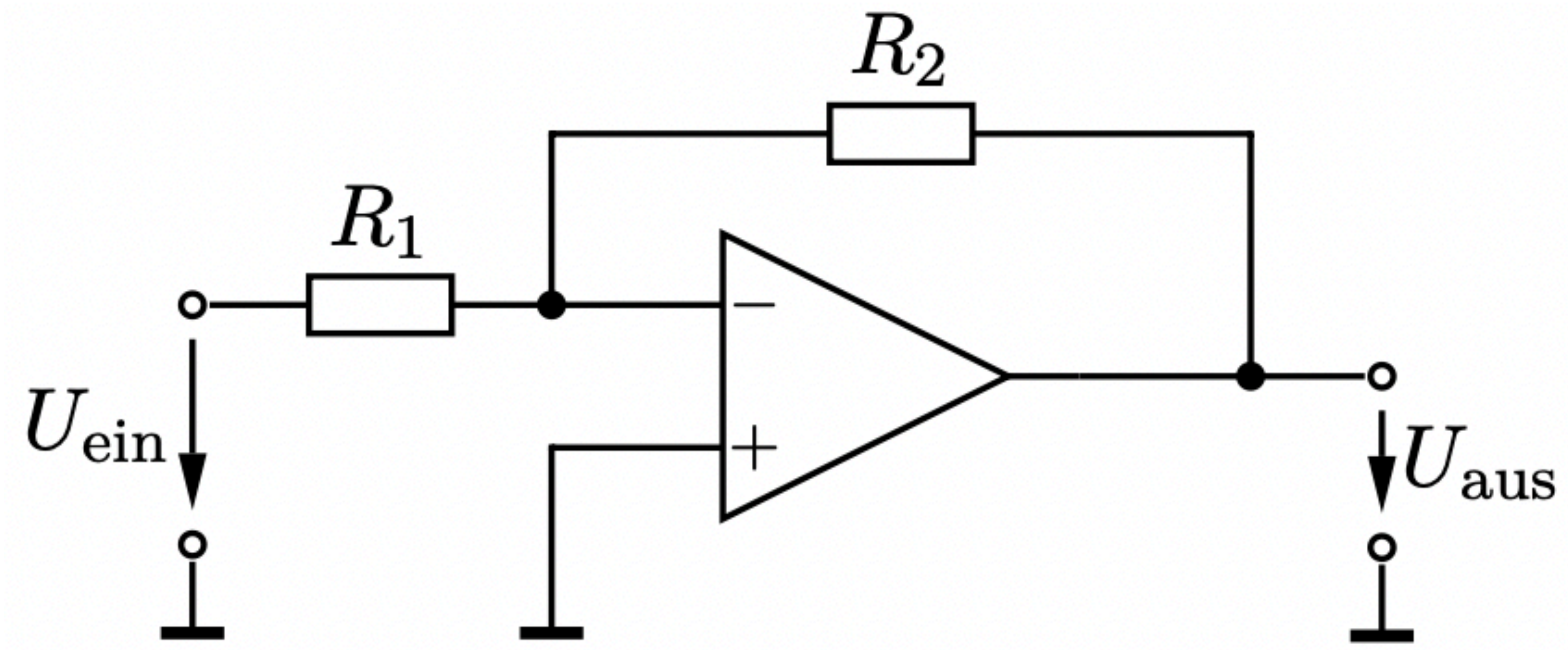
- Parameters w/o feedback

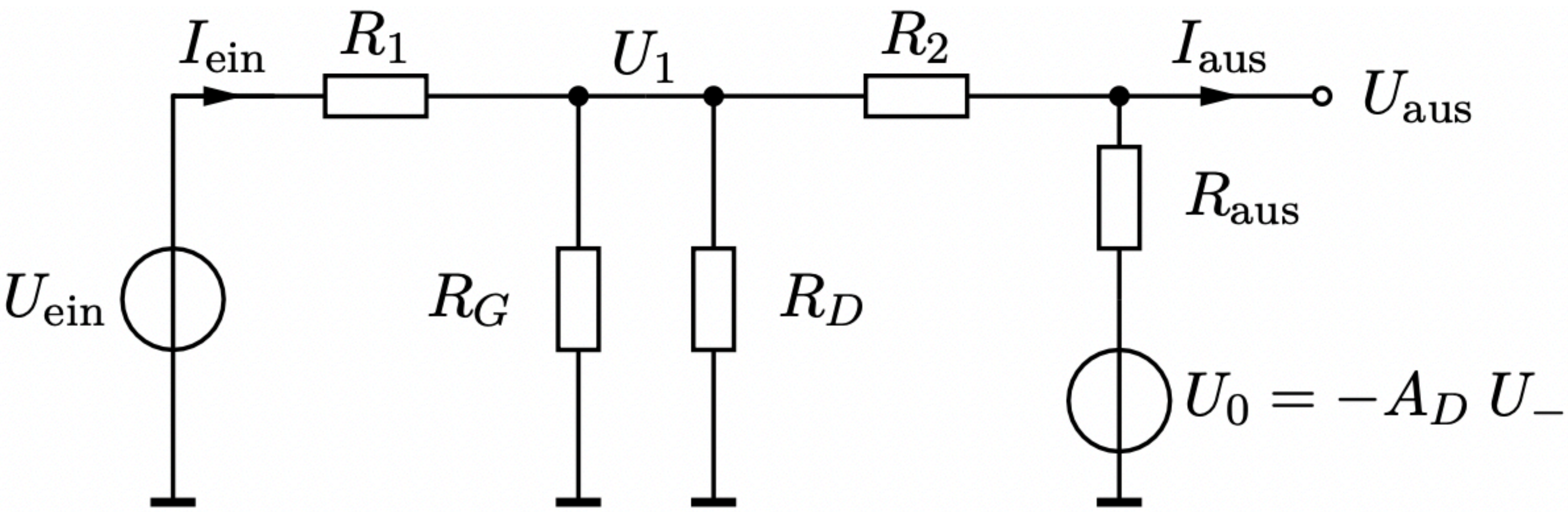
Amplification of $U_+ - U_-$

Amplification of $U_+ + U_-$

Equivalent Circuit

Inverting Amplifier





$U_- = U_1$ Voltage on inverting input

Input Impedance?

Output Impedance?

Definitions:

$$Z_{ein} = \frac{dU_{ein}}{dI_{ein}}$$

for open output $I_{aus} = 0$

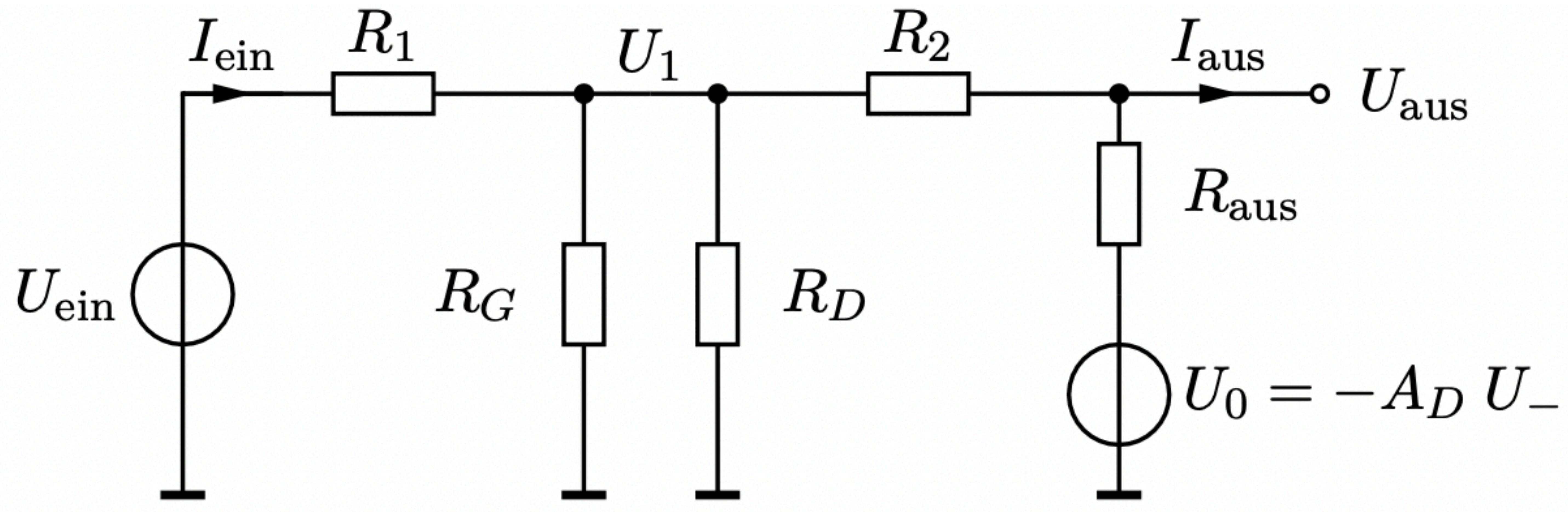
$$Z_{aus} = \frac{dU_{aus}}{dI_{aus}}$$

short circuit on input

$U_{ein} = 0$

Equivalent Circuit

Inverting Amplifier



$$I_{ein} = \frac{U_{ein} - U_-}{R_1}$$

$(R_2 + R_{aus}) = R$

can be calculated with superposition principle, assuming $R_G >> R_D >> R_1, R_2$

- not done in detail here -

$$U_- = U_{ein}\frac{R}{R_1 + R} - A_D U_- \frac{R_1}{R + R_1}$$

=>

$$U_- = U_{ein}\frac{R}{R + R_1 + R_1 A_D}$$

A_D is very large. Consequence: $U_- << U_{ein}$. => ~ 0 V ("virtual mass"), see 1st Golden Rule. Results in: $Z_{ein} = R_1$

Reminder: $Z_{ein} = \frac{dU_{ein}}{dI_{ein}}$

Next Lectures:

Digital - Tuesday, December 12

Analog 08 - Chapter 04 & 05 - Thursday, Dec. 14

Time Plan for Next Lectures

A few Changes coming up!

Calender Week	Tuesday	Thursday
45	07.11. **Analog**	09.11. **Digital**
46	14.11. **Analog**	16.11. **Digital**
47	21.11. **Digital**	23.11. **Analog**
48	28.11. **Digital**	30.11. **Digital**
49	05.12. **Digital**	07.12. **Analog**
50	12.12. **Digital**	14.12. **Analog**
51	19.12. **Analog**	21.12. **Digital**
2	09.01. **Analog**	11.01. **Analog**
3	16.01. **Digital**	18.01. **Digital**
4	23.01. **Analog**	25.01. **Digital**
5	30.01. **Analog**	01.02. **Digital**
6	06.02. **Analog**	08.02. **Digital**
7	13.02. **Analog**	15.02. **Digital**