# Electronics for Physicists

## *Analog Electronics*

*Chapter 5; Lecture 09*

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Karlsruhe Institute of Technology

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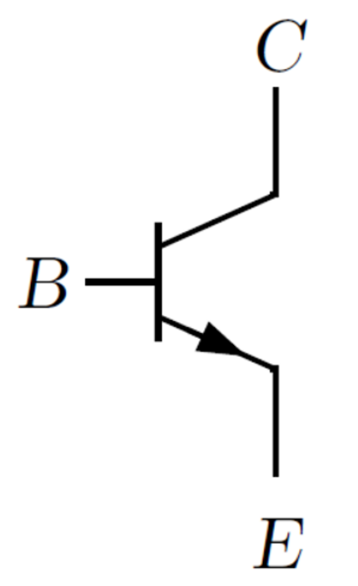
Institute for Data Processing and Electronics

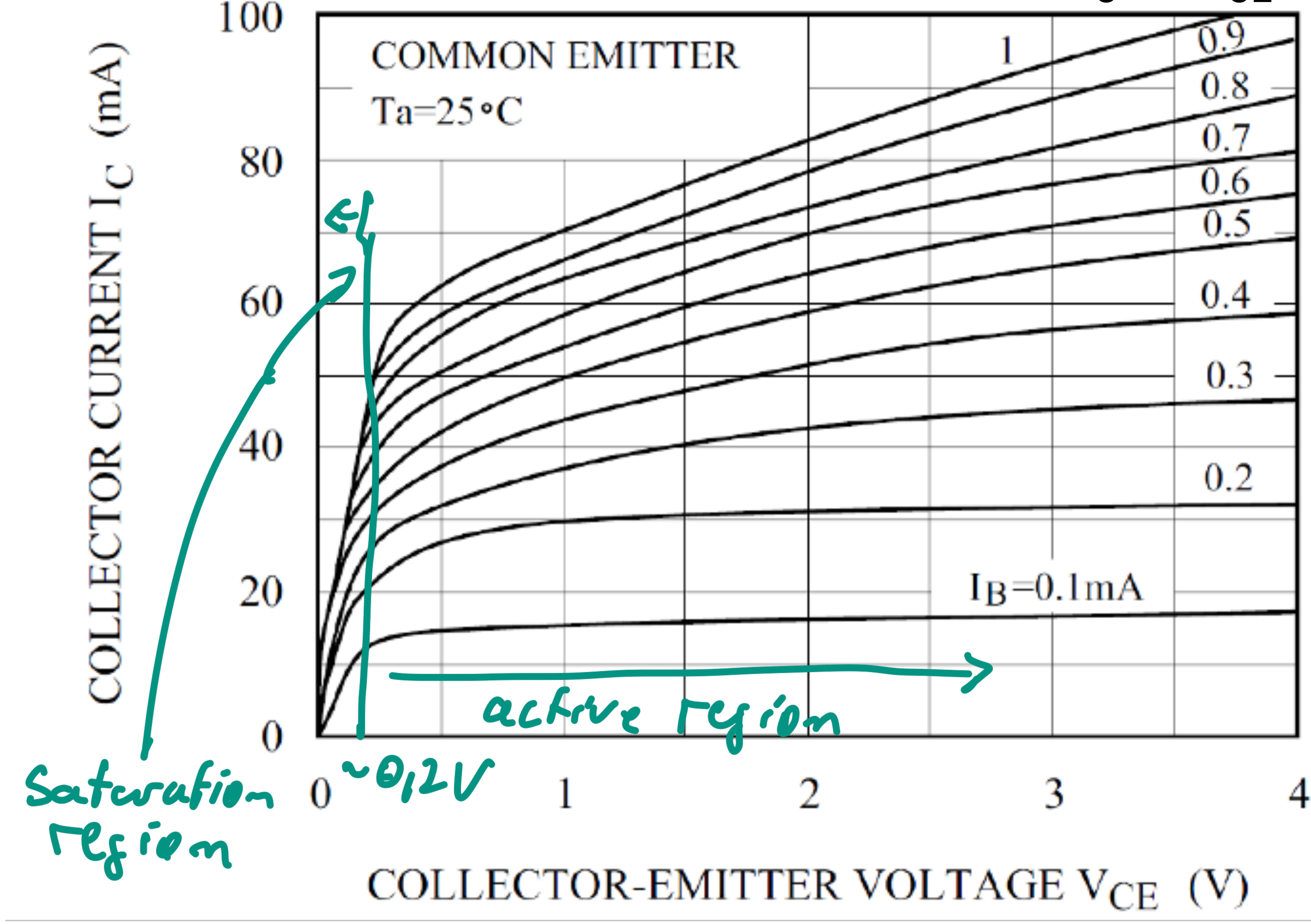
# IV Curves of Transistors

*Transistor-Kennlinien*

- With 3 terminals there is a wide range of possibilities: $I_C$ vs $U_{BE}$, $I_B$ vs $U_{BE}$, $I_C$ vs $U_{CE}$, …
  Describe the behavior of the component: All that is needed for circuit design.




$I_C$ vs $U_{CE}$

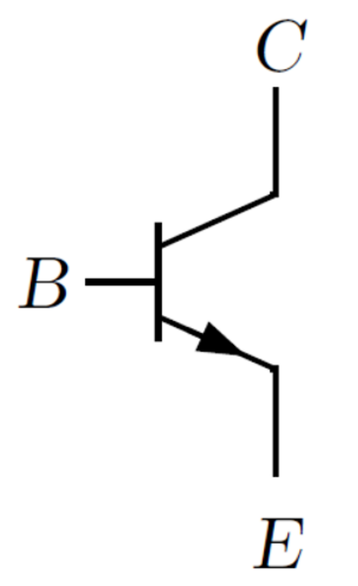



In general: watch power limit:
$I_C * U_{CE} < P_{max}$

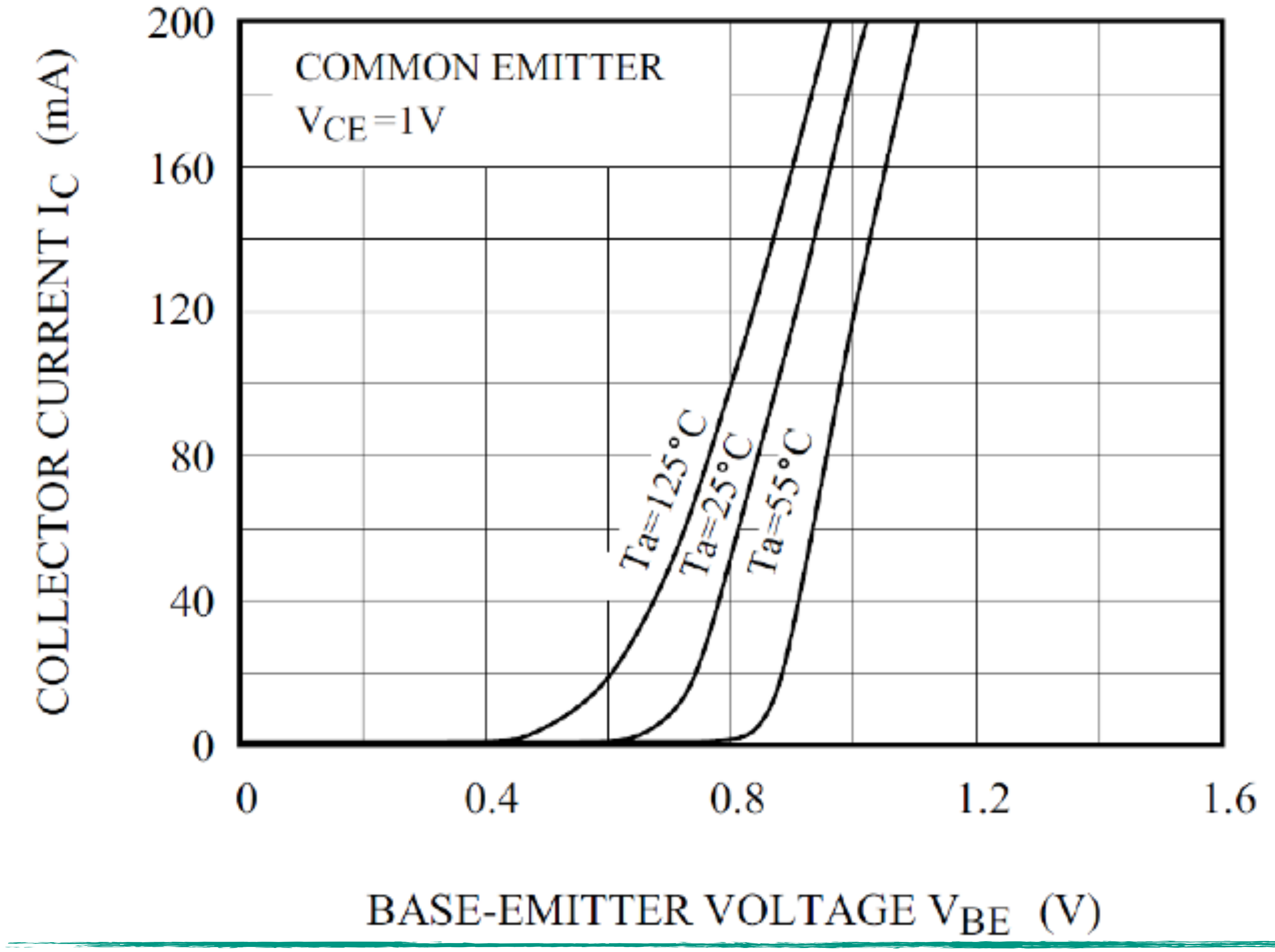
Minimal $U_{CE}$ (~ 0.2V) has to be exceeded so that collector-emitter current $I_C$ flows: "active region
Below: saturation region - not enough electrons from base to collector.

In active region:
Weak dependence of $I_C$ on $U_{CE}$:
large differential resistor $R_{CE}$

- With 3 terminals there is a wide range of possibilities: $I_C$ vs $U_{BE}$, $I_B$ vs $U_{BE}$, $I_C$ vs $U_{CE}$, …
  Describe the behavior of the component: All that is needed for circuit design.




$I_C$ vs $U_{BE}$




$U_{BE}$ determines the collector current.

Exponential dependence, just as for a diode:

Transistor as active component with amplification

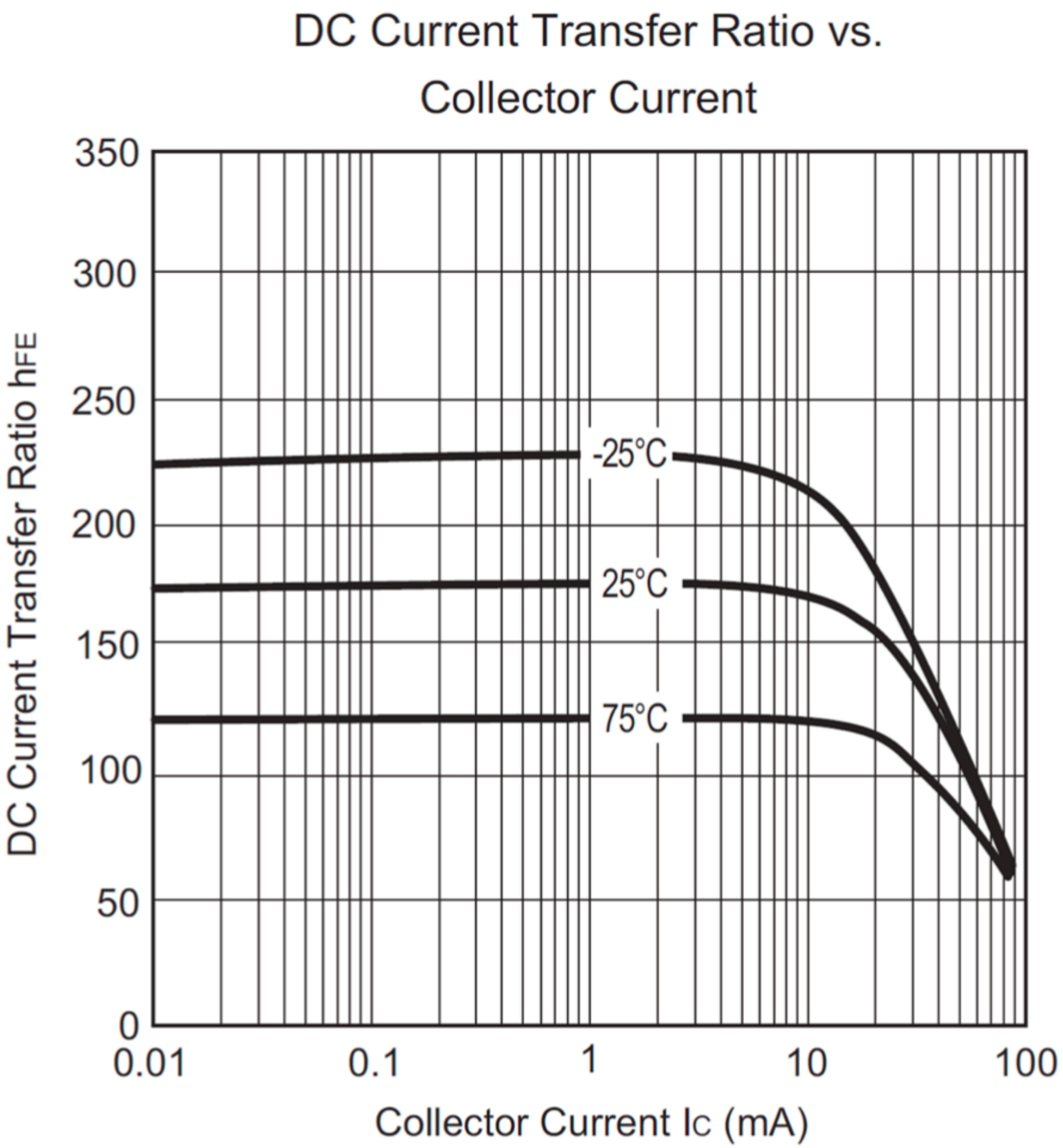
$$I_C = \beta I_S \left(e^{\frac{U_{BE}}{U_T}} - 1\right)$$

$I_S$, $U_T$ as for a diode (reverse bias saturation current, voltage due to thermal excitation durch therm. Anregung)

β: current amplification

# Current Amplification

*Stromverstärkung*




$$\beta = \frac{dI_C}{dI_B}\Big|_{U_{CE}=const}$$

- $I_C \sim \beta\, I_B$ over a wide range in current

typical values: β ~ 100 [(significantly) larger values also possible]

=> BJT as current-driven current source.

# Transistor Equations

*Behavior of Transistors*

Karlsruhe Institute of Technology

- Normally most relevant: Dependence of base current $I_B$ and collector current $I_C$ on the voltages $U_{BE}$, $U_{CE}$

$$dI_C\left(U_{BE}, U_{CE}\right) = \frac{\partial I_C}{\partial U_{BE}} dU_{BE} + \frac{\partial I_C}{\partial U_{CE}} dU_{CE} = S\, dU_{BE} + \frac{1}{R_{CE}} dU_{CE}$$

$$dI_B\left(U_{BE}, U_{CE}\right) = \frac{\partial I_B}{\partial U_{BE}} dU_{BE} + \frac{\partial I_B}{\partial U_{CE}} dU_{CE} \approx \frac{1}{R_{BE}} dU_{BE}$$

(2. term neglegted)

with:

$$R_{BE} = \frac{\partial U_{BE}}{\partial I_B}$$

differential base-emitter resistance

$$R_{CE} = \frac{\partial U_{CE}}{\partial I_C}$$

differential collector-emitter resistance

$$S = \frac{\partial I_C}{\partial U_{BE}}$$

transconductance (“Steilheit”)

# Simple Transistor Model

*Based on Diode Behavior*

- Considering the base-emitter diode

$$I_B = I_S \left[ e^{\frac{U_{BE}}{U_T}} - 1 \right] \approx I_S \, e^{\frac{U_{BE}}{U_T}}$$ Shockley equation

in addition: $U_T = \frac{k\,T}{q_e}$ (see Chapter 3) and $I_C = \beta \, I_B$ Amplification of base current in collector (see S 15)

results in: $R_{BE} = \frac{\partial U_{BE}}{\partial I_B} = \frac{k\,T}{q_e \, I_B}$

$$S = \frac{\partial I_C}{\partial U_{BE}} = I_C \frac{q_e}{k\,T} = \frac{I_C}{U_T}$$

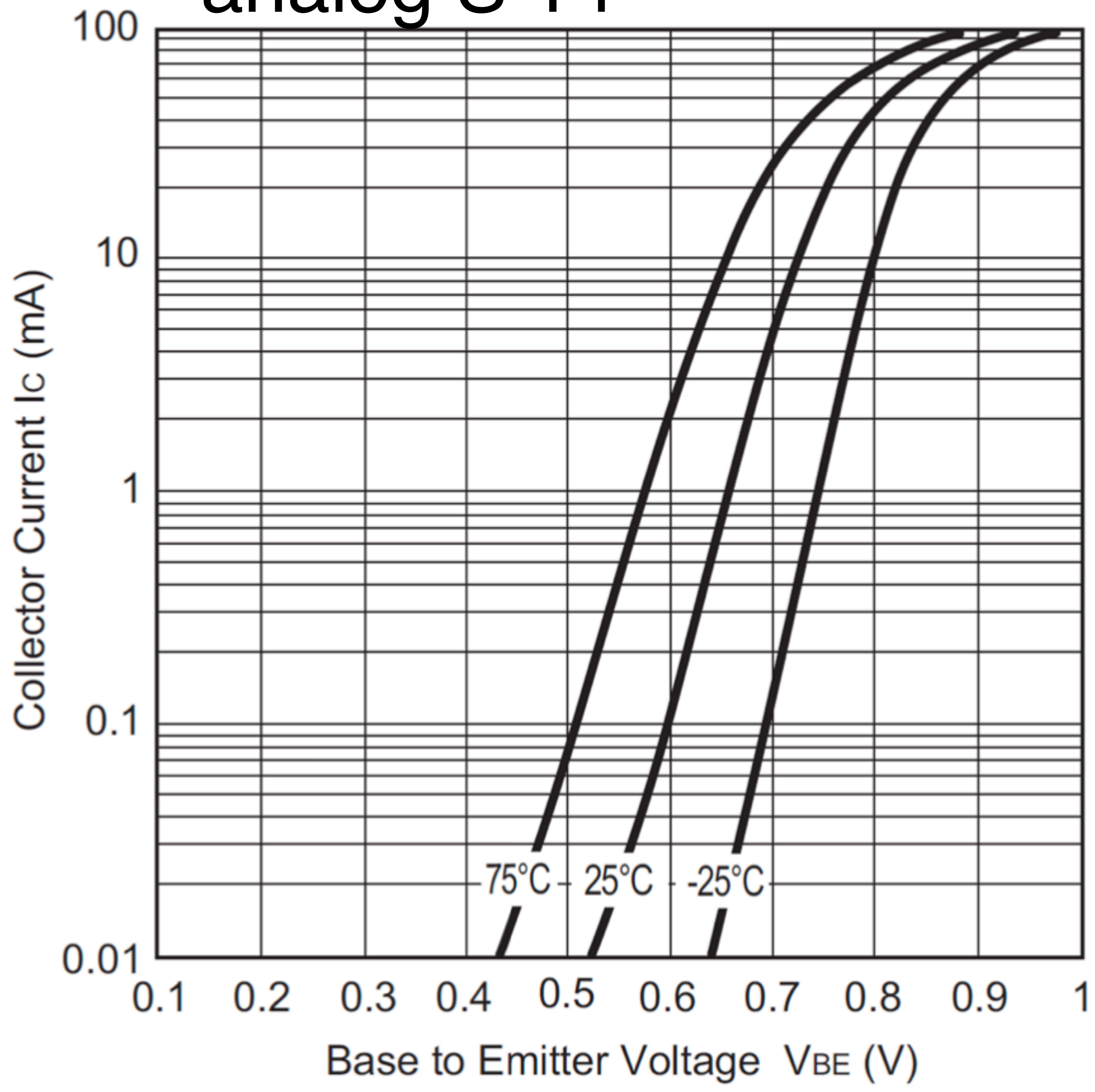
with kT/$q_e$ = 25 mV at room temperature: $S[\frac{1}{\Omega}] = 40 \, I_C[\mathrm{A}]$

=> Transconductance independent of transistor details, depends on $I_C$, T

Connection of transconductance and current amplification:

$$S = \frac{\partial I_C}{\partial U_{BE}} = \beta \frac{\partial I_B}{\partial U_{BE}} = \frac{\beta}{R_{BE}} \quad \text{und} \quad R_{BE} = \frac{k\,T}{q_e \, I_B} = \frac{U_T}{I_B}$$

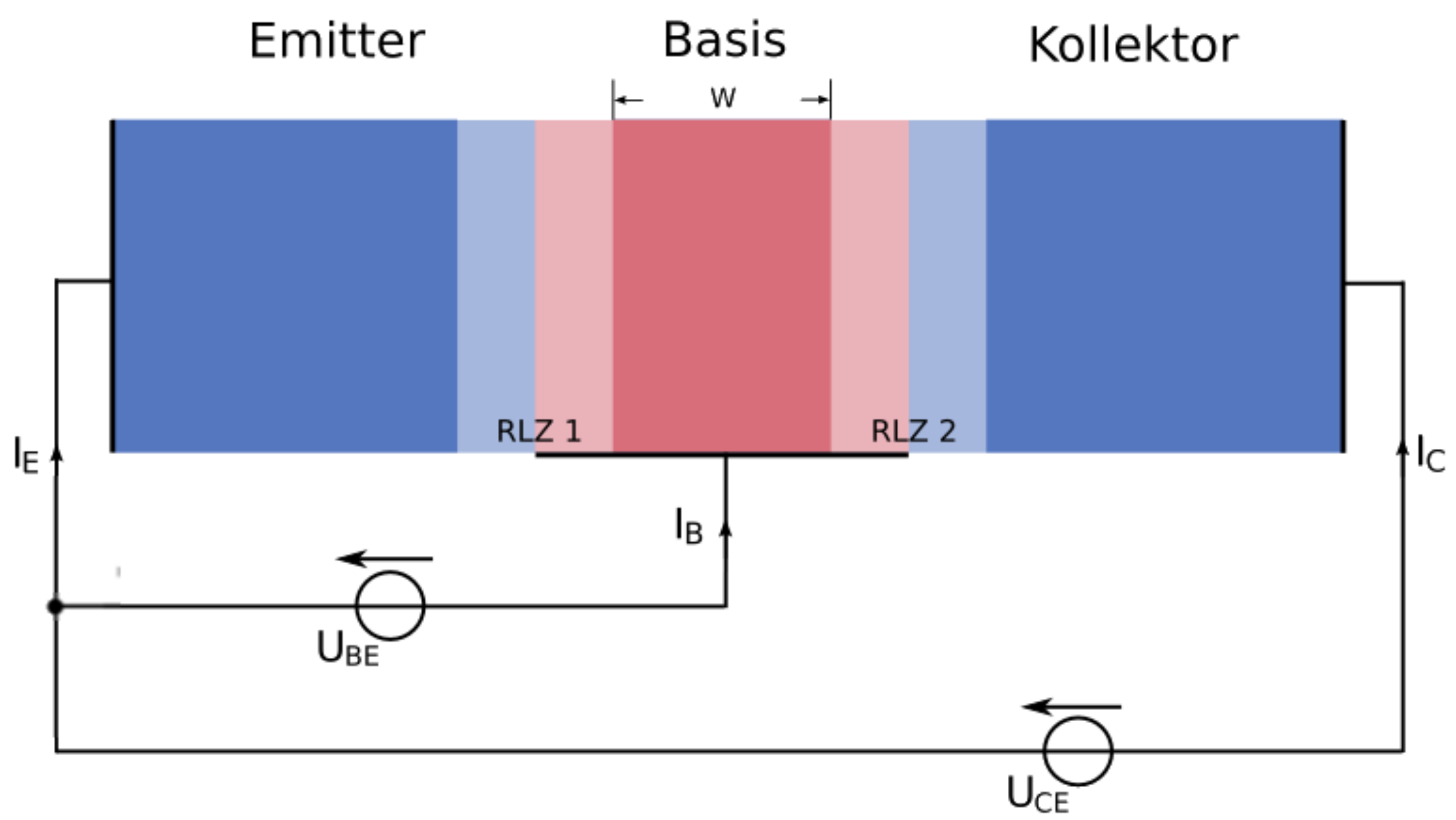
analog S 14



# Early Effect, Early Voltage

*Early Effekt, Early Spannung*

- Ideal current source:
  $I_C$ in actuve region independent of $U_{CE}$, only depends on $I_B$
- In reality: Space charge region increases with increasing voltage, which reduces the thickness of the base: Increase if amplification β - Slope of $I_C$-$U_{CE}$ curve -> ***Early Effect***




Slope of IV curve in active region: $1/R_{CE}$, with $R_{CE} = \frac{\partial U_{CE}}{\partial I_C}$

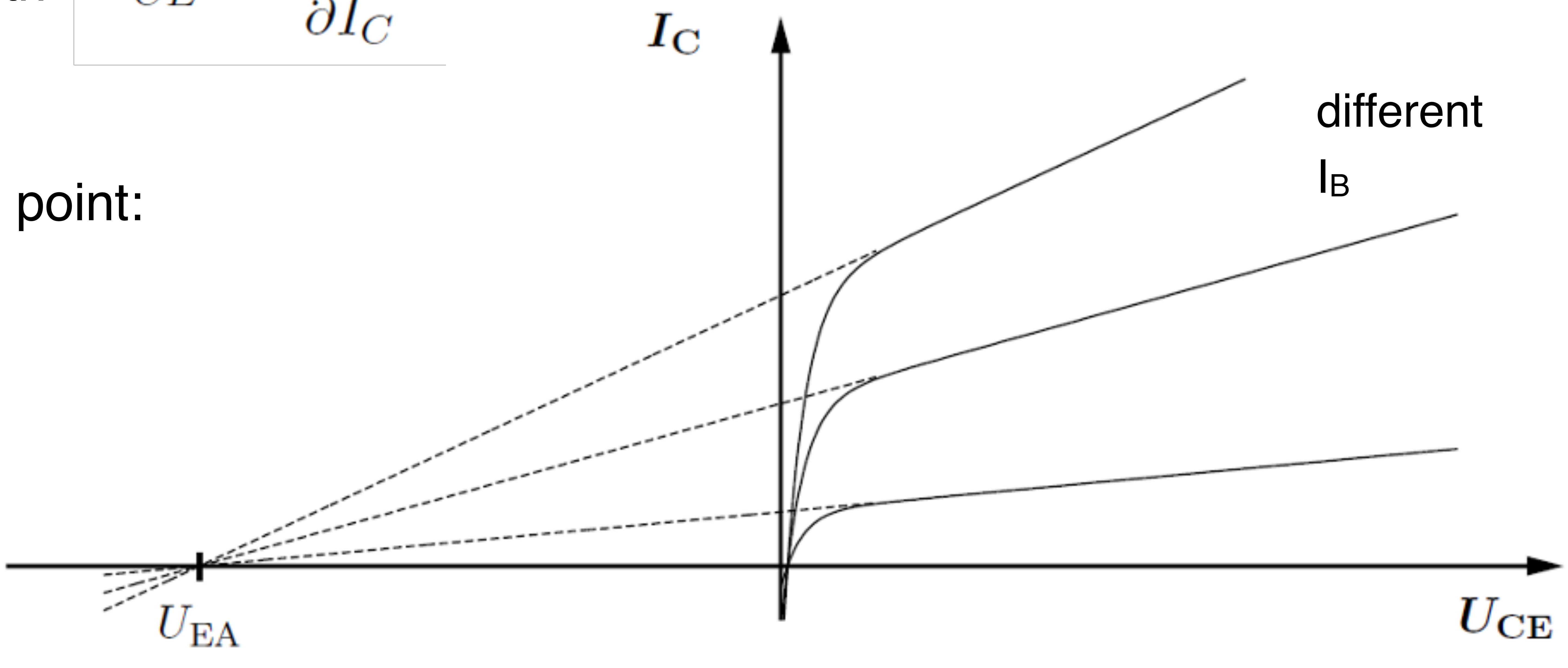
depends on $I_B$ - steeper slope for larger $I_B$

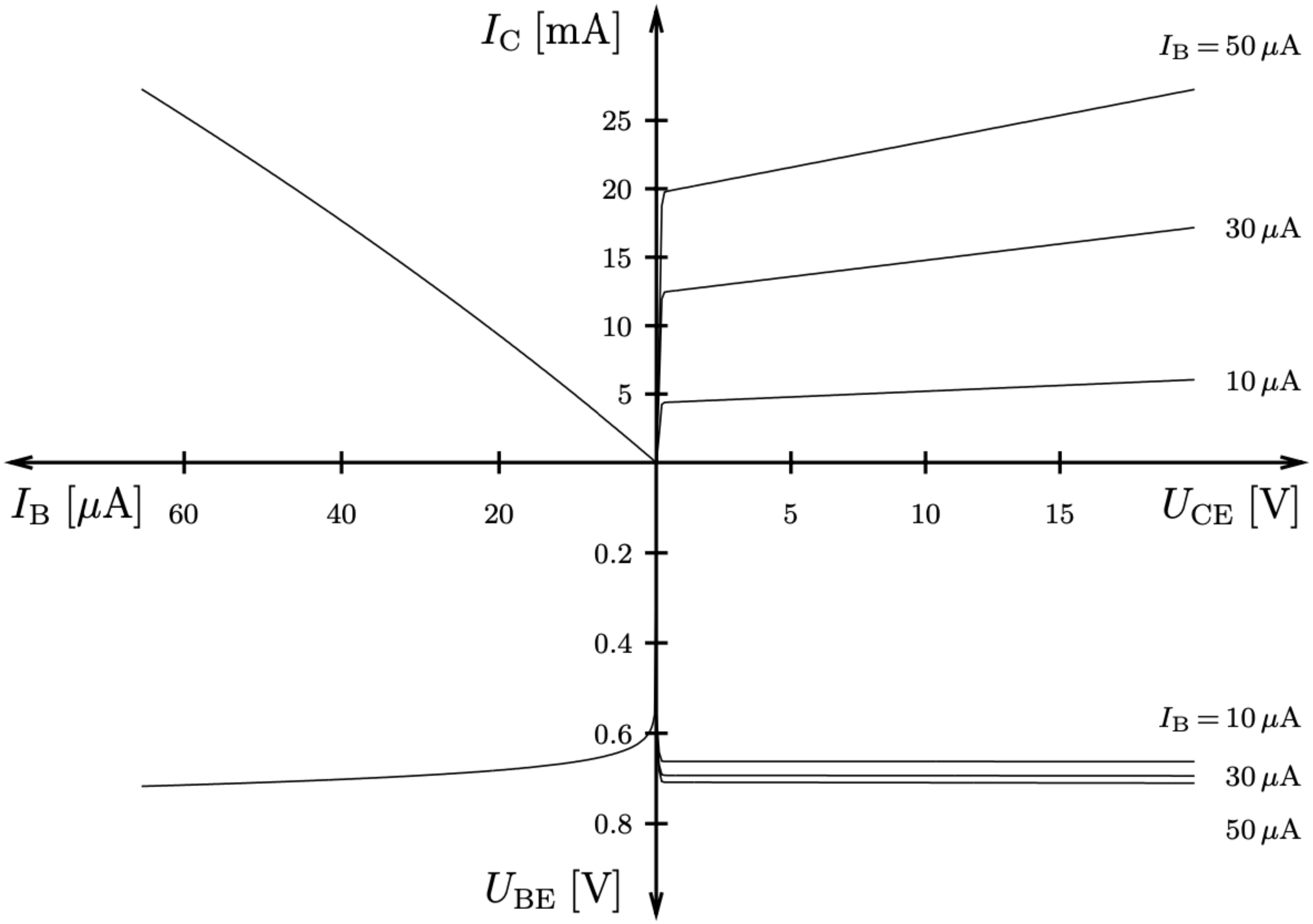
Extrapolated IV curves meet all meet in one point:
$U_{EA}$ ***Early Voltag***

with that: $R_{CE} = \frac{U_{EA}}{I_{C0}}$

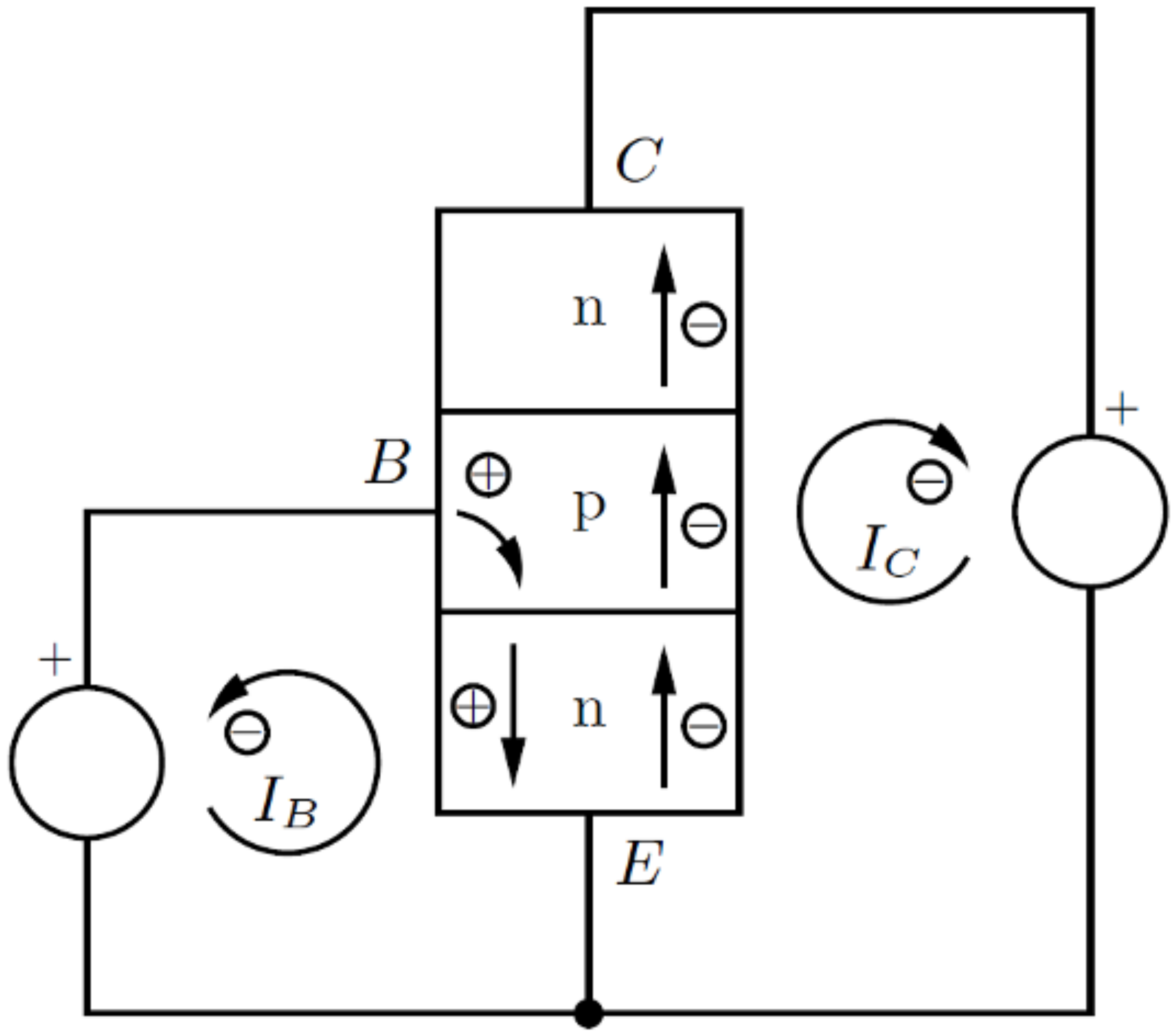
Typical values: 15 - 150 V

$R_{CE}$ relatively large, ~ 100 kΩ

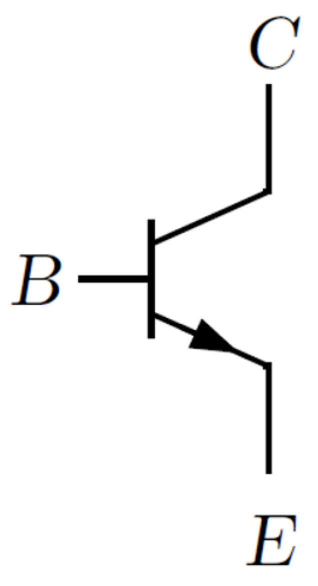




$I_C$ [mA]
25
20
15
10
5
$I_B = 50\,\mu A$
$30\,\mu A$
$10\,\mu A$
$I_B$ [$\mu A$]
60
40
20
5
10
15
$U_{CE}$ [V]
0.2
0.4
0.6
0.8
$I_B = 10\,\mu A$
$30\,\mu A$
$50\,\mu A$
$U_{BE}$ [V]

C
n
B
p
n
E
$I_C$
$I_B$
+
+

C
B
E

# Basic Transistor Circuits

*In: Chapter 5: Transistor Basics*

# Transistors as Part of a Circuit

*Concepts*

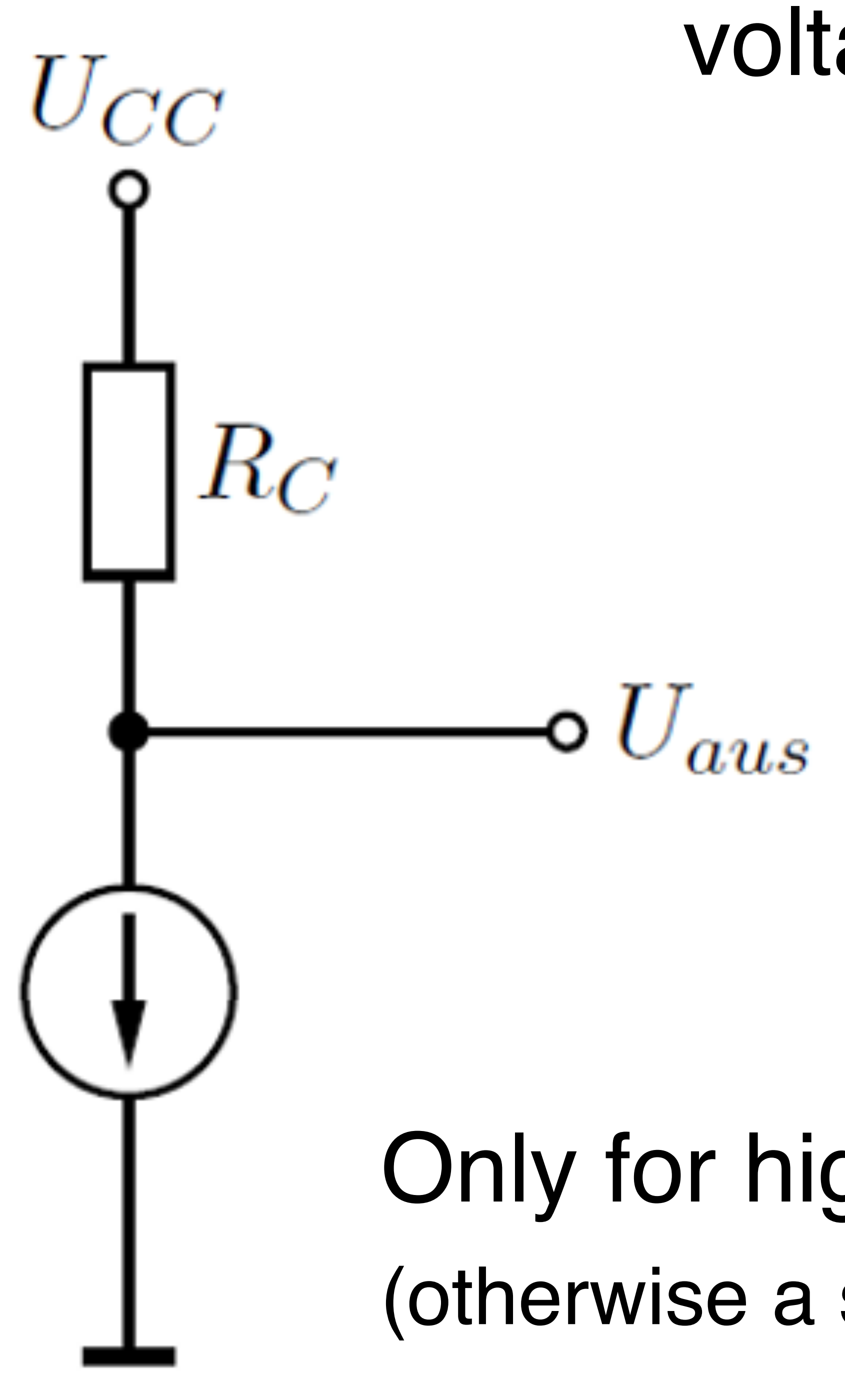
- Transistors as current- or voltage-driven current source:
  $I_C$ depending on $I_B$ (or $U_B$)

As current drain /
voltage amplifier

The transistor current results in a voltage drop: $R_C I_C$

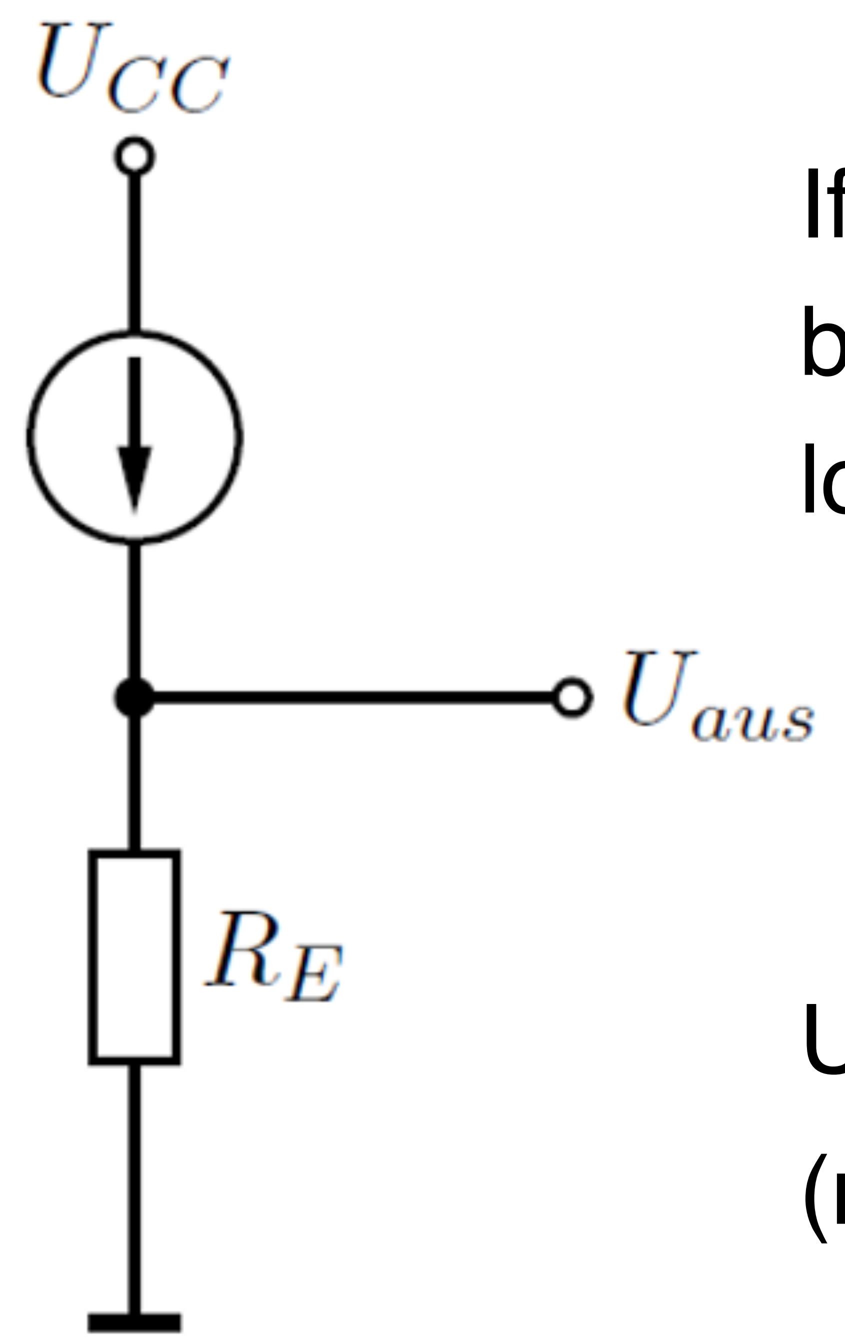
With that: $dU_{aus} = - R_C I_C$

Circuit behaves as inverting voltage amplifier.




Only for high load impedance
(otherwise a simple voltage divider)

Als current source /
current amplifier/
voltage buffer

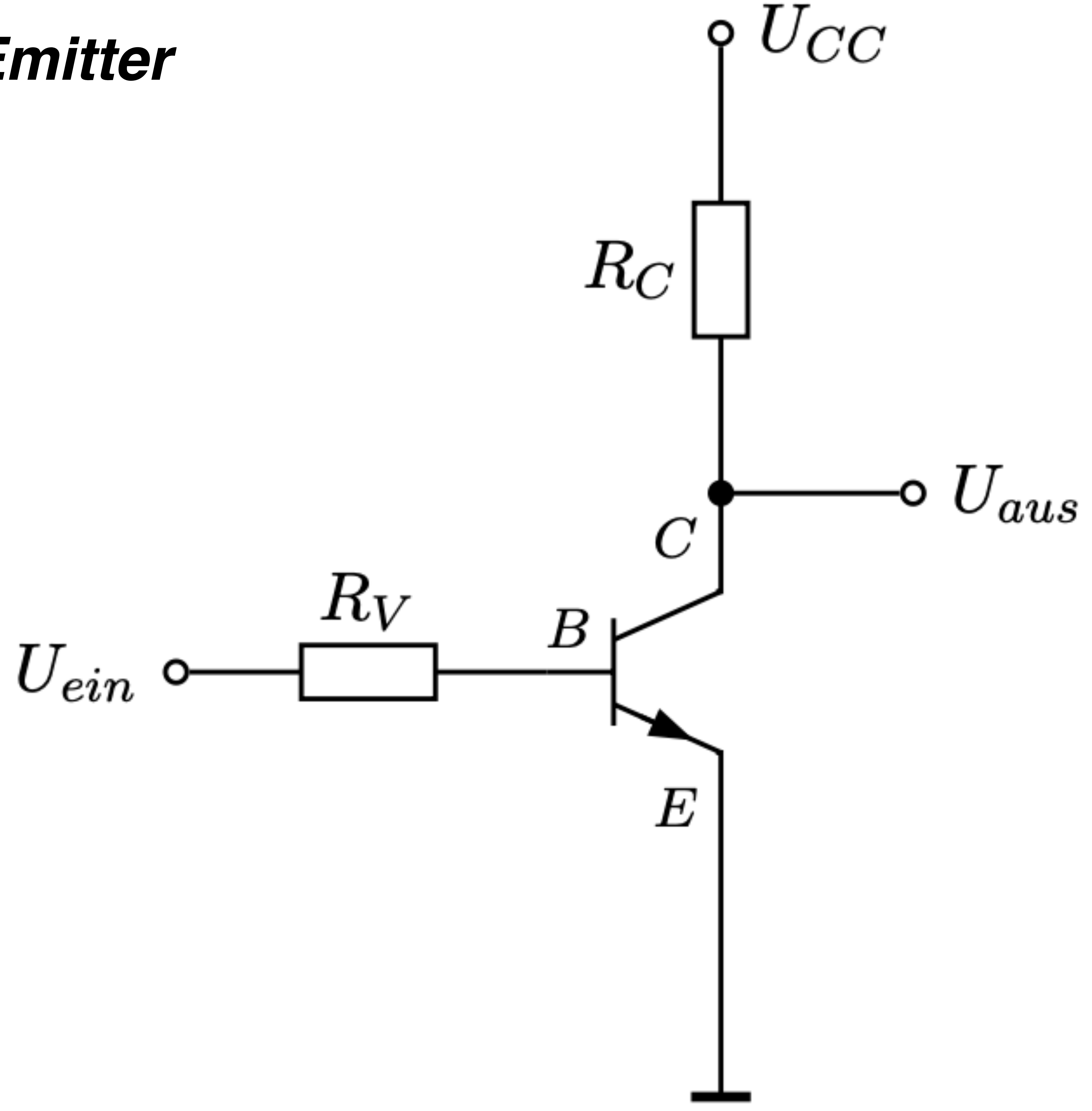



If load << $R_E$: voltage buffer, current through load

$U_{aus}$ depends on $U_B$ (minus diode drop)

- The "naming scheme": The basic circuits are named after the terminal that is connected to ground or to the supply voltage, and is neither input nor output.

Here: ***Common Emitter***




Components:

Resistor (protection) $R_V$ on input

Collector resistor $R_C$

Inverting voltage amplifier

Also: (moderate) current amplification

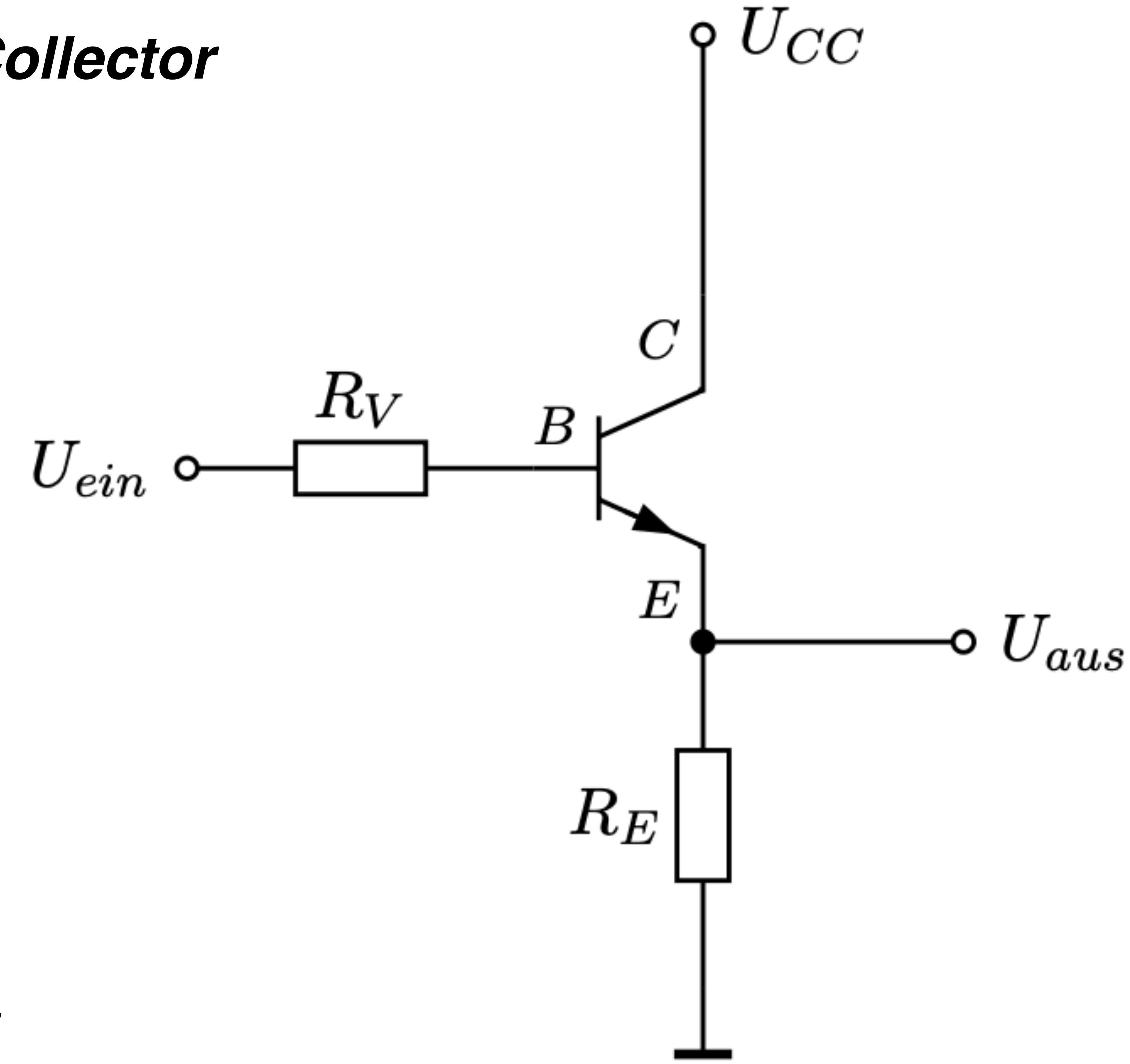
*Emitter-Schaltung*

# Basic Transistor Circuits

*Common Collector Amplifier*

- The "naming scheme": The basic circuits are named after the terminal that is connected to ground or to the supply voltage, and is neither input nor output.

Here: ***Common Collector***




Components:

Resistor (protection) $R_V$ on input

Emitter resistor $R_E$

Large current amplification (voltage buffer)

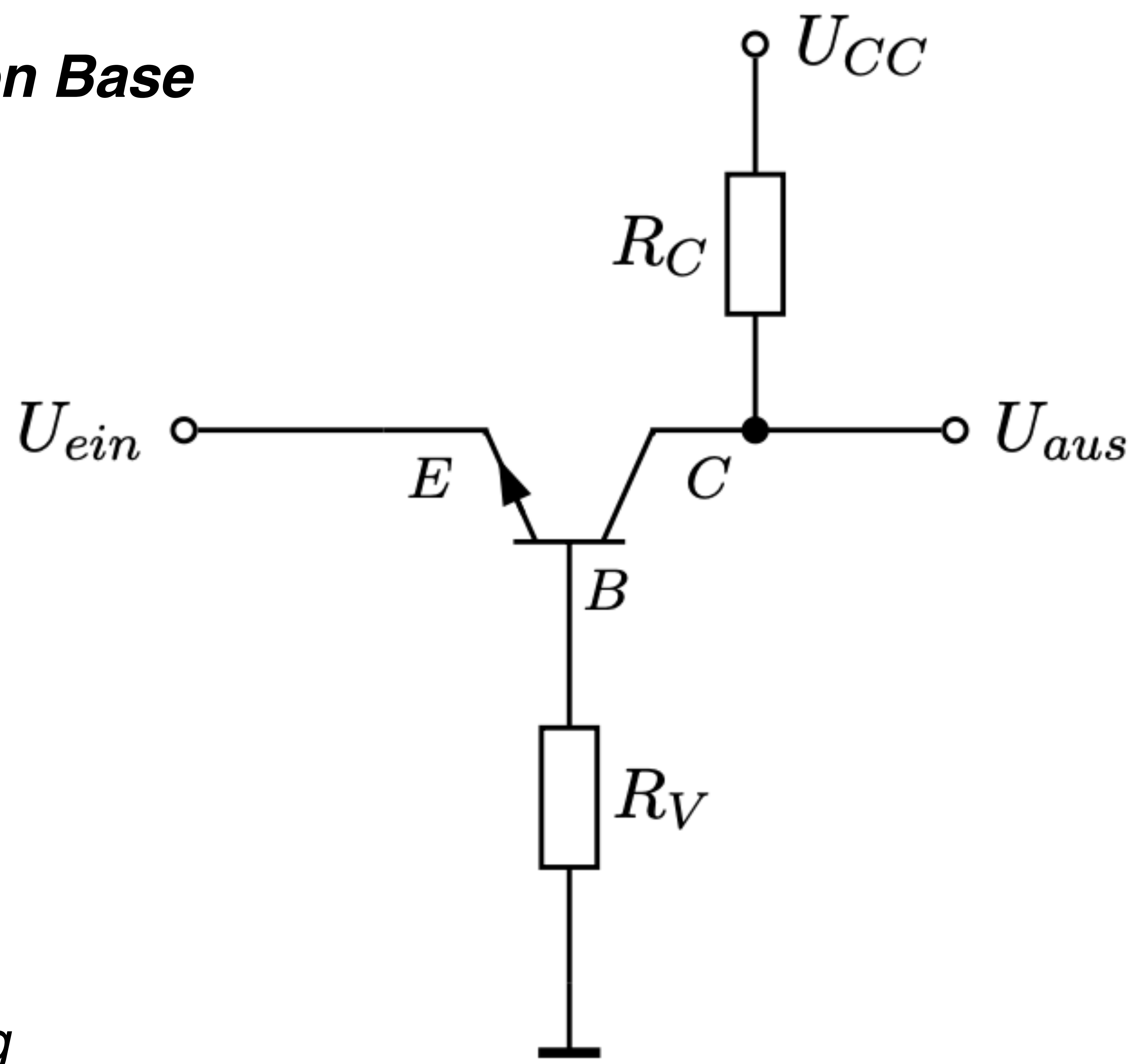
No voltage amplification (< 1)

*Kollektor Schaltung*

in practice the most common circuit

- The “naming scheme”: The basic circuits are named after the terminal that is connected to ground or to the supply voltage, and is neither input nor output.

Here: ***Common Base***




Components:

Resistor (protection) $R_V$ on base

Resistor $R_C$ on collector

Voltage amplification

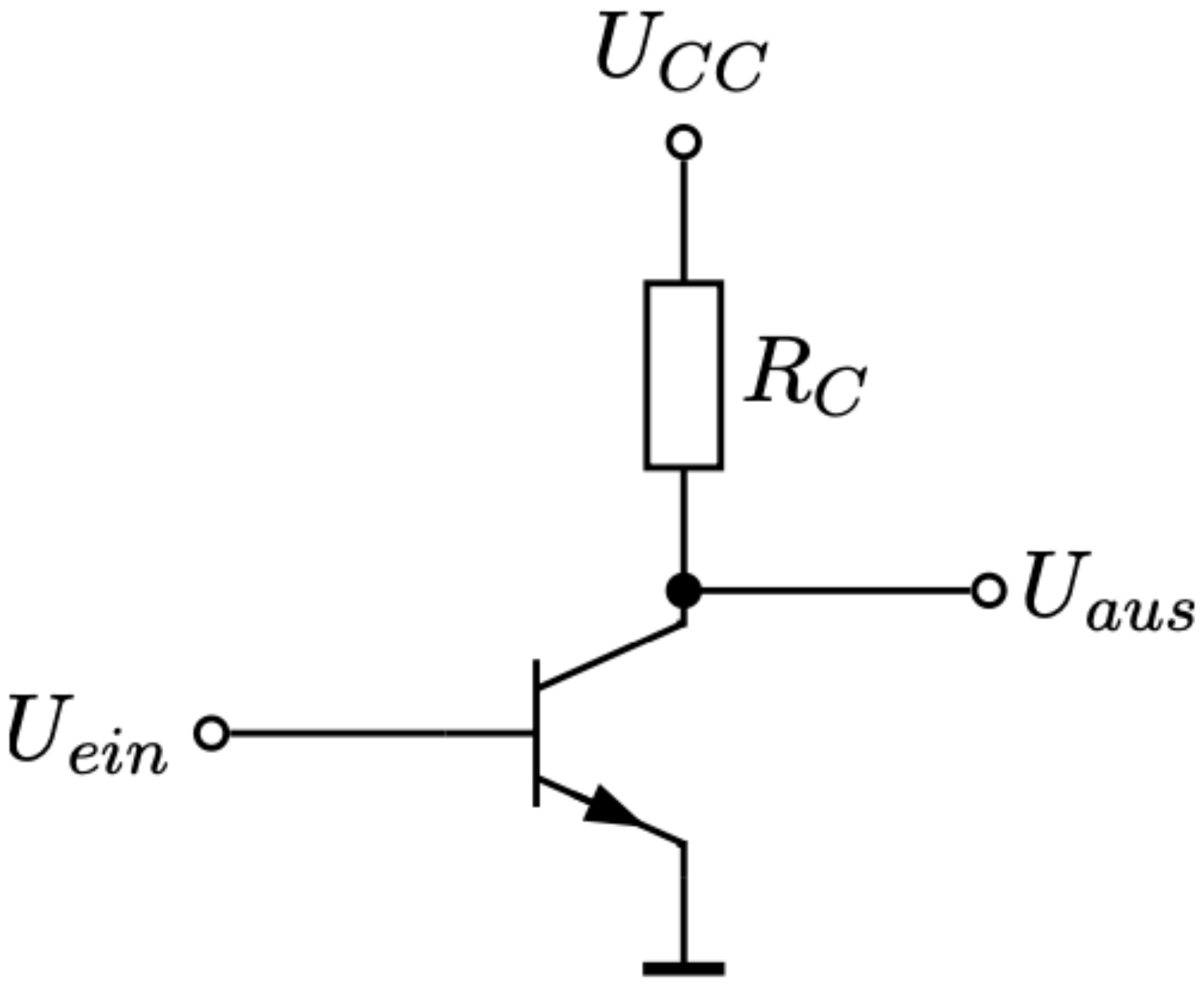
No current amplification

*Basis-Schaltung*

in practice the rarest circuit.

# The Common Emitter

*Understanding the Circuit*




- Consider the voltages:

from S 16

$$S = \frac{\partial I_C}{\partial U_{BE}} = \beta \frac{\partial I_B}{\partial U_{BE}} = \frac{\beta}{R_{BE}}$$

$$U_{aus} = U_{CC} - I_C R_C$$

Amplification:

$$V_U = \frac{dU_{aus}}{dU_{ein}} = -R_C \frac{dI_C}{dU_{ein}} = -S R_C = -\beta \frac{R_C}{R_{BE}}$$

(Neglecting $R_{CE}$ and load)

➯ Output voltage inverted

➯ High voltage gain with large $R_C$

➯ High transconductance S = high gain: $S = 40\, I_C / V$ (S 17)

# Circuit Analysis

*Small Signals*

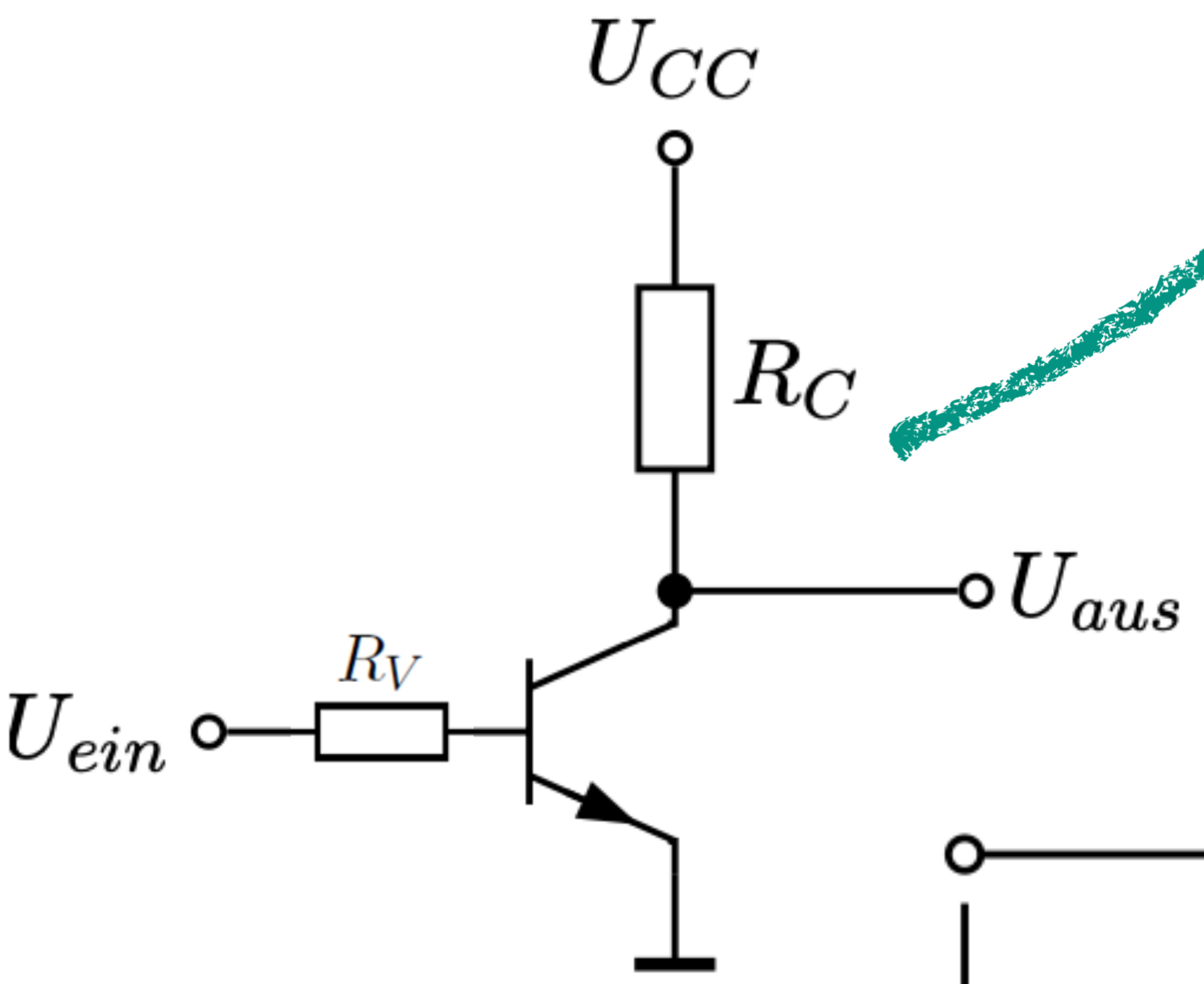
- Analyzing circuits with non-linear components can be difficult - a solution for limited signal ranges is the small-signal equivalent circuit (“Kleinsignal-Ersatzschaltbild”): Considering a small range of the physically possible signals - typically with small perturbations around a stable condition - for example a DC current.

*The recipe:*

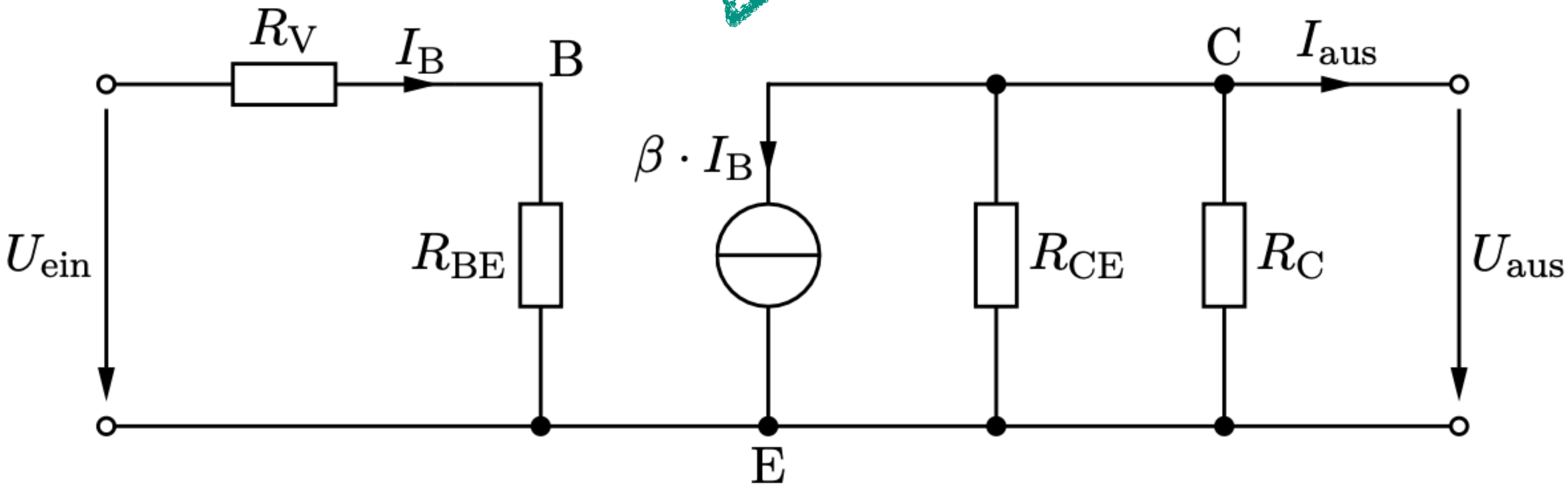
- Voltage and current sources are replaced with their internal resistance:
  - Current source: open circuit (infinite resistance)
  - Voltage source: short circuit (resistance 0)
- Non-linear components are replaced with their differential impedance at the working point
- Transistors are replaced with current- or voltage-driven current sources

# Common Emitter

*Small-Signal Equivalent Circuit*

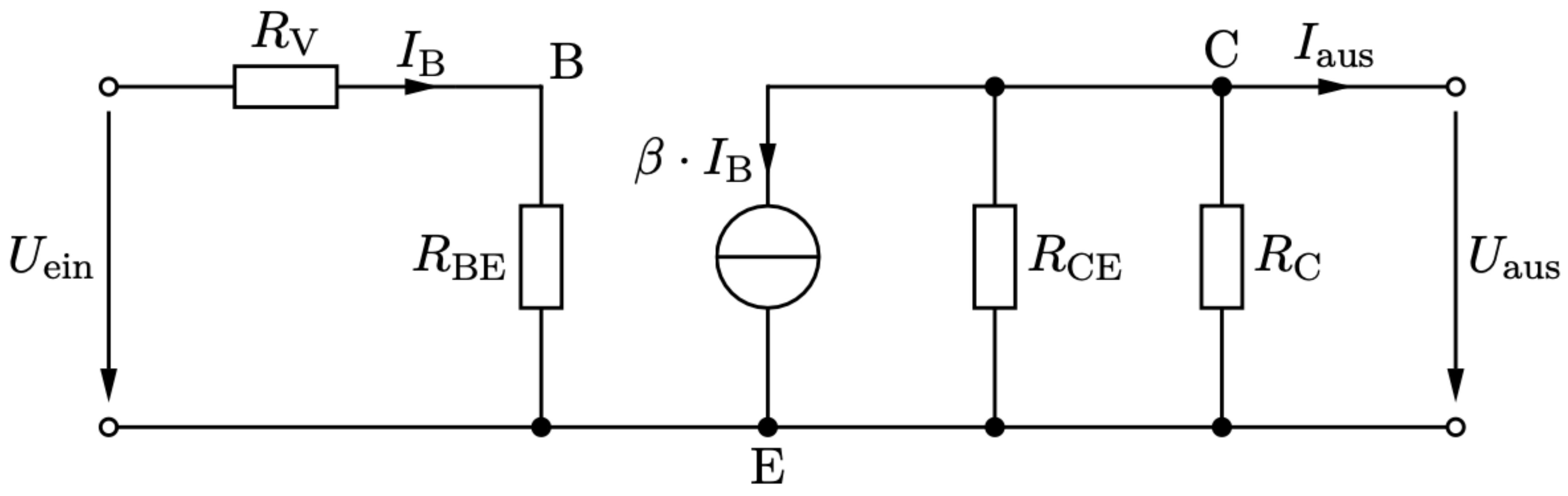



- Applying the recipe



# Common Emitter - Equivalen Circuit

*An Example*




Typical values

$$\beta = 100,\ I_C = 1\,\mathrm{mA}\ \text{und}\ R_{CE} = 100\,\mathrm{k}\Omega$$

$$R_C = 5\,\mathrm{k}\Omega$$

Resulting in:

$$R_{ein} = R_V + R_{BE}$$ and $$R_{BE} = \frac{k\,T}{q_e\,I_B} = \frac{25}{10}\frac{\mathrm{mV}}{\mu\mathrm{A}} =$$ 2.5 kΩ (see S 17)

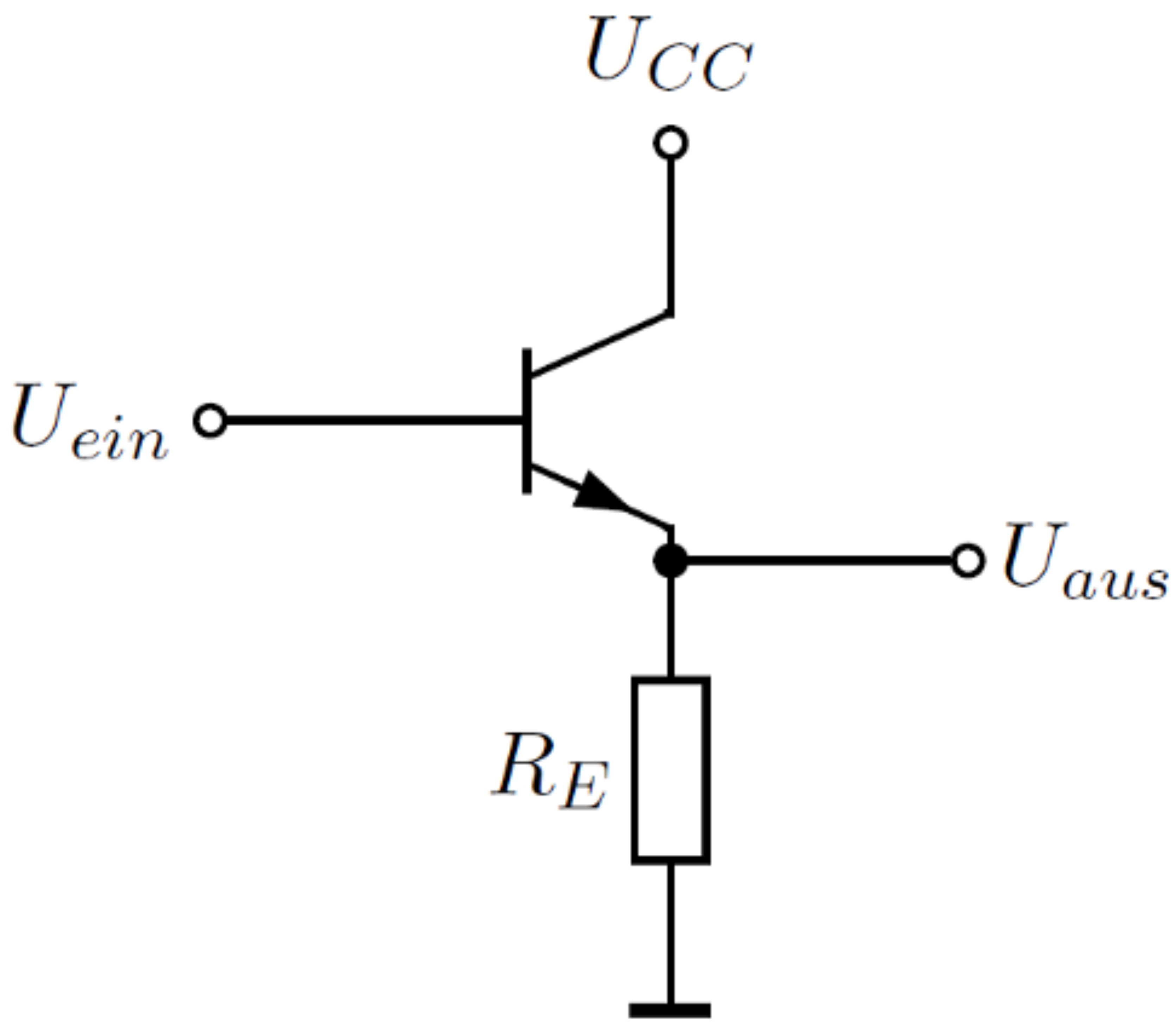
Output impedance $R_{aus} = R_C \,||\, R_{CE} \approx R_C = 5\,\mathrm{k}\Omega$

$$U_{aus} = -\beta\,I_B\ R_C \,||\, R_{CE} \approx -\beta\,I_B\ R_C$$ and $$U_{ein} = I_B\,(R_V + R_{BE}) \approx I_B\,R_{BE}$$

Gain of the circuit: $$V = \frac{dU_{aus}}{dU_{ein}} = -\frac{\beta\,R_C}{R_V + R_{BE}}$$ ~ -200

# Common Collector

*Basic Functionality*




- Behavior:

$$U_{ein} - U_{aus} = U_B - U_E = U_{BE} \approx 0,7\,\mathrm{V}$$

resulting in:

$dU_{ein} = dU_{aus}$ => voltage gain = 1

The output voltage “follows” the input voltage.

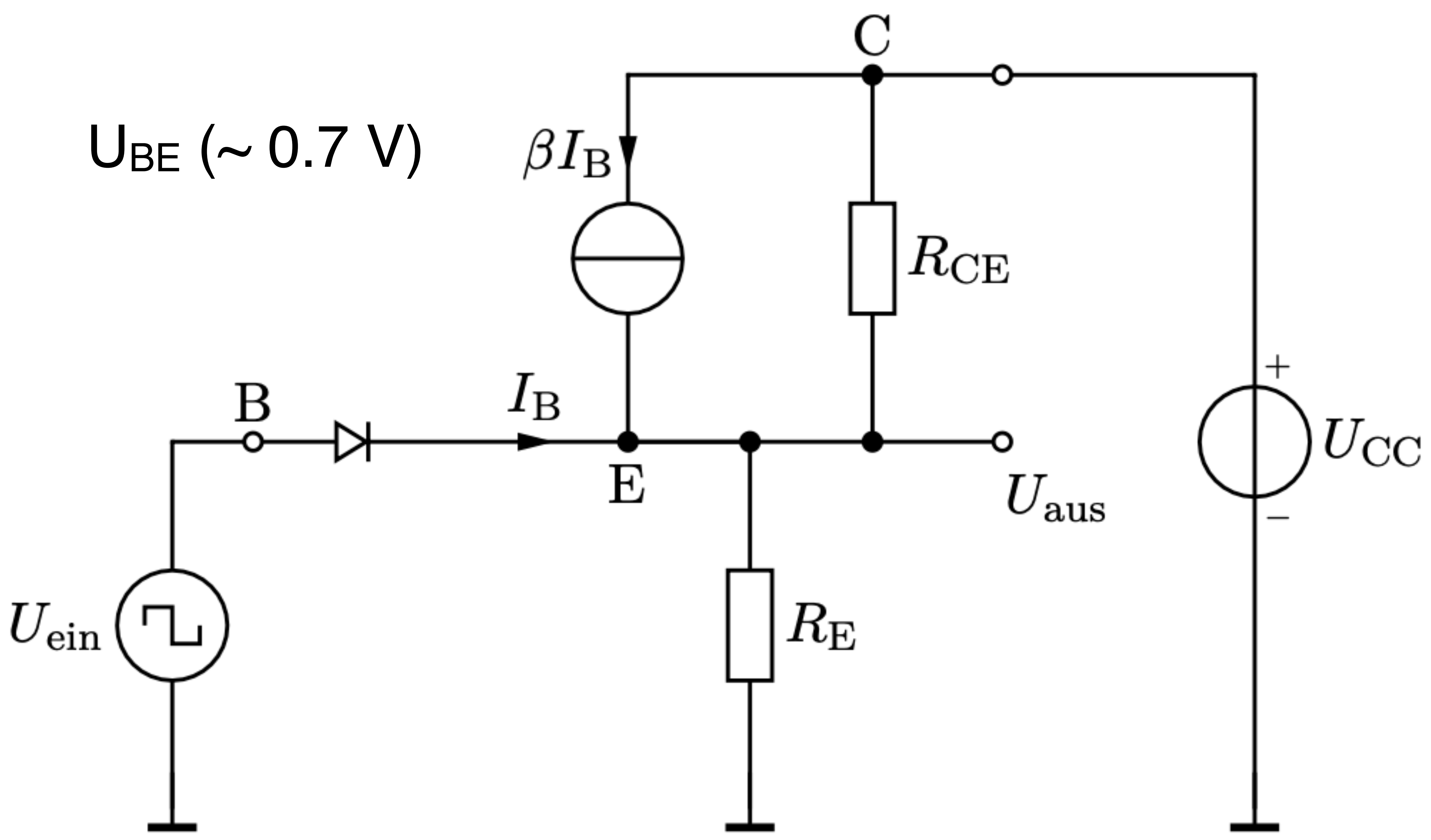
Thus also known as: “***emitter follower***”.

What’s the point?

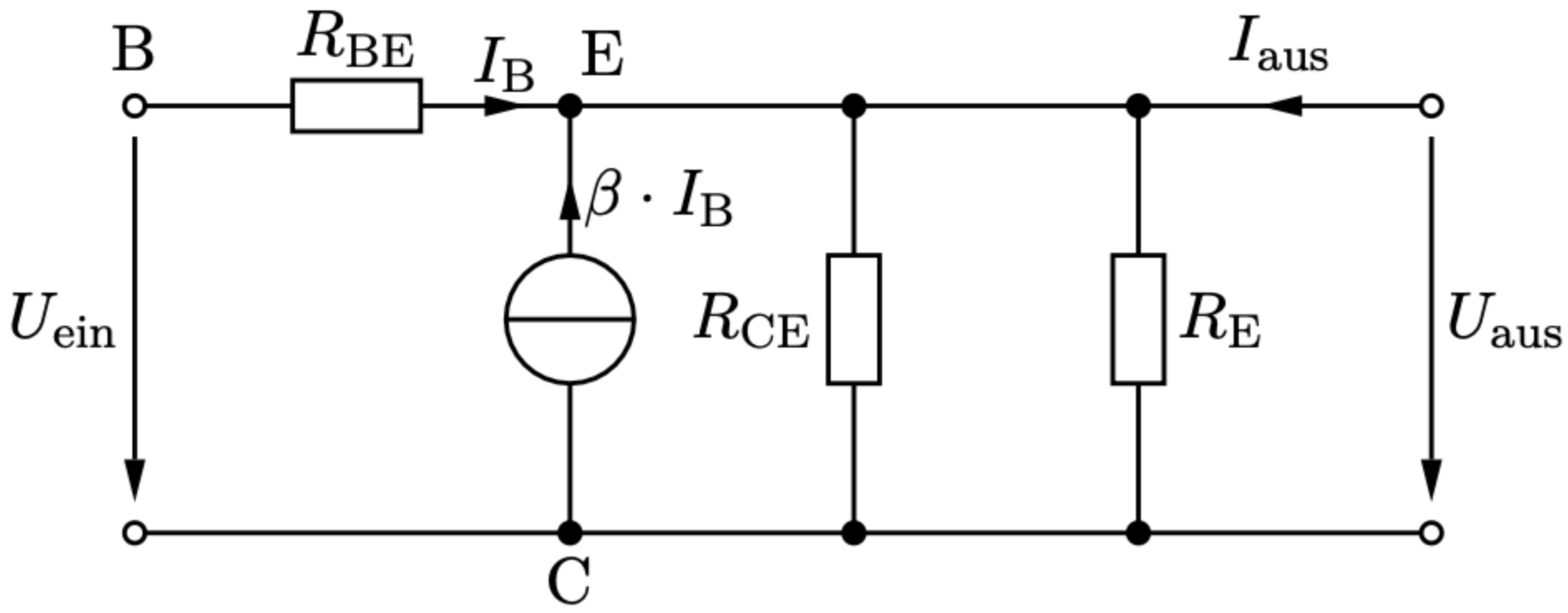
High current gain:

$I_C = \beta I_B$, and $I_E = (\beta + 1) I_B$

$U_{BE}$ (~ 0.7 V)



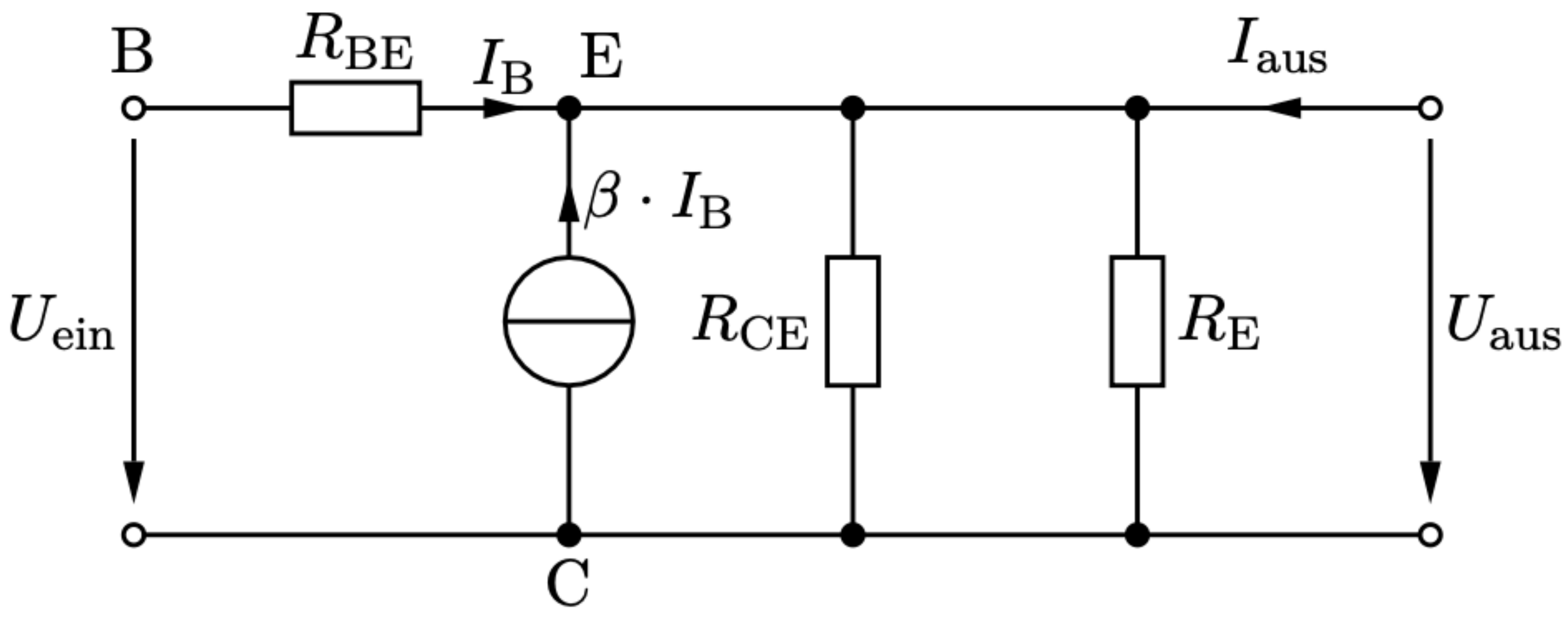

- In large signal analysis:
  External and internal voltage sourced not yet replaced (includes the base-emitter diode)




- Transformation into small-signal analysis: absorbs the “diode drop” - for the analysis of small changes

# Understanding the Common Collector

*Input Impedance*




Input impedance: Effect of a small input current on the input voltage

$$Z_{ein} = \frac{dU_{ein}}{dI_{ein}}$$

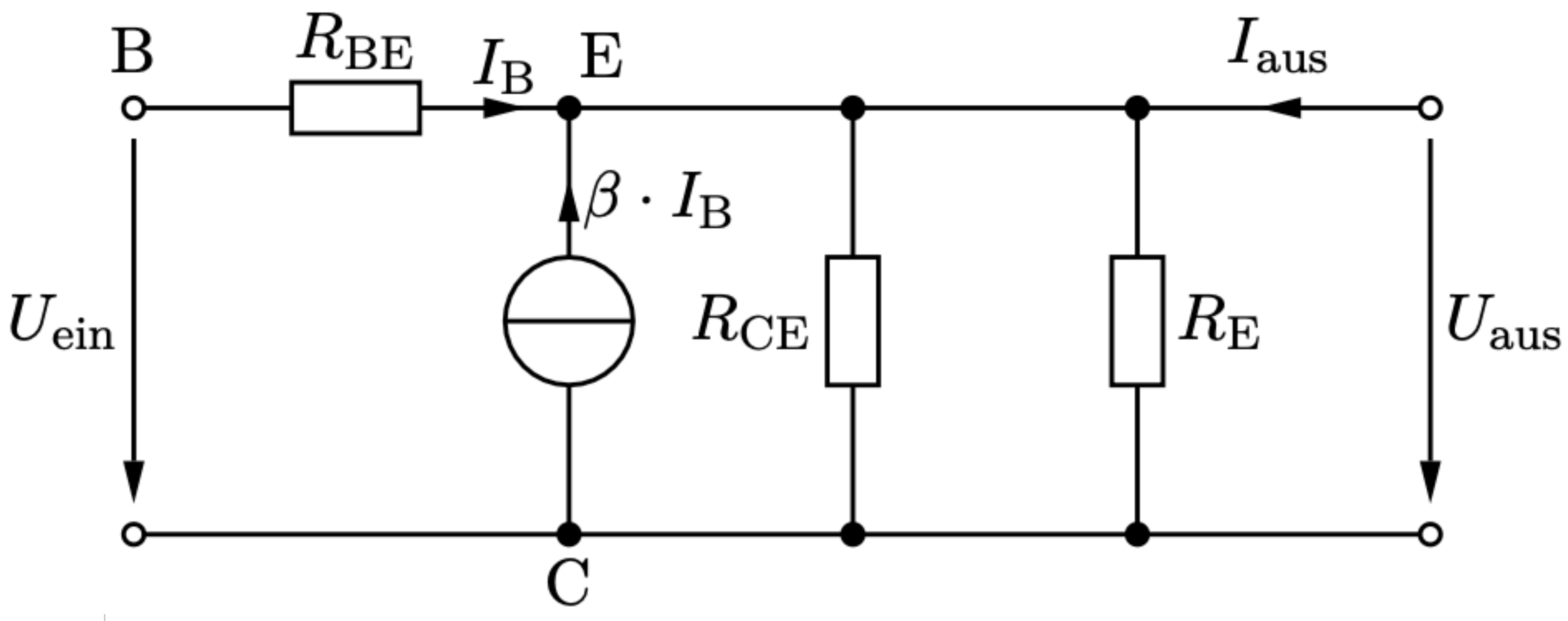
$$\mathrm{d}U_{\mathrm{ein}} = R_{\mathrm{BE}} \cdot \mathrm{d}I_{\mathrm{ein}} + (\beta + 1)\, \mathrm{d}I_{\mathrm{ein}} \cdot R_{\mathrm{E}} \| R_{\mathrm{CE}}$$

(Input current results in amplified emitter current)

$$Z_{\mathrm{ein}} = \frac{\mathrm{d}U_{\mathrm{ein}}}{\mathrm{d}I_{\mathrm{ein}}} = R_{\mathrm{BE}} + (\beta + 1)\, R_{\mathrm{E}} \approx (\beta + 1)\, R_{\mathrm{E}}$$

(Neglecting $R_{CE}$ relative to $R_E$)

➩ High input impedance (Typical values for $R_E$ 100Ω - 1 kΩ)
(For high load impedance - For small values: Input impedance depends on load!)




Output impedance: Effect of a change in output voltage on output current for constant $U_{ein}$

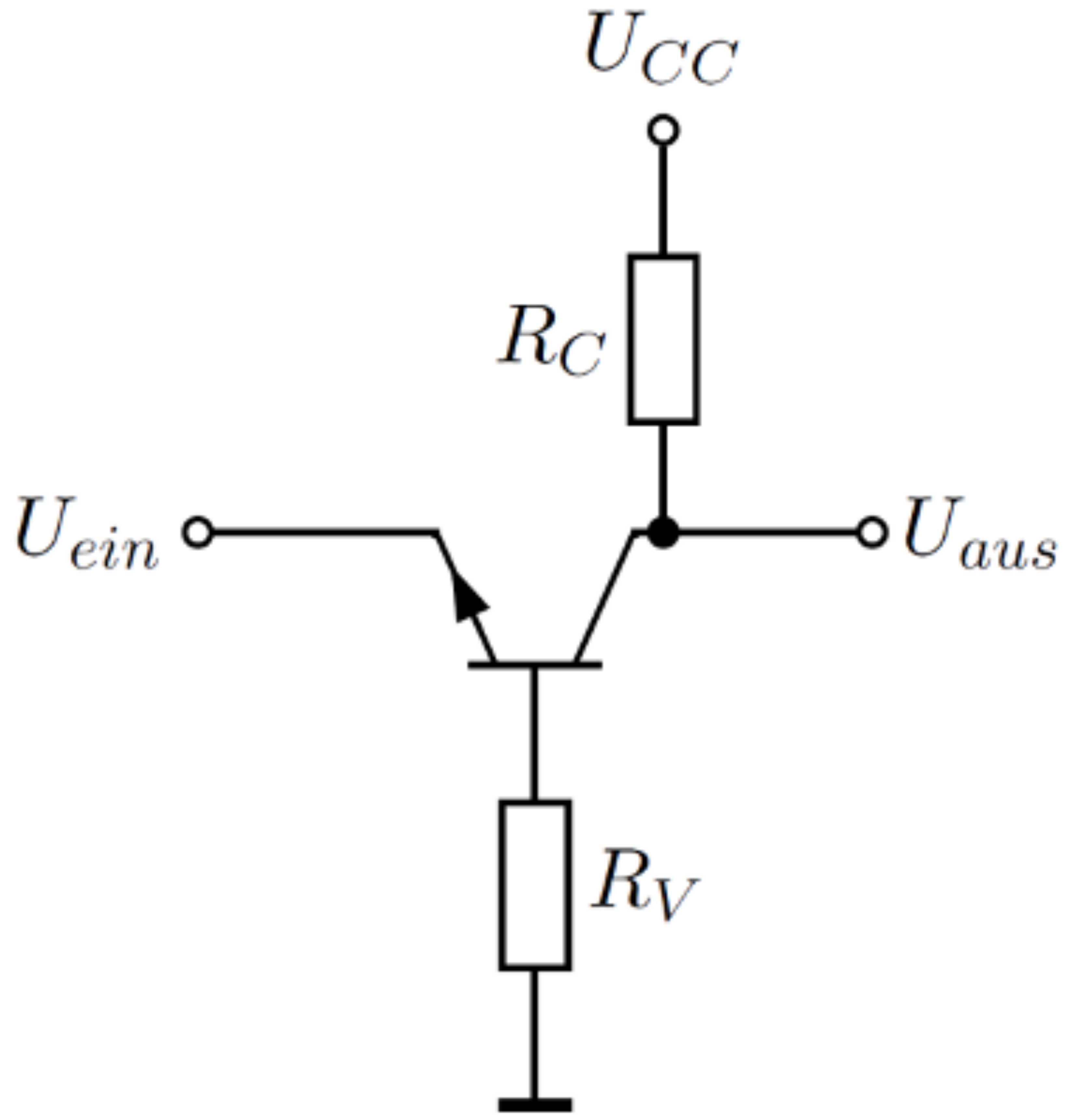
$$Z_{\mathrm{aus}} = \frac{\mathrm{d}U_{\mathrm{aus}}}{\mathrm{d}I_{\mathrm{aus}}}$$

$$\mathrm{d}I_{\mathrm{aus}} + \mathrm{d}I_{\mathrm{B}}\,(\beta+1) = \frac{dU_{\mathrm{aus}}}{R_{\mathrm{CE}}\|R_{\mathrm{E}}}$$

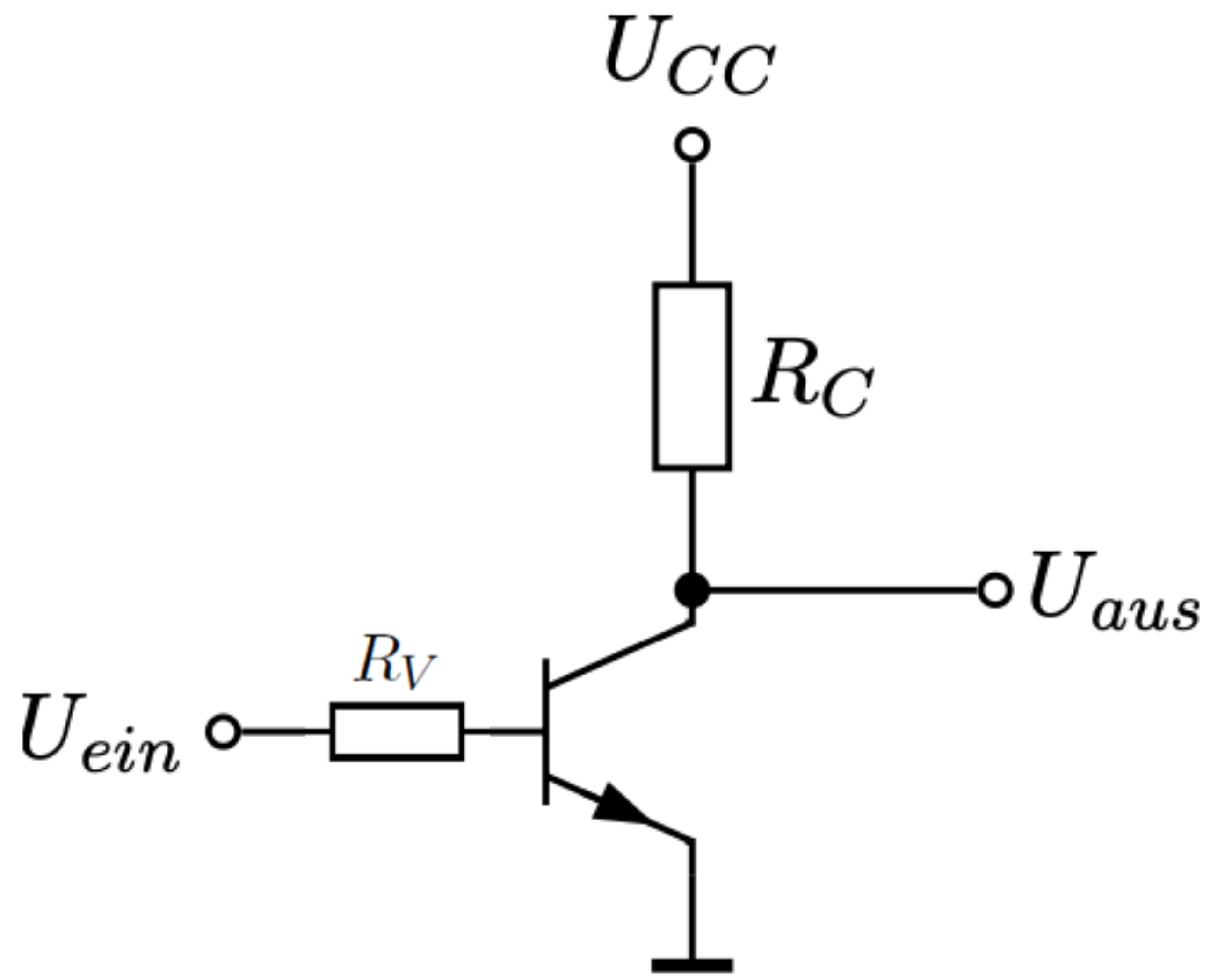
$$dI_{\mathrm{B}} = \frac{dU_{\mathrm{BE}}}{R_{\mathrm{BE}}} = -\frac{dU_{\mathrm{aus}}}{R_{\mathrm{BE}}}$$

results in: $\mathrm{d}I_{\mathrm{aus}} = \frac{\beta+1}{R_{\mathrm{BE}}}dU_{\mathrm{aus}} + \frac{dU_{\mathrm{aus}}}{R_{\mathrm{CE}}\|R_{\mathrm{E}}}$ and $Z_{\mathrm{aus}} = \frac{\mathrm{d}U_{\mathrm{aus}}}{\mathrm{d}I_{\mathrm{aus}}} = \frac{R_{\mathrm{BE}}}{\beta+1}\|R_{\mathrm{CE}}\|R_{\mathrm{E}} \approx \frac{R_{\mathrm{BE}}}{\beta+1}$

➩ Low output impedance
Together with high input impedance: Well suited as impedance converter: Little impact on input signal, low-impedance transmission.




In general: similar to common emitter:




Voltage gain:

$$V_U = \frac{dU_{aus}}{dU_{ein}} = R_C \frac{dI_C}{dU_{ein}} = S\, R_C$$

Current gain ~ 1 (small base current, $I_E \sim I_C$)

Output impedance $Z_{aus} \sim R_C$ (as for common emitter)

but: small input impedance!

(smaller by 1/β compared to common emitter)

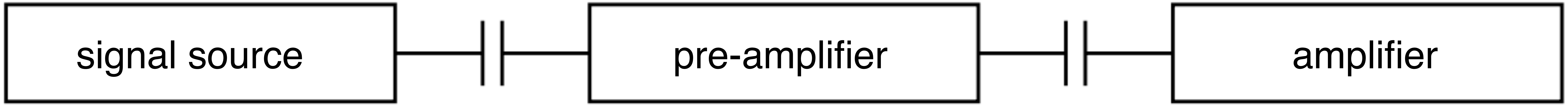
$$Z_{ein} \approx \frac{dU_{BE}}{dI_E} \approx \frac{dU_{BE}}{dI_C} = \frac{1}{S} = \frac{k\,T}{q_e\, I_C}$$

(25 Ω bei 25C, $I_C$ = 1 mA)

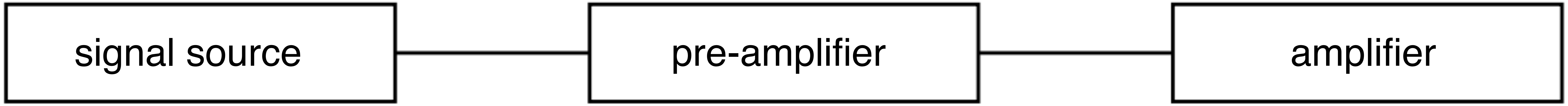
# Couping: AC or DC

*Short Excursion*

- How do I get the signal into the amplifier?




(a)



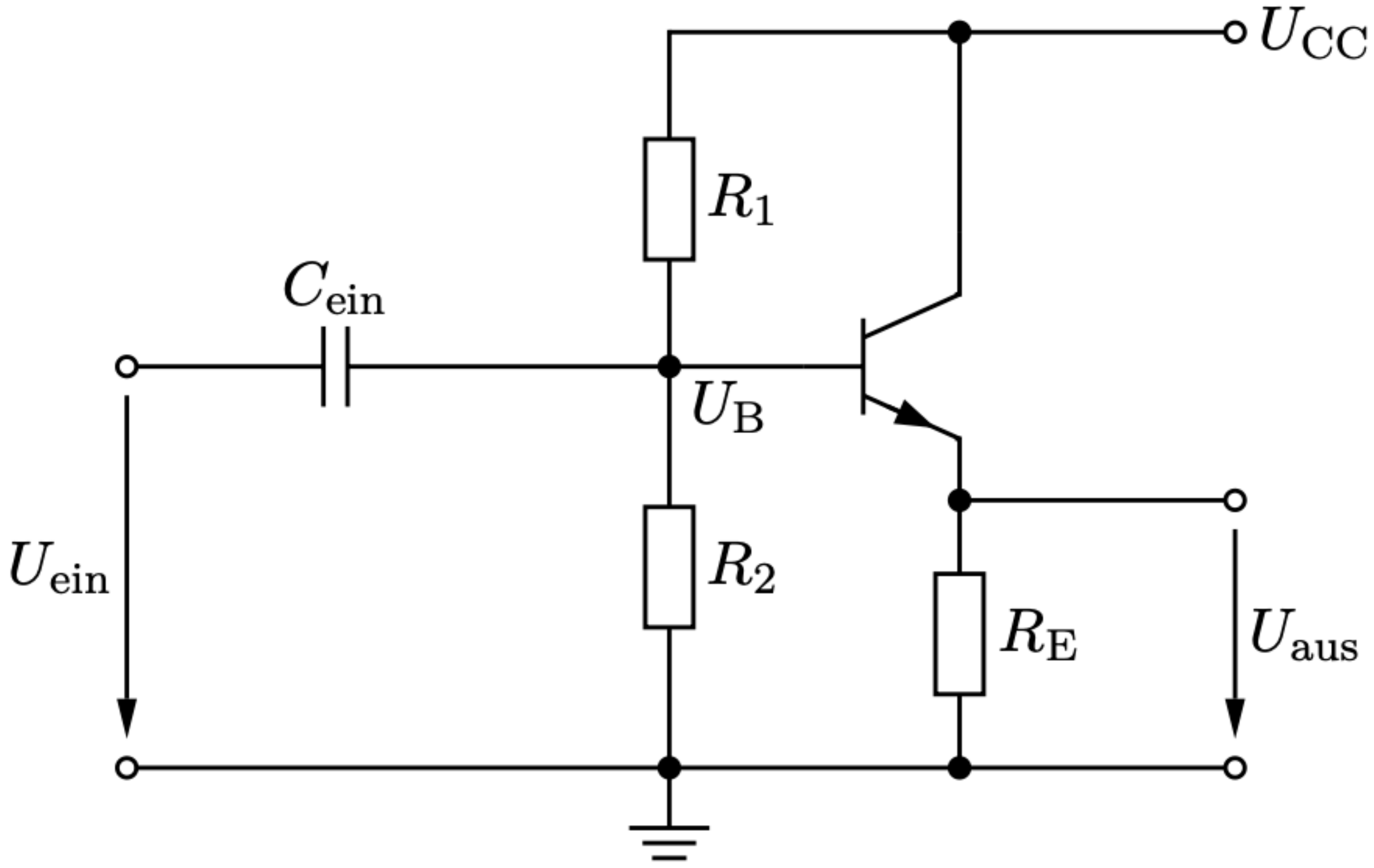

(b)

# Working Point

*Adjusting Amplifiers*

- The circuit has to be designed such that a useful output is possible over the range of expected input signals (as a minimum: transistor is conductive).
  High freedom with AC coupling:

  Here: common collector




One strategy:
Set target values for $I_C$ and $U_{CE}$

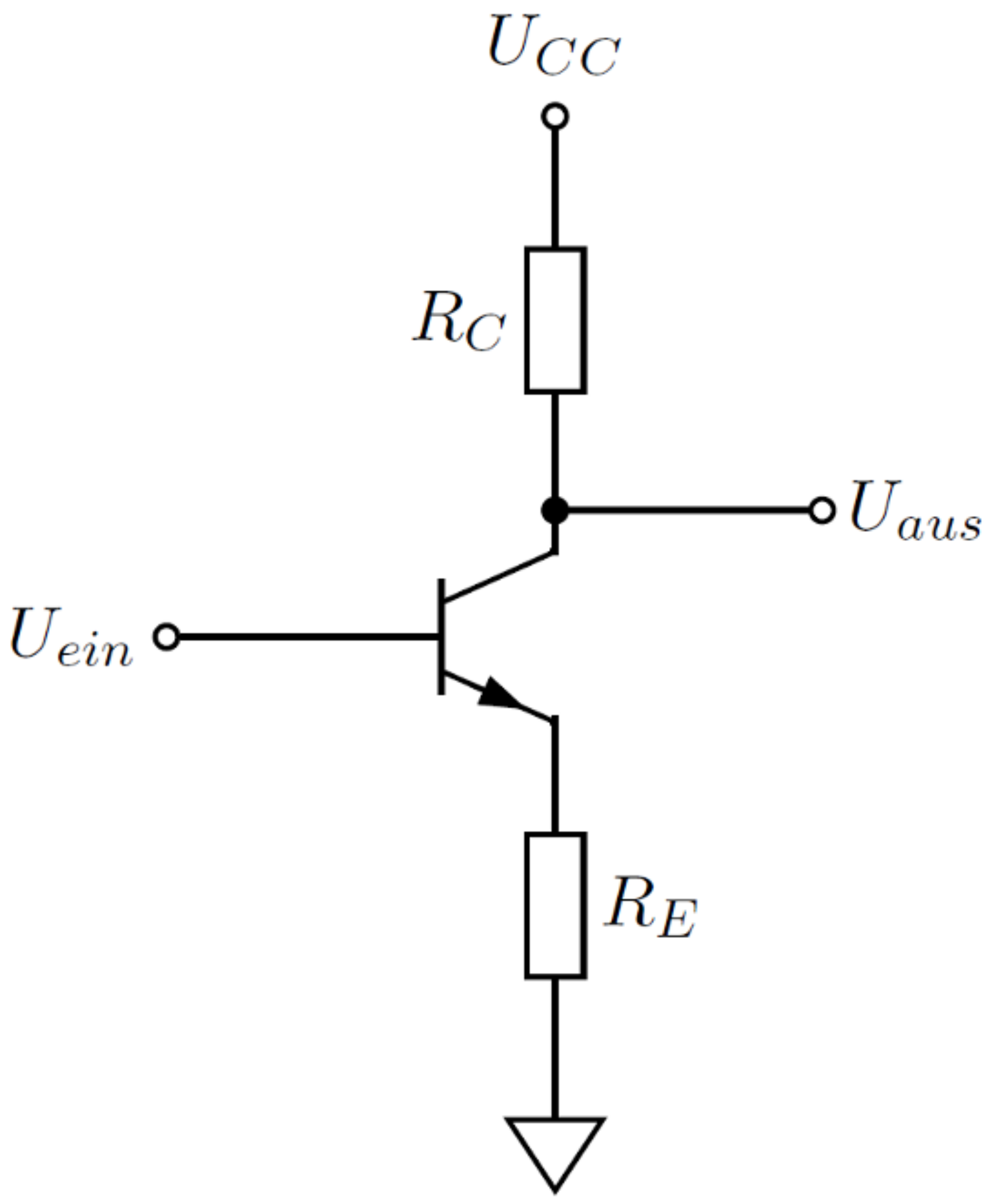
Defines: Choice of $R_{E:}$ $(U_{CC} - U_{CE}) / I_C$

Choice of $R_1$ and $R_2$ such that $U_B$ without input signal is set as
$U_B = U_{CC} - U_{CE} + 0.7V$

$C_{ein}$ sufficiently large that all relevant input frequencies are passed.

# Feedback in Transistor Circuits

*Common Emitter with Current Feedback*




"*Stromgegenkopplung*"

- One additional resistor relative to the "normal" common emitter amplifier: $R_E$

How it works:

$$U_{aus} = U_C = U_{CC} - I_C\,R_C$$
$$dU_{aus} = -\,dI_C\,R_C$$

just as for the "normal" circuit

in addition:

$$dI_E = \frac{dU_E}{R_E} \quad , \quad dU_E = dU_{ein} \quad \text{und} \quad I_E = I_C - I_L + I_B \approx I_C$$

$$dU_{aus} = -\,dI_C\,R_C \approx -\,dI_E\,R_C = -\,dU_E\,\frac{R_C}{R_E} = -\,dU_{ein}\,\frac{R_C}{R_E}$$

Gain:

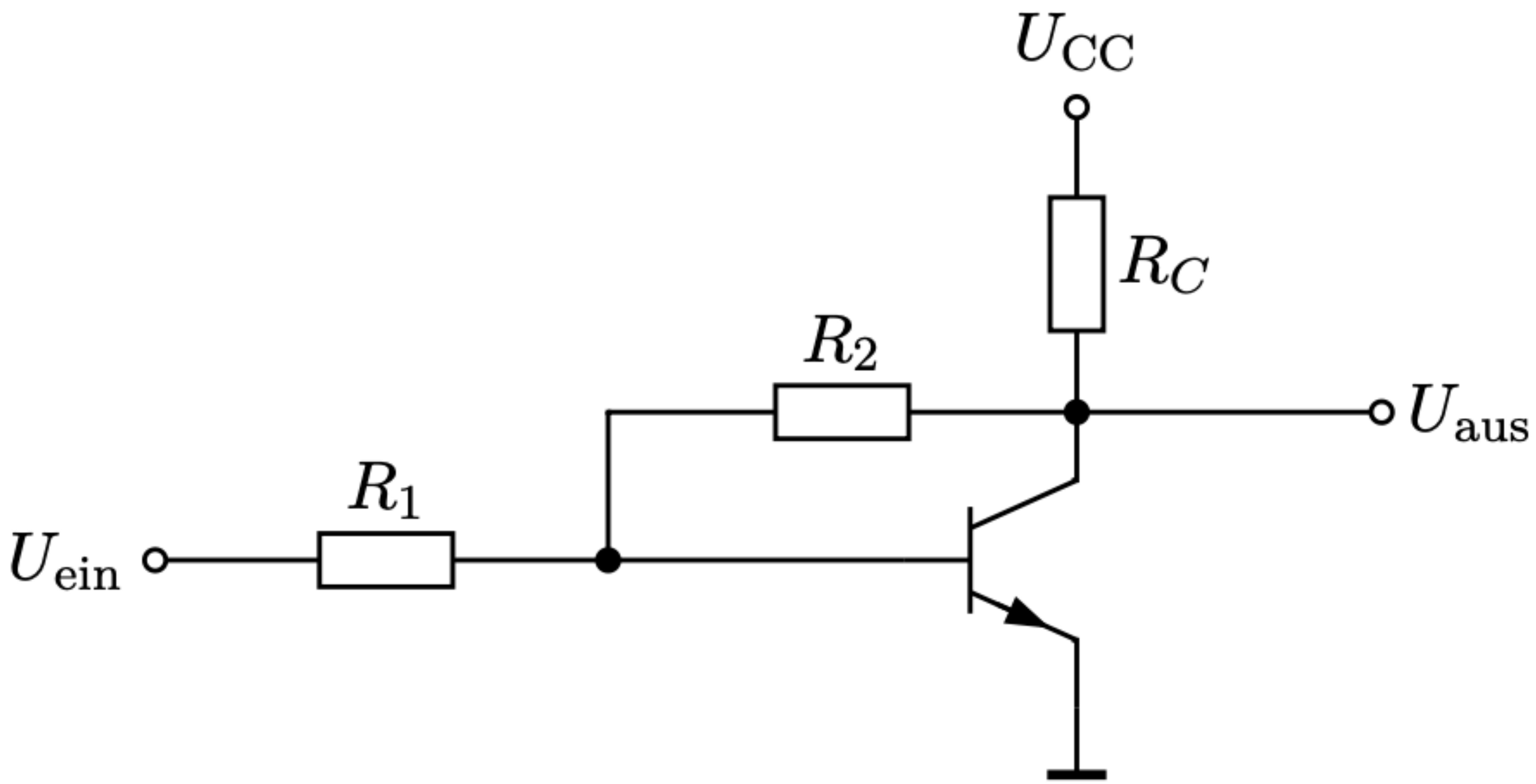
$$V_U = \frac{dU_{aus}}{dU_{ein}} = -\,\frac{R_C}{R_E}$$

Comparing to the "normal" circuit:

$$V_U = -\,S\,R_C = -\,\beta\,\frac{R_C}{R_{BE}}$$

# Feedback in Transistor Circuits

*Common Emitter with Voltage Feedback*




- The same principle as negative feedback in the inverting amplifier with an OpAmp:

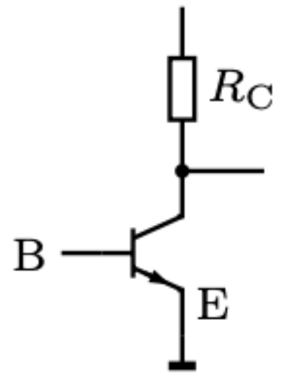
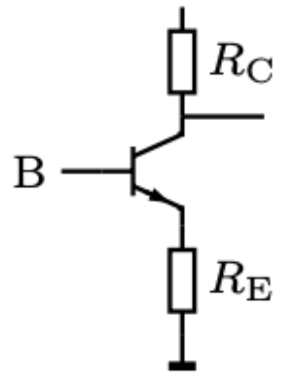
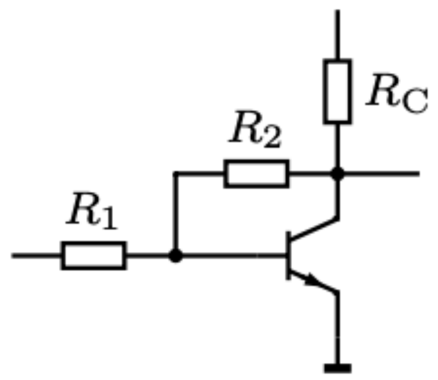
$$V_U = \frac{U_{aus}}{U_{ein}} = -\frac{R_2}{R_1}$$

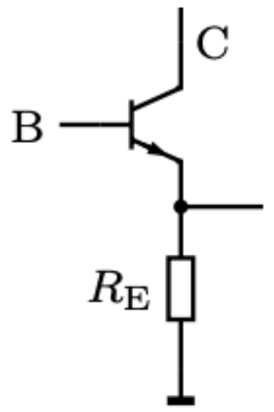
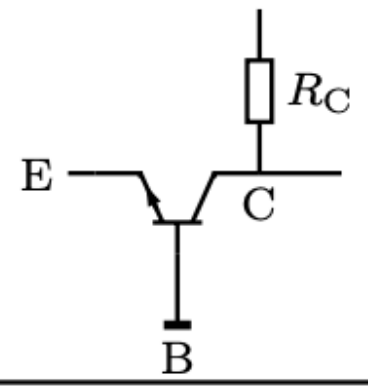
Also here: Gain independent of gain of the transistor.

*"Spannungsgegenkopplung"*

# Basic Transistor Circuits

*Summary*

| circuit | $V_U$ | $V_I$ | $Z_{\text{ein}}$ | $Z_{\text{aus}}$ für $\mathrm{d}U_{\text{ein}} = 0$ |
|---|---|---|---|---|
| common emitter  | $-S\,R_{\text{C}} \parallel R_{\text{CE}}$ | $\beta$ | $R_{\text{BE}}$ | $R_{\text{C}} \parallel R_{\text{CE}}$ |
| c. emitter /w current feedback  | $-\frac{R_{\text{C}}}{R_{\text{E}}}$ | $\beta$ | $R_{\text{BE}} + \beta\,R_{\text{E}}$ | $R_{\text{C}} \parallel R_{\text{CE}}$ |
| c. emitter /w voltage feedback  | $-\frac{R_2}{R_1}$ | $\frac{R_2}{R_{\text{C}}}$ | $R_1$ | $R_{\text{C}} \parallel \frac{1}{S}\left(1 + \frac{R_2}{R_1}\right)$ |

| circuit | $V_U$ | $V_I$ | $Z_{\text{ein}}$ | $Z_{\text{aus}}$ für $\mathrm{d}U_{\text{ein}} = 0$ |
|---|---|---|---|---|
| common collector  | $1$ | $\beta$ | $\beta\,R_{\text{E}}$ | $R_{\text{E}} \parallel \left(\frac{1}{S} + \frac{R_{\text{Signal}}}{\beta}\right)$ |
| common base  | $S\,R_{\text{C}}$ | $1$ | $\frac{1}{S}$ | $R_{\text{C}}$ |

$V_U$: voltage gain

$V_I$: current gain

## Next Lectures:

Digital - Thursday, December 21

Analog 10 - Chapters 05, 06 - Tuesday, January 9, 2024

**Merry Christmas and a Happy New Year!**

# Time Plan for Next Lectures

*A few Changes coming up!*

| Calender Week | Tuesday | Thursday |
|---|---|---|
| 45 | 07.11. **Analog** | 09.11. **Digital** |
| 46 | 14.11. **Analog** | 16.11. **Digital** |
| 47 | 21.11. **Digital** | 23.11. **Analog** |
| 48 | 28.11. **Digital** | 30.11. **Digital** |
| 49 | 05.12. **Digital** | 07.12. **Analog** |
| 50 | 12.12. **Digital** | 14.12. **Analog** |
| 51 | 19.12. **Analog** | 21.12. **Digital** |
| 2 | 09.01. **Analog** | 11.01. **Digital** |
| 3 | 16.01. **Digital** | 18.01. **Digital** |
| 4 | 23.01. **Analog** | 25.01. **Digital** |
| 5 | 30.01. **Analog** | 01.02. **Digital** |
| 6 | 06.02. **Analog** | 08.02. **Analog** |
| 7 | 13.02. **Analog** | 15.02. **Digital** |

# Electronics for Physicists

## *Analog Electronics*

**Frank Simon**
**Institute for Data Processing and Electronics**

Karlsruhe Institute of Technology

***KIT, Winter 2023/24***

Institute for Data Processing and Electronics