Electronics for Physicists

Analog Electronics

Chapter 8; Lecture 14

Frank Simon
Institute for Data Processing and Electronics

08.02.2024

Karlsruhe Institute of Technology

KIT, Winter 2023/24

Institute for Data Processing and Electronics

Chapter 8

Additional Topics

- Filters and Voltage Regulators
- Noise

Overview

1. Basics
2. Circuits with R, C, L with Alternating Current
3. Diodes
4. Operational Amplifiers
5. Transistors - Basics
6. 2-Transistor Circuits
7. **Field Effect Transistors**
8. Additional Topics
   - Filters
   - Voltage Regulators
   - Noise

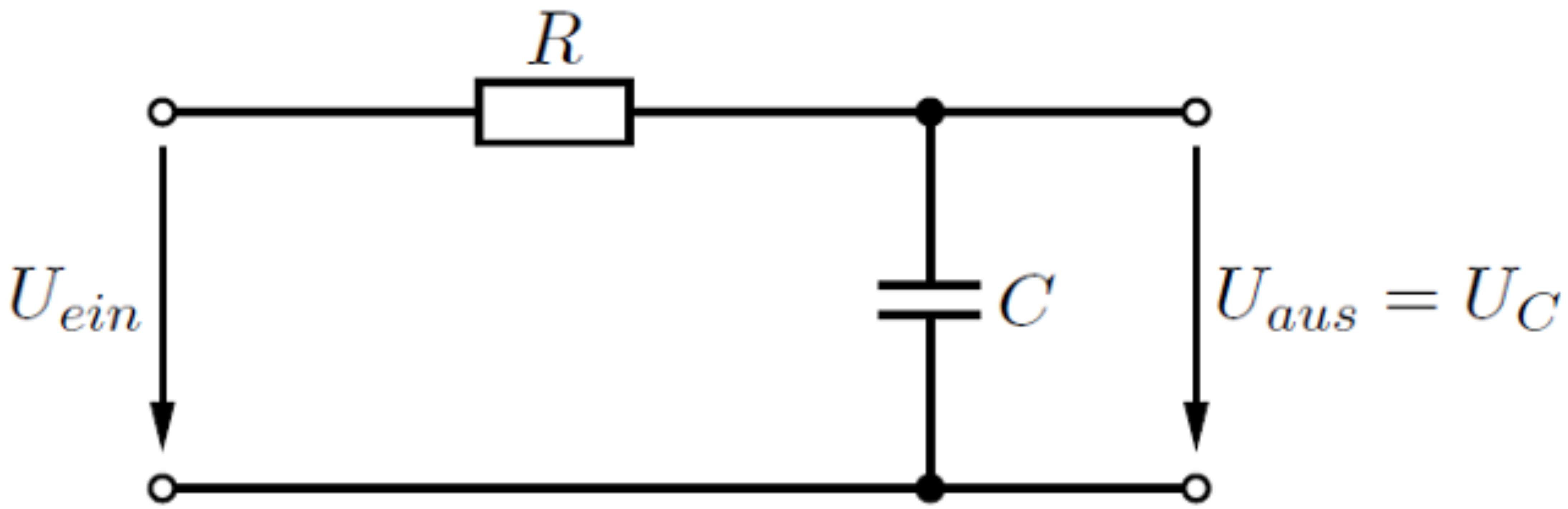
Active Filters

In: Chapter 8 - Additional Topics

Passive vs active Filters

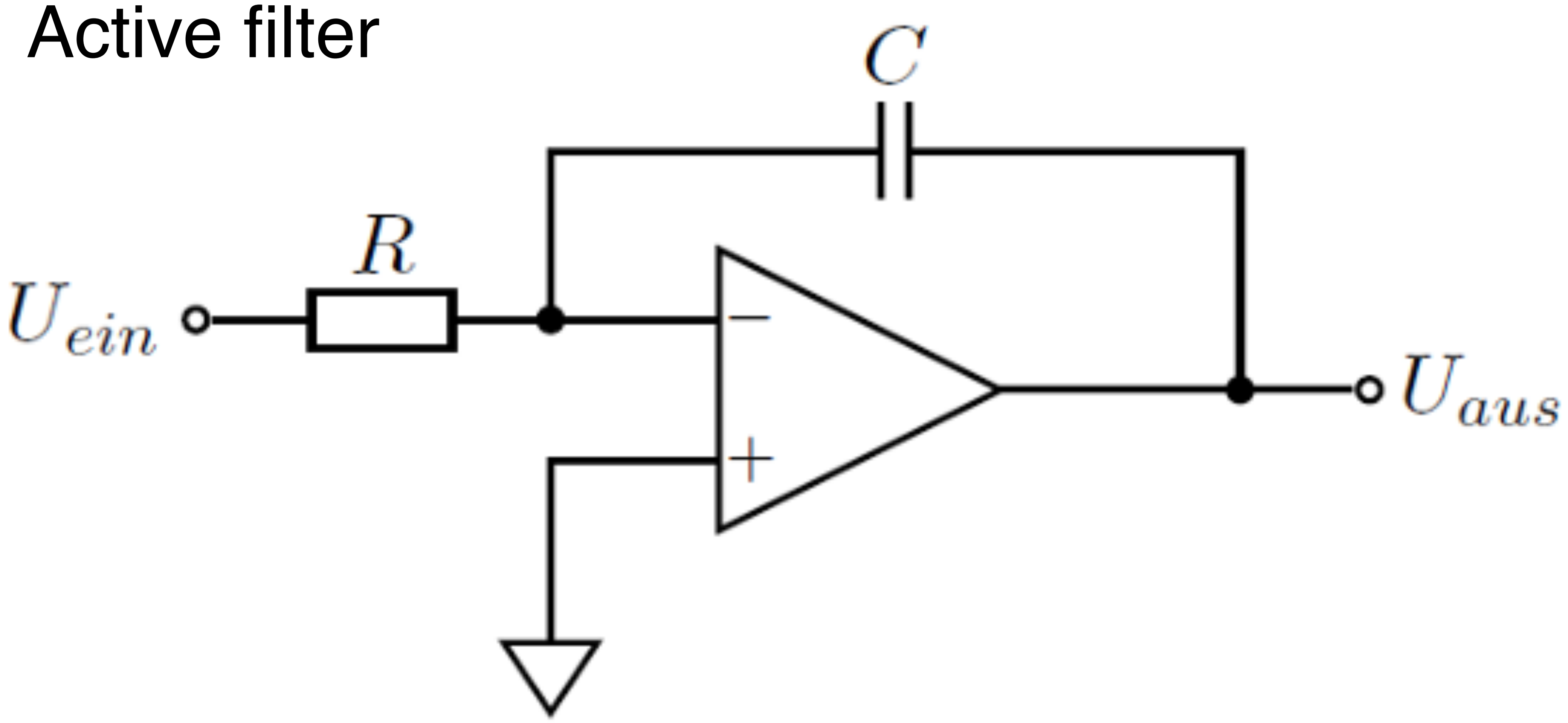
Introduction

- Passive filter



Low pass: $\frac{U_{aus}}{U_{ein}} = -\frac{Z}{R} = -\frac{1}{\omega RC}$

- Active filter

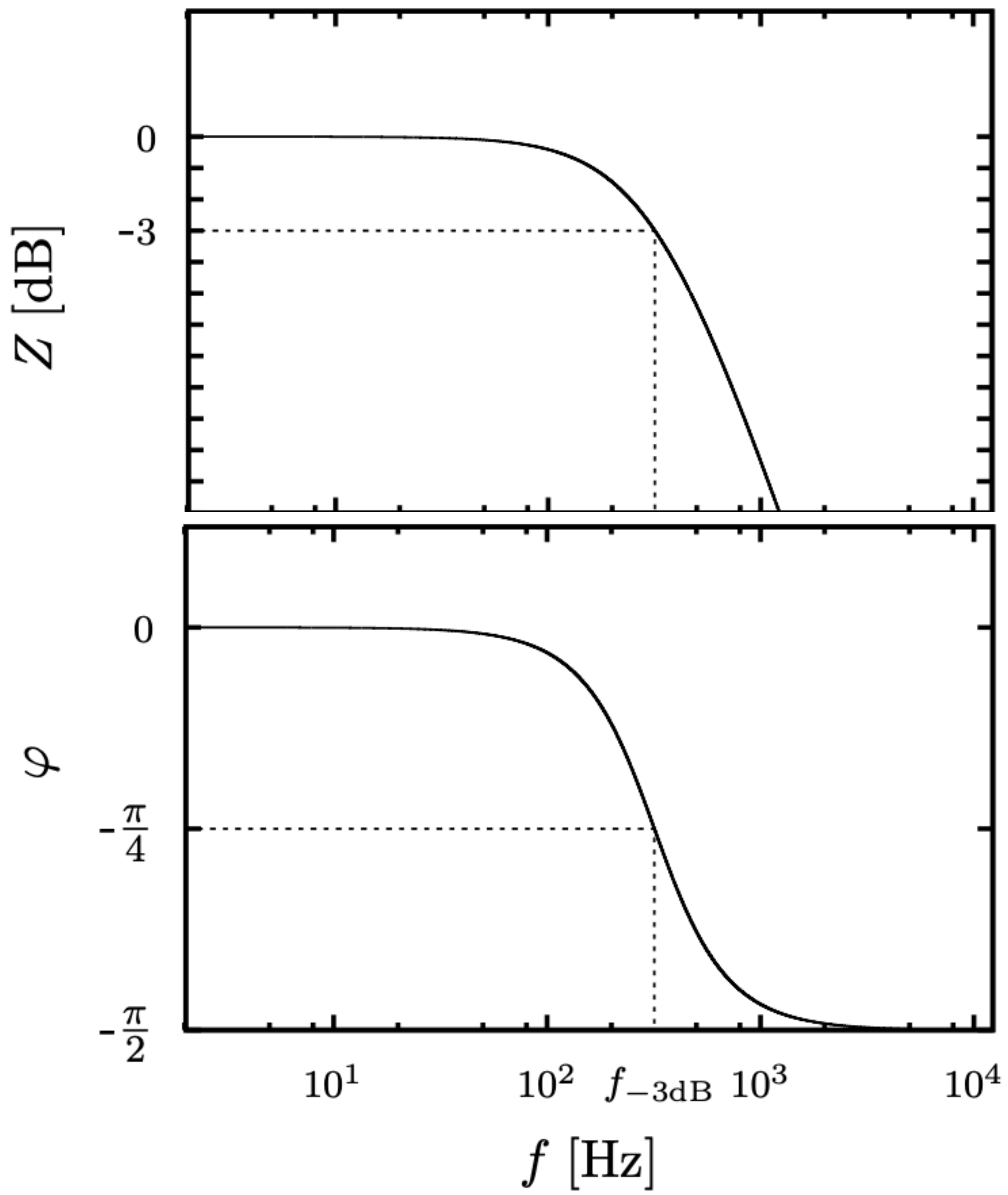


Integrator: $U_{aus} = -\frac{1}{RC}\int U_{ein}\,dt$

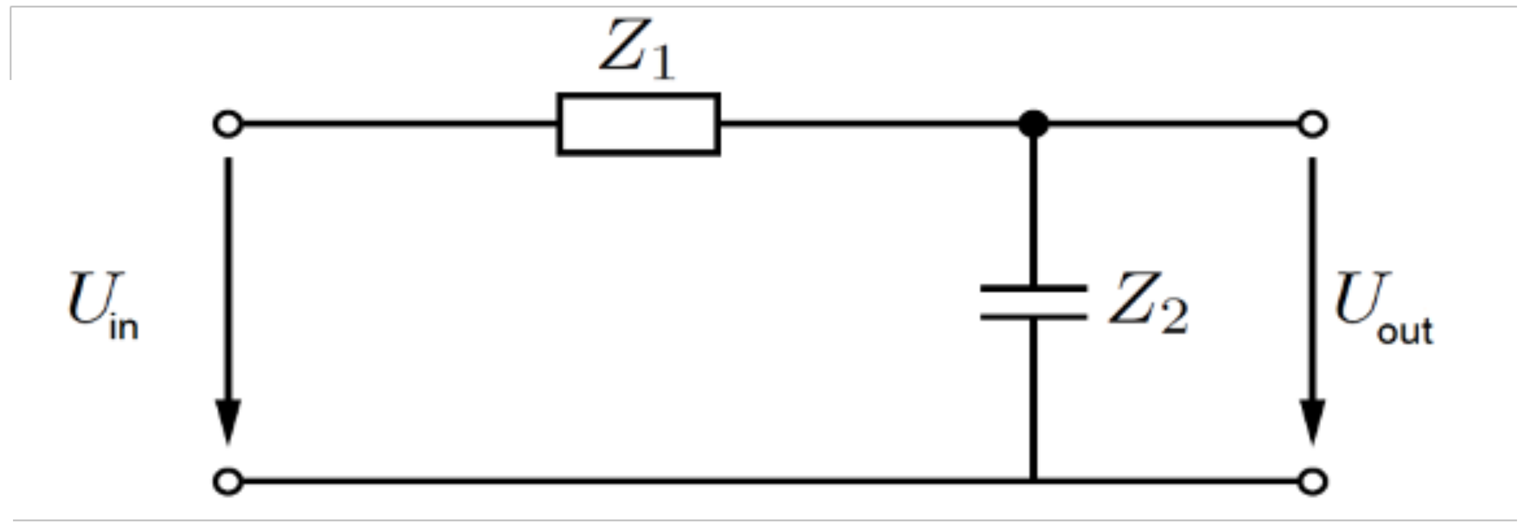
- Active filters: Op-Amps, R, C
  - Requires power - but built-in amplification prevents load on input / intermediate stages for multi-stage filters.

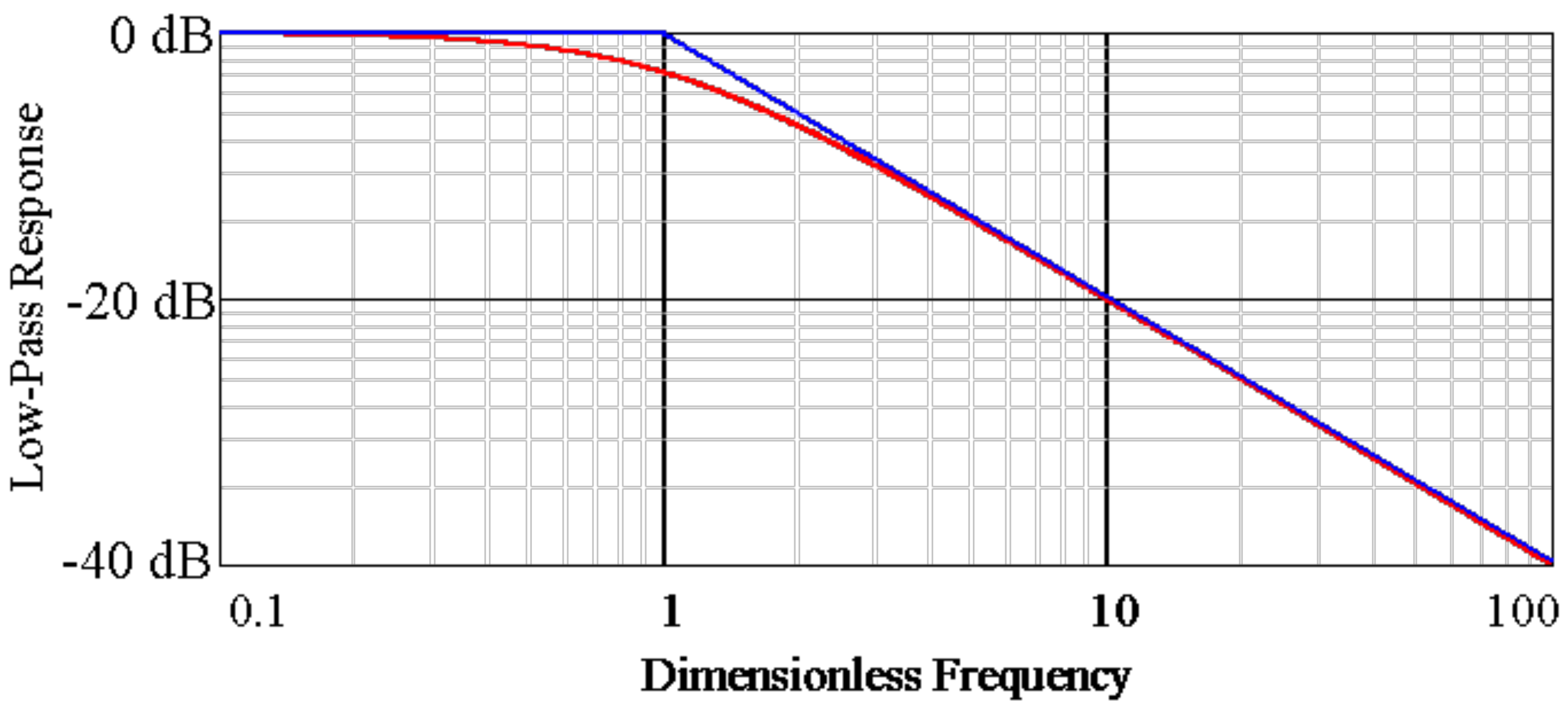
- Order of filters: Defines loss of amplification / dampening ("rolloff rate") for frequencies far above (low pass) / far below (high pass) the critical frequency.

Reminder: Chapter 2



Low pass





For high frequencies:

Z drops with 20 dB per frequency decade / 6 dB / octave (frequency x2 / x0.5)

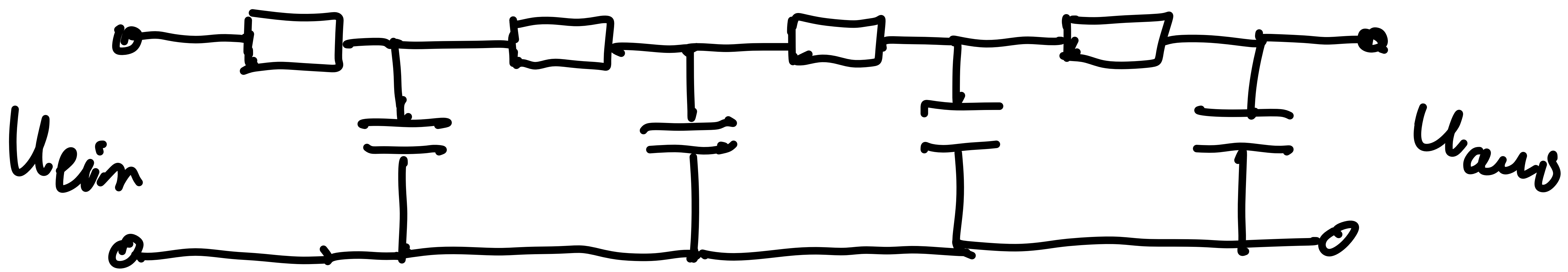
=> Filter of 1st order !

Filter Order

Background Info

- Order of filters: Defines loss of amplification / dampening (“rolloff rate”) for frequencies far above (low pass) / far below (high pass) the critical frequency.

The easiest path to higher orders - and steeper flanks / higher rolloff rate:
Cascading of RC low passes



In general:
dampening of n x 20 dB / decade

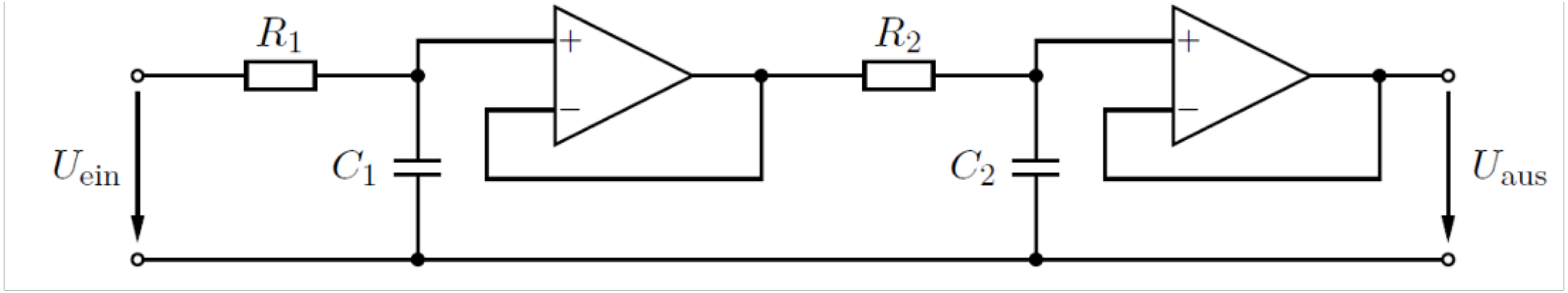
n = order of filter

Active Filter: 2^{nd} Order Filter

The naive Approach - Cascading

- Cascading of filters provides higher orders - lossless due to active components
  - Enables compact construction - inductors can be replaced with capacitors and OPVs

Naive solution for a 2^{nd} order low pass

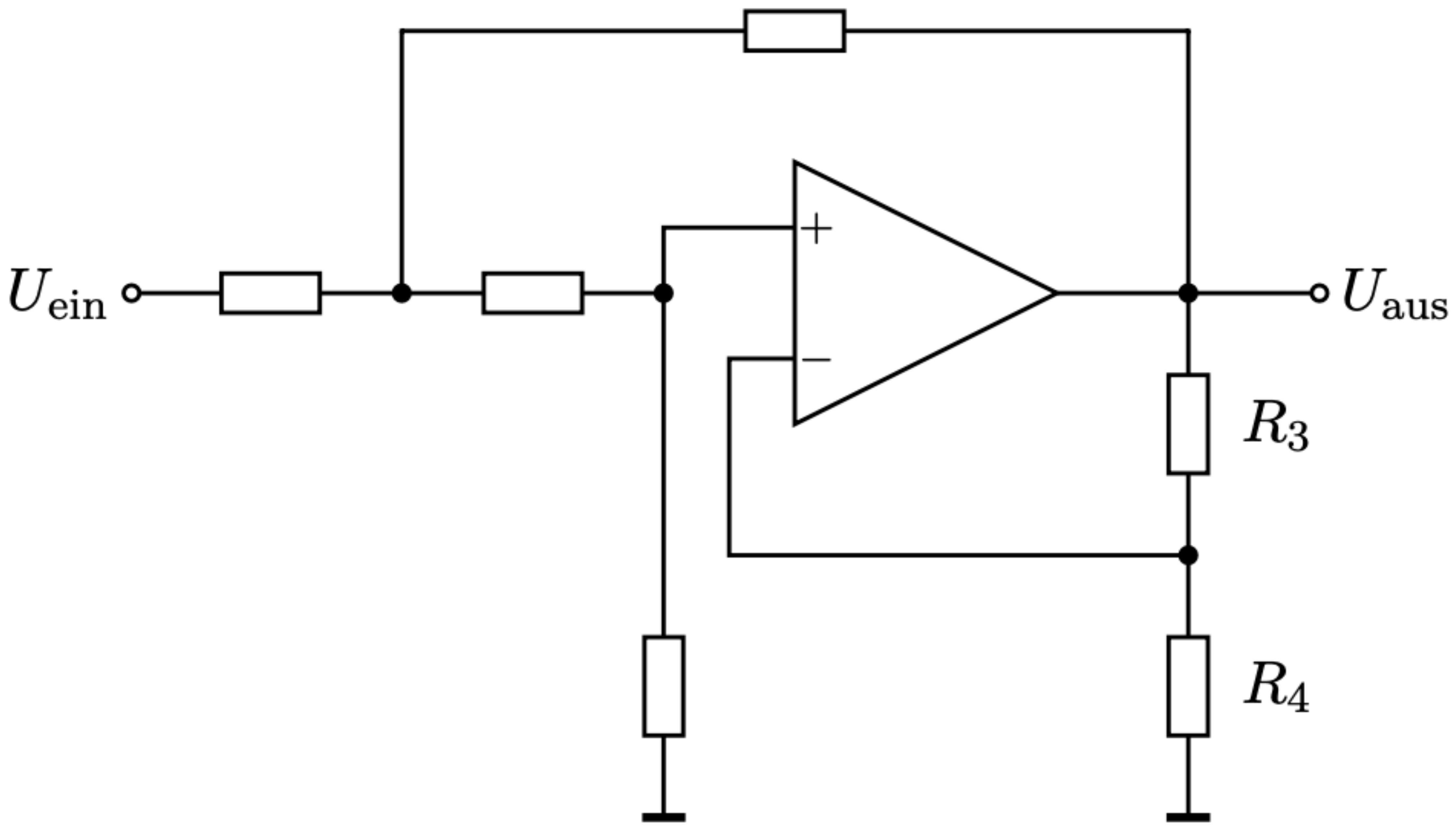


2x passive RC filter, 2 x OpAmp as buffer

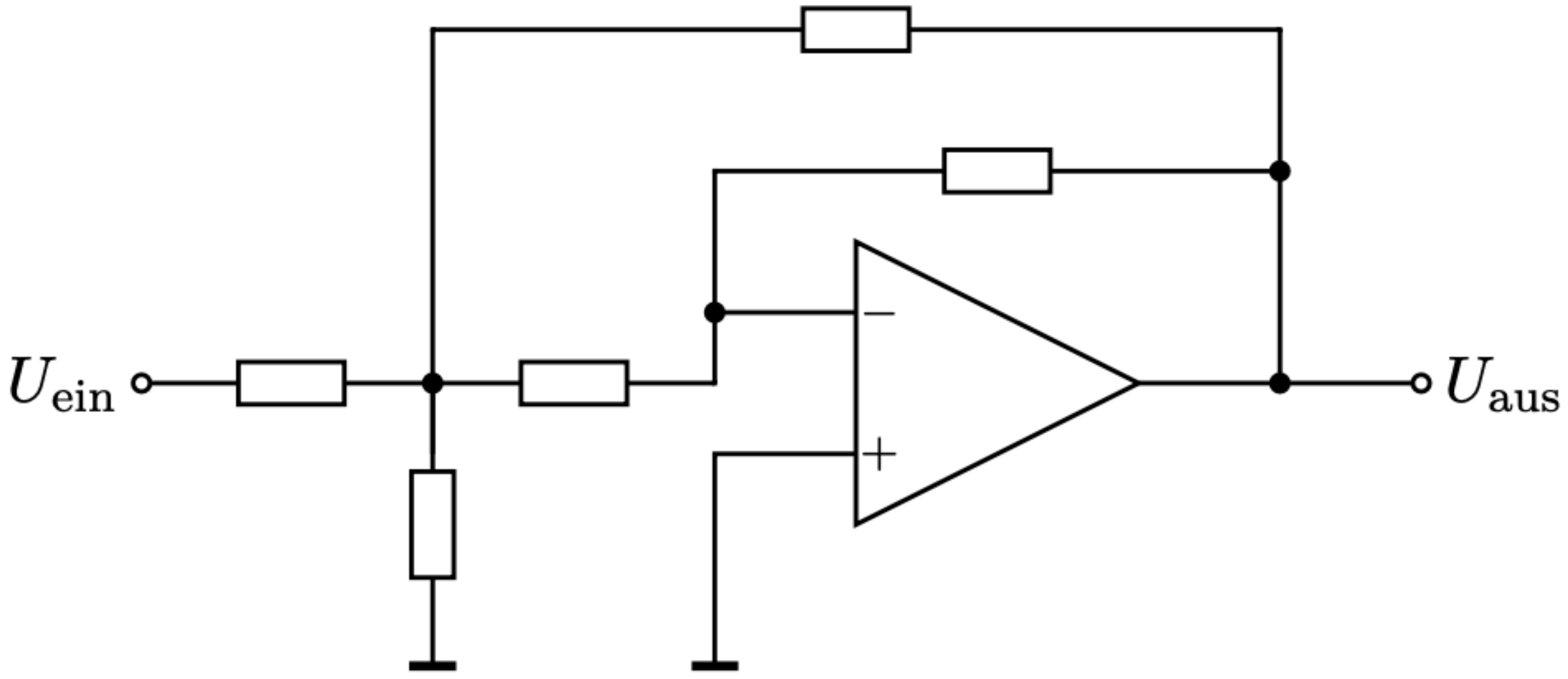
Frank Simon (frank.simon@kit.edu)

Active Filter: 2nd Order Filter

Two common Topologies



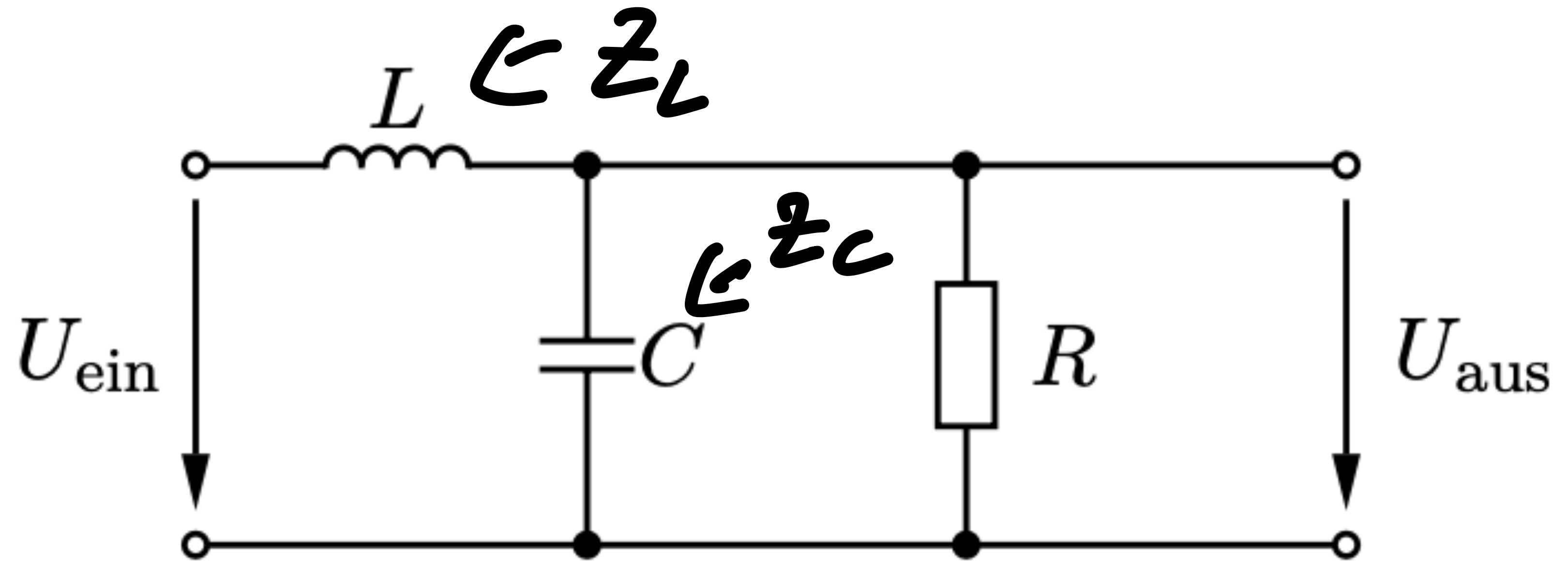
- Generic Sallen-Key filter
  (R. P. Sallen and E. L. Key, MIT, 1955)
  - Simple structure, minimal number of components.



- Generic multi-feedback filter

Turning these into filters: two of the unnumbered resistors replaced by capacitors - the choice determines the type of filter.

- Passive low pass 2nd order



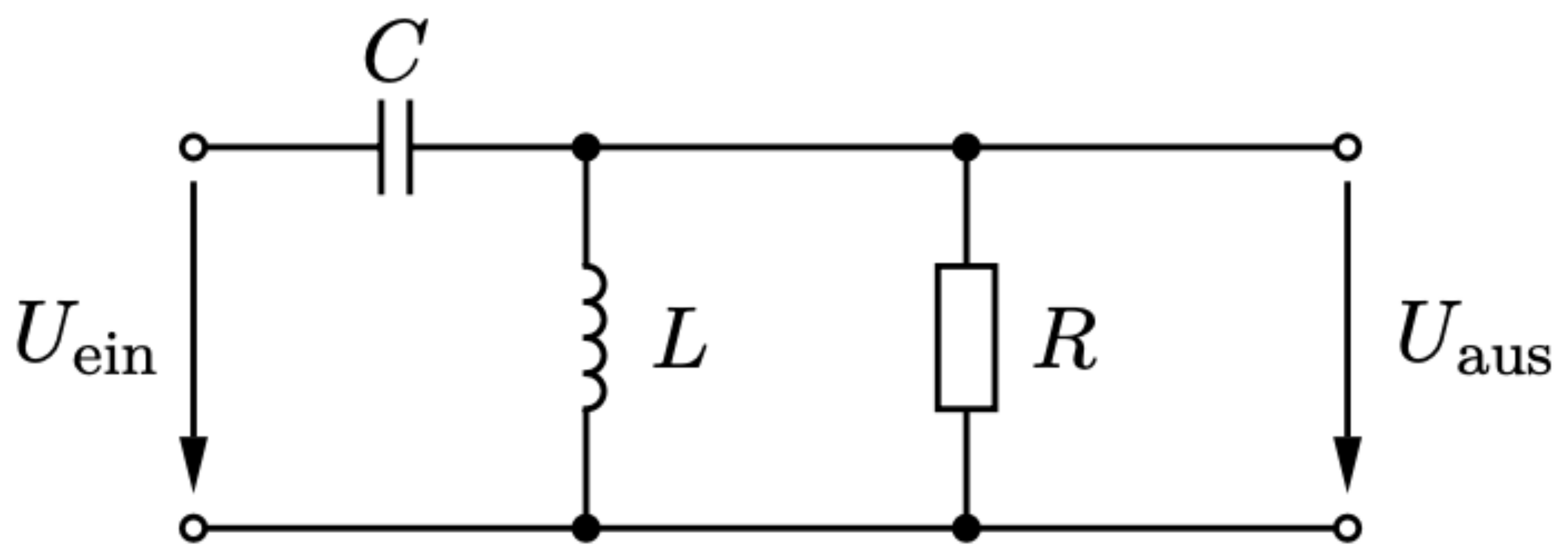
Def.: $s \hat{=} j\omega$

Transfer function:

$$Z = \frac{Z_C \parallel R}{Z_L + Z_C \parallel R} = \frac{\frac{R\frac{1}{sC}}{R+\frac{1}{sC}}}{sL + \frac{R\frac{1}{sC}}{R+\frac{1}{sC}}} = \frac{R}{s^2RCL + sL + R}$$

General: $$Z(s) = \frac{1}{as^2 + bs + 1}$$

- Passive high pass 2nd order

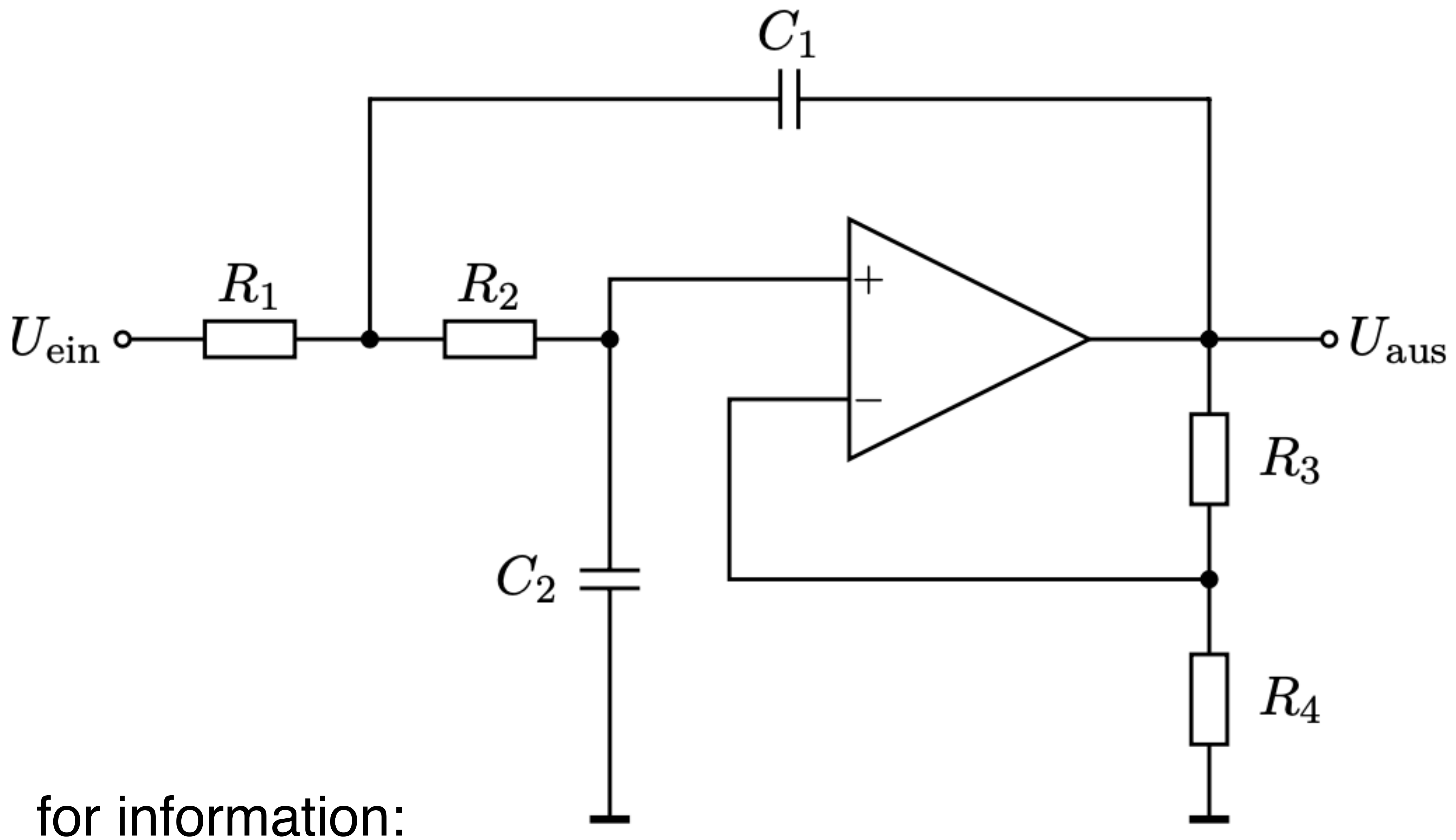


Same principle:

$$Z = \frac{sL \parallel R}{\frac{1}{sC} + sL \parallel R} = \frac{s^2RCL}{s^2RCL + sL + R}$$

$$Z(s) = \frac{s^2}{s^2 + as + b}$$

Sallen-Key Low Pass



- Why does this circuit act as a low pass?

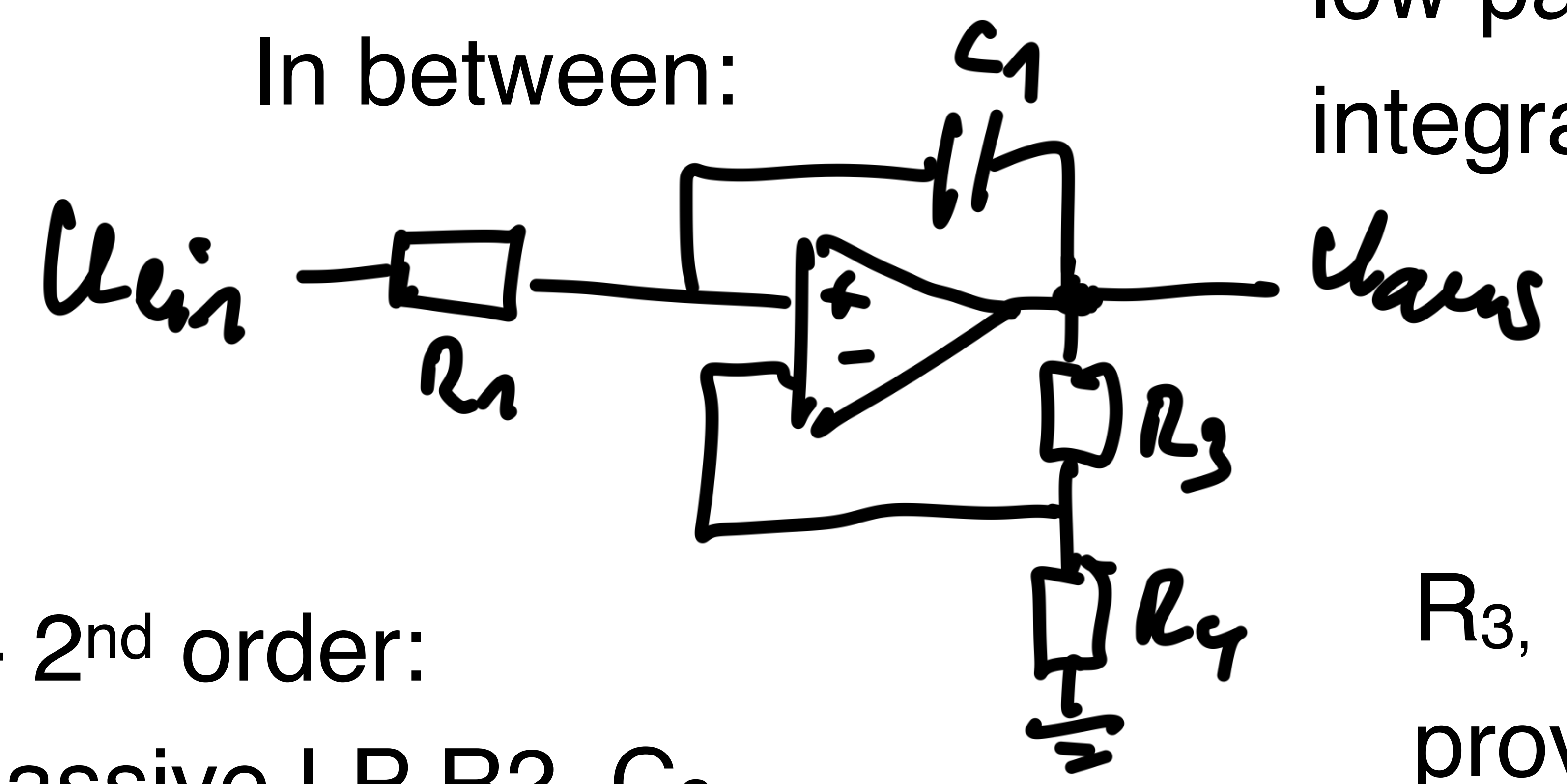
For small frequencies: C_1, C_2 “open”:

Op-Amp “passing through” => Z = 1

For high frequencies: C_2 “shorted”:

Op-Amp “+” input on ground, with golden rules U_{aus} = 0.

In between:



low pass 1st order.: integrator with C_1, R_1

+ 2nd order: passive LP R2, C_2

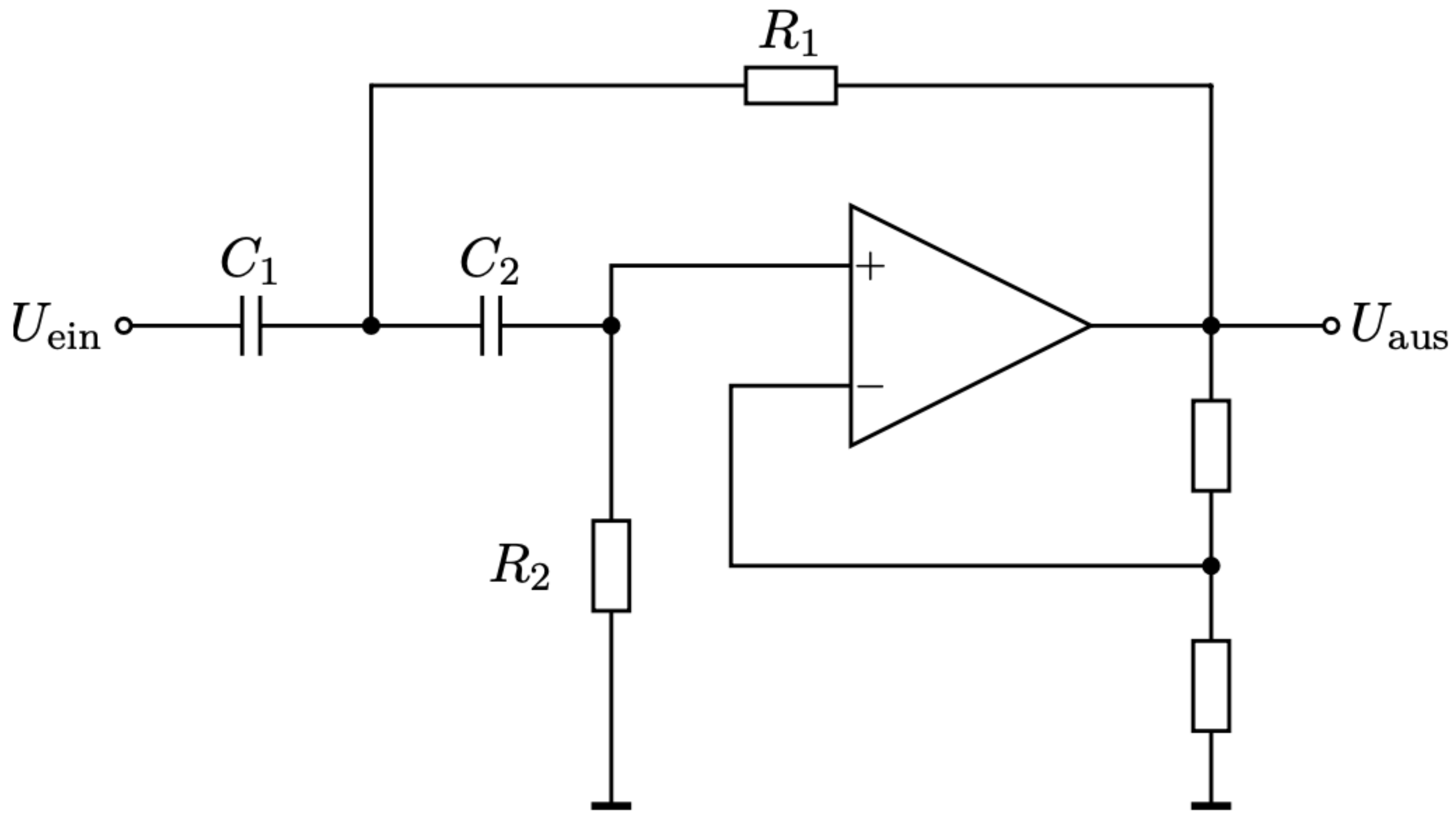
R_3, R_4: provide amplification

for information:

$$Z(s) = \frac{1}{as^2 + bs + 1} \qquad Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2(R_1 + R_2)} \qquad 2\pi f_c = \omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$a = C_1 C_2 R_1 R_2 = \frac{1}{\omega_C^2} \quad , \quad b = C_2 \cdot (R_1 + R_2) = \frac{1}{\omega_C Q}$$

Sallen-Key High Pass



- Analoguous to low pass:
  High frequencies passed by C_1 and C_2.
  Low frequencies blocked by C_1, R_1 and C_2, R_2.

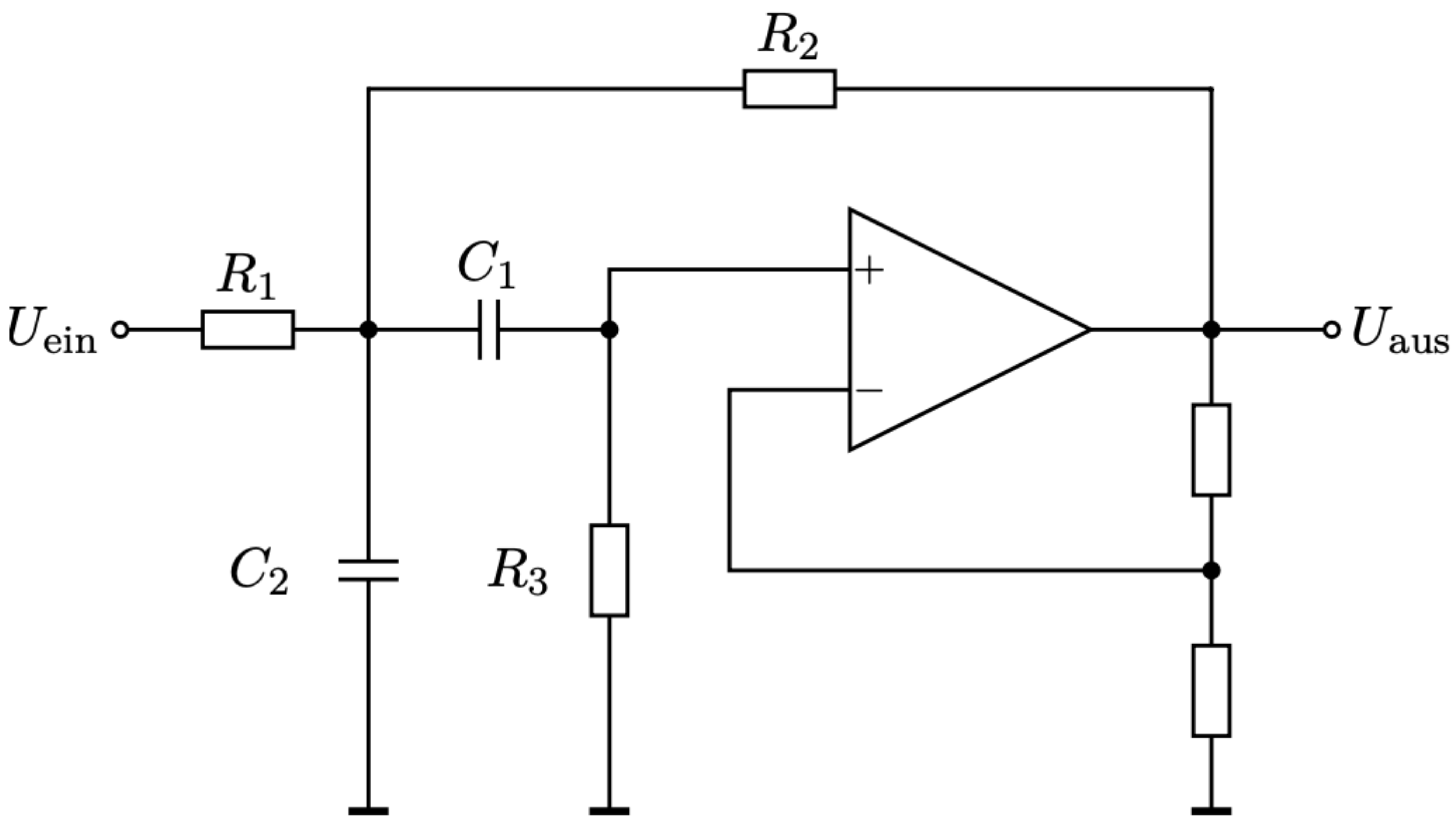
For information:

$$Z(s) = \frac{s^2}{s^2 + as + b}$$

$$a = \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} \quad , \quad b = \frac{1}{R_1 R_2 C_1 C_2}$$

$$2\pi f_c = \omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad \text{und} \quad Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_2 (C_1 + C_2)}$$

Sallen Key Band Pass



- High pass via C_1
- Low pass via C_2

Mathematical Description of Filters

Just for Information…

- The behavior of filters is described by polynomes of corresponding order in s (= jω)

Generalized complex transfer function:

$$Z(s) = Z_0 \cdot \frac{s^n + \alpha_{n-1}s^{n-1} \dots + \alpha_1 s + \alpha_0}{s^n + \beta_{n-1}s^{n-1} \dots + \beta_1 s + \beta_0}$$

Highest power of s defines filter order *n*

Filter of higher order can be built by cascading several filters of lower order. Mathematically: Decomposition into polynomes with the appropriate factors:

$$= \frac{(s^2 + a_1 s + b_1)(s^2 + a_2 s + b_2)\dots}{(s^2 + c_1 s + d_1)(s^2 + c_2 s + d_2)\dots}$$

$$= \frac{(s - z_0)(s - z_1)(s - z_2)\dots(s - z_n)}{(s - p_0)(s - p_1)(s - p_2)\dots(s - p_n)}$$

As example:

Filter of 4^{th} order:
=> two 2^{nd} order filters

5^{th} order:
=> add one more 1^{st} order filter

Filter Types

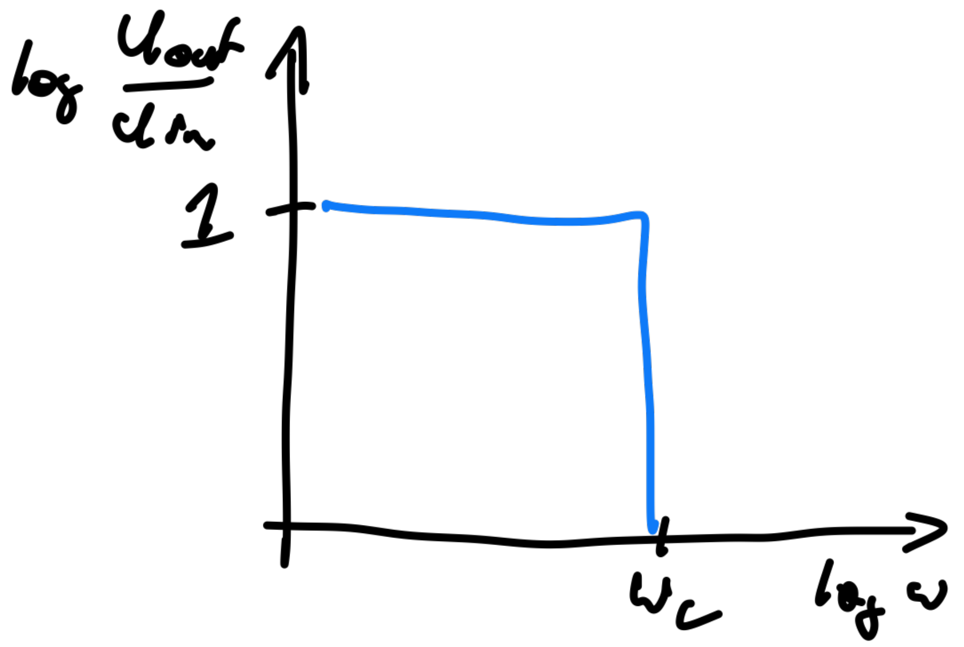
General Considerations

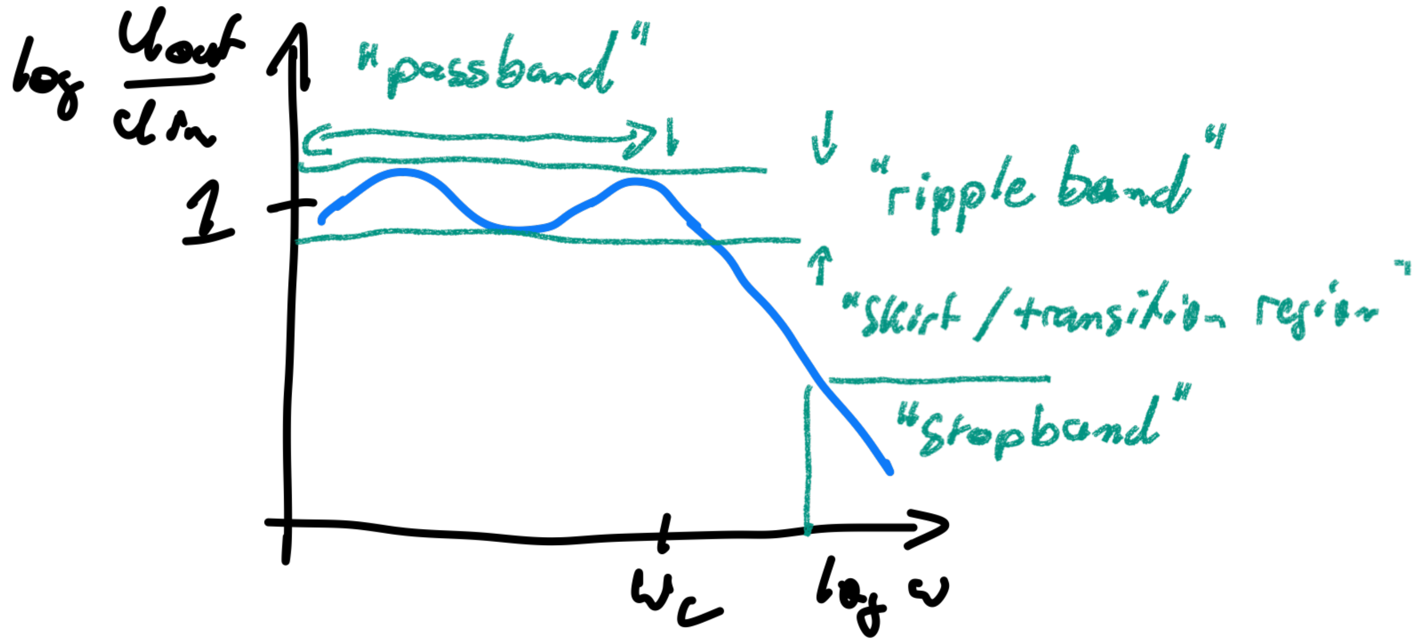
- The choice of the coefficients a_i, b_i,… define the behavior of the filter.
  Components have to be chosen accordingly for the circuit implementation.

idealized expectation:

real situation:

Example:
low pass





pass band ("*Durchlassbereich*")
ripple in the pass band
skirt / transition region
stop band ("*Sperrbereich*")

Central property (independent of concrete implementation):

- The *cutoff frequency* ω_c defines the end of the pass band (typ. -3 dB)
- In the stop band the amplitude drops as *n* x 20 dB per frequency decade

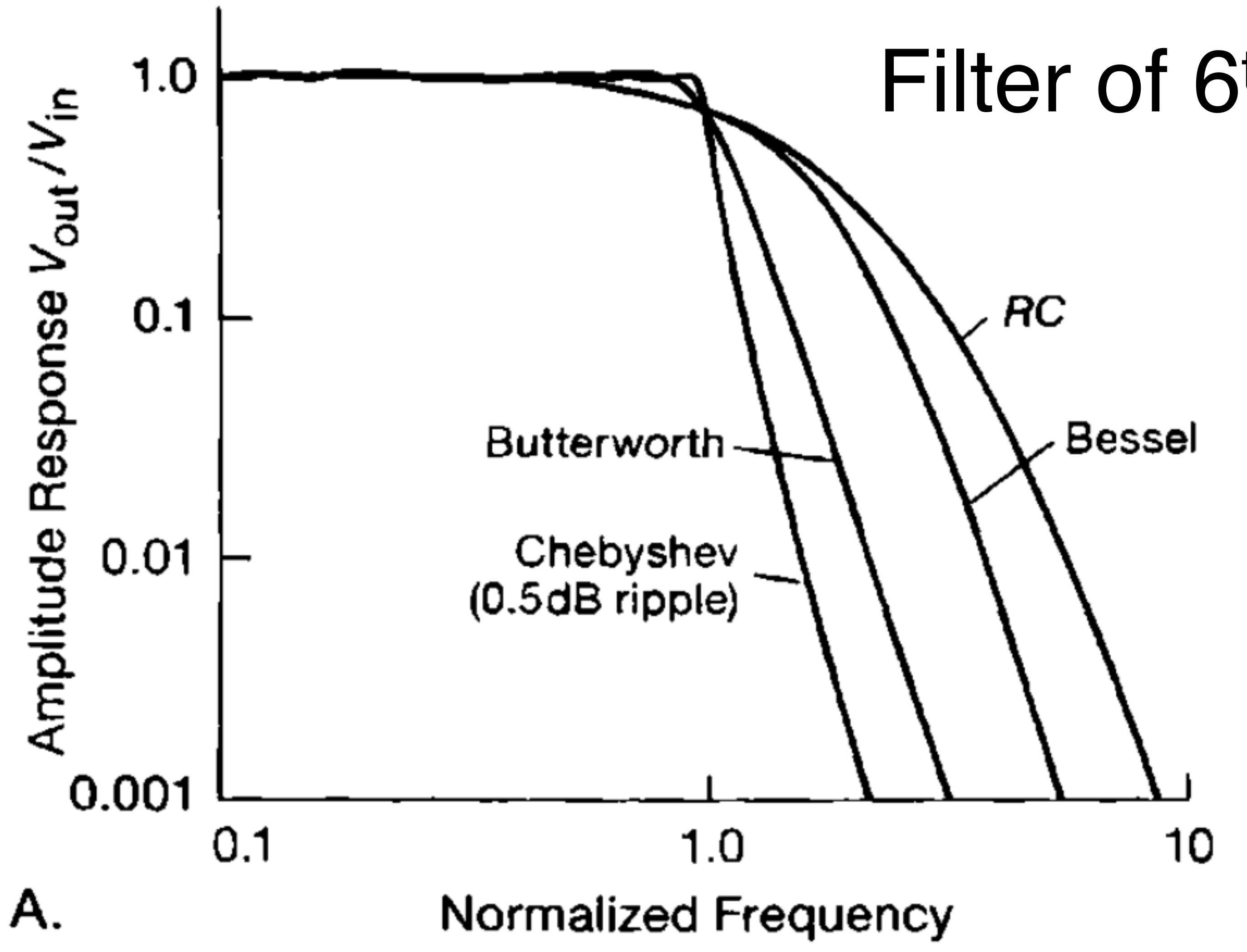
The concrete implementation defines - among others:

- Exact shape of the transfer function as a function of frequency (and the phase phase shift)
- Transition from pass band to stop band

- Differences in behavior in different regions and in phase
  - ***Butterworth Filter:*** Minimal ripple in pass band, smooth transition, monotone dampening in stop band.
  - ***Tschebyscheff Filter:*** Ripple in pass band, steep transition.
  - ***Bessel Filter:*** Slow transition, phase depends linearly on frequency: Minimal distortions.
  - ***Elliptic Filter:*** Very steep transition, but ripple in pass band and stop band.

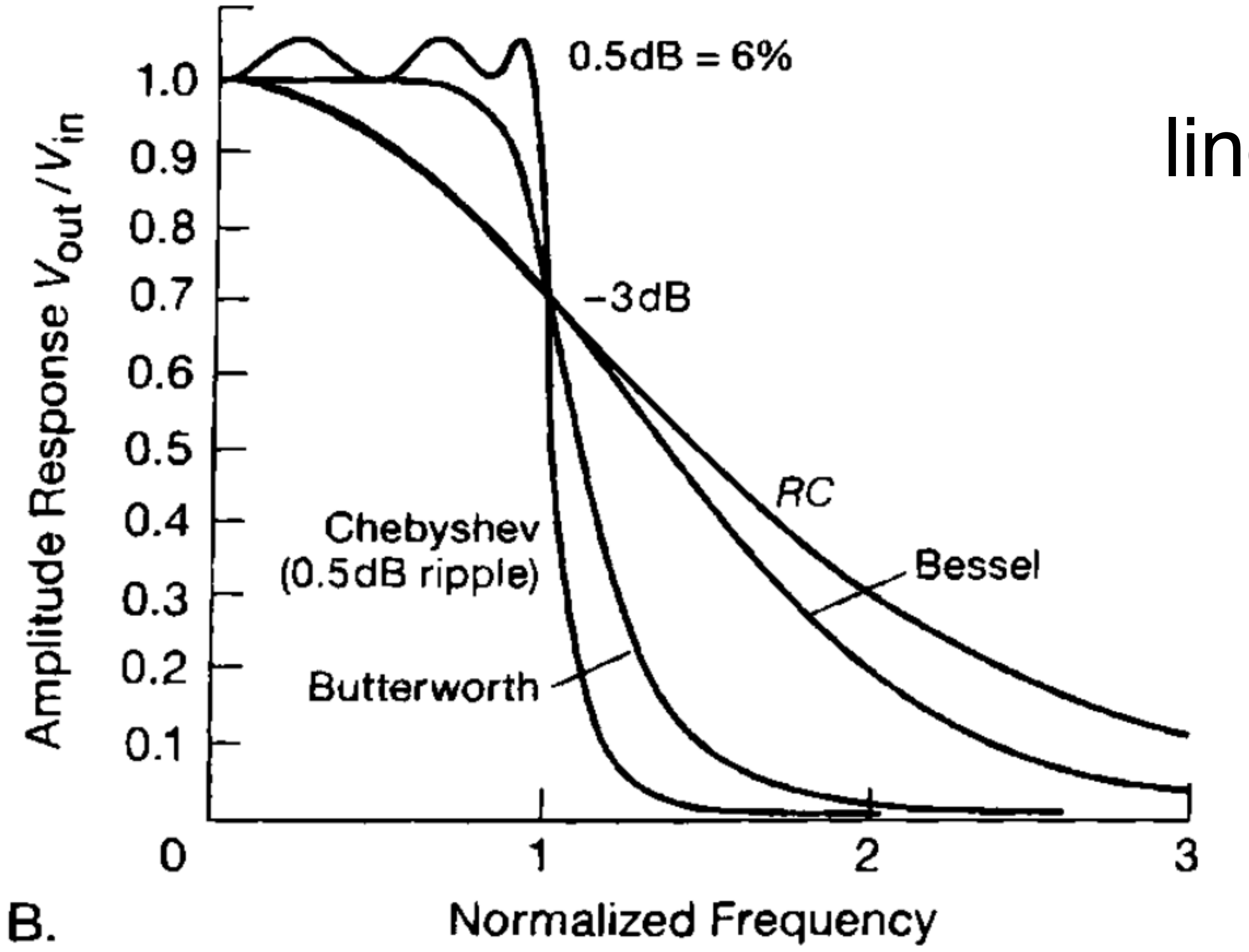
Filter of 6th order

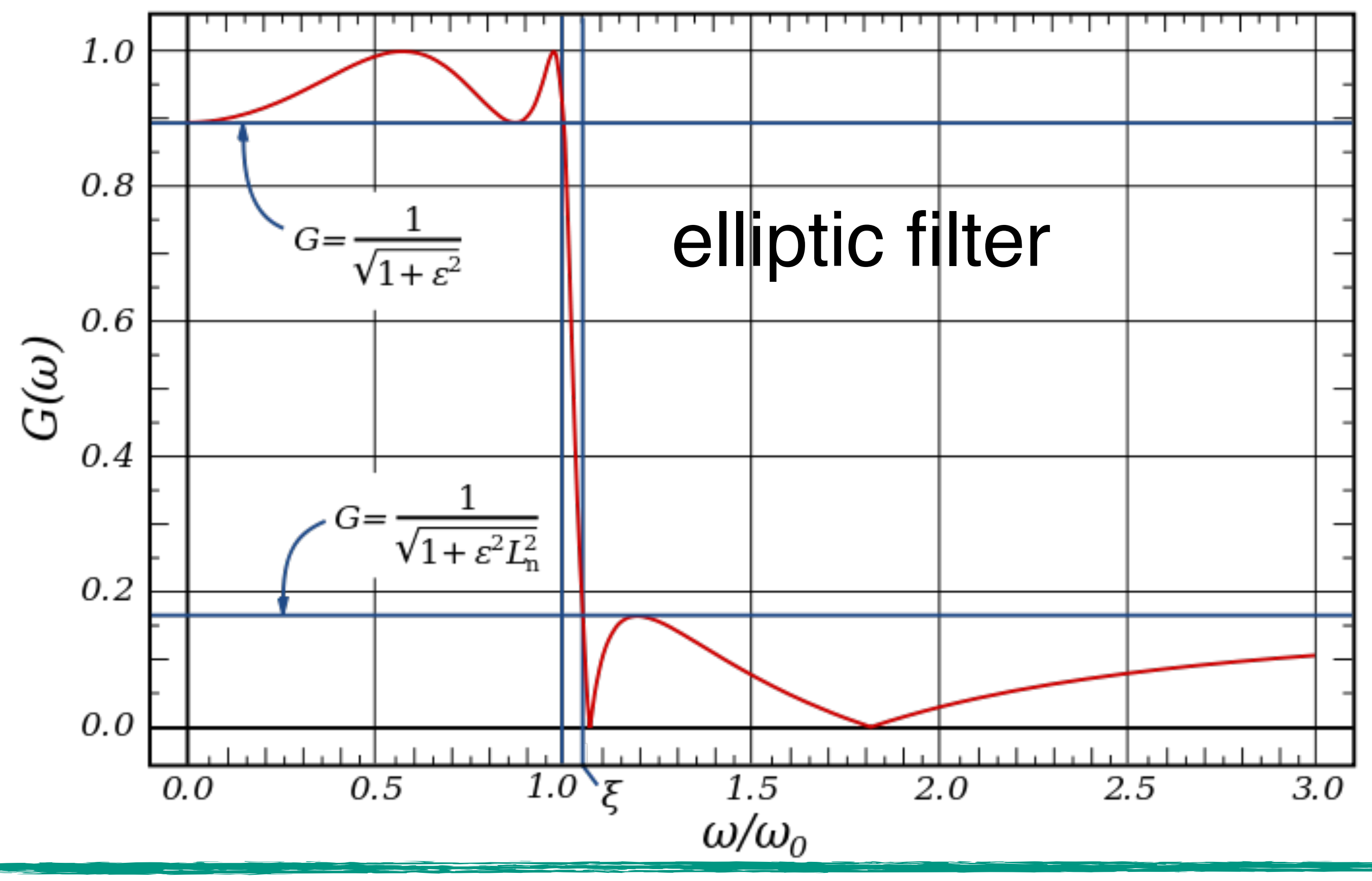
log



from: Horrowitz, Hill “The art of electronics”

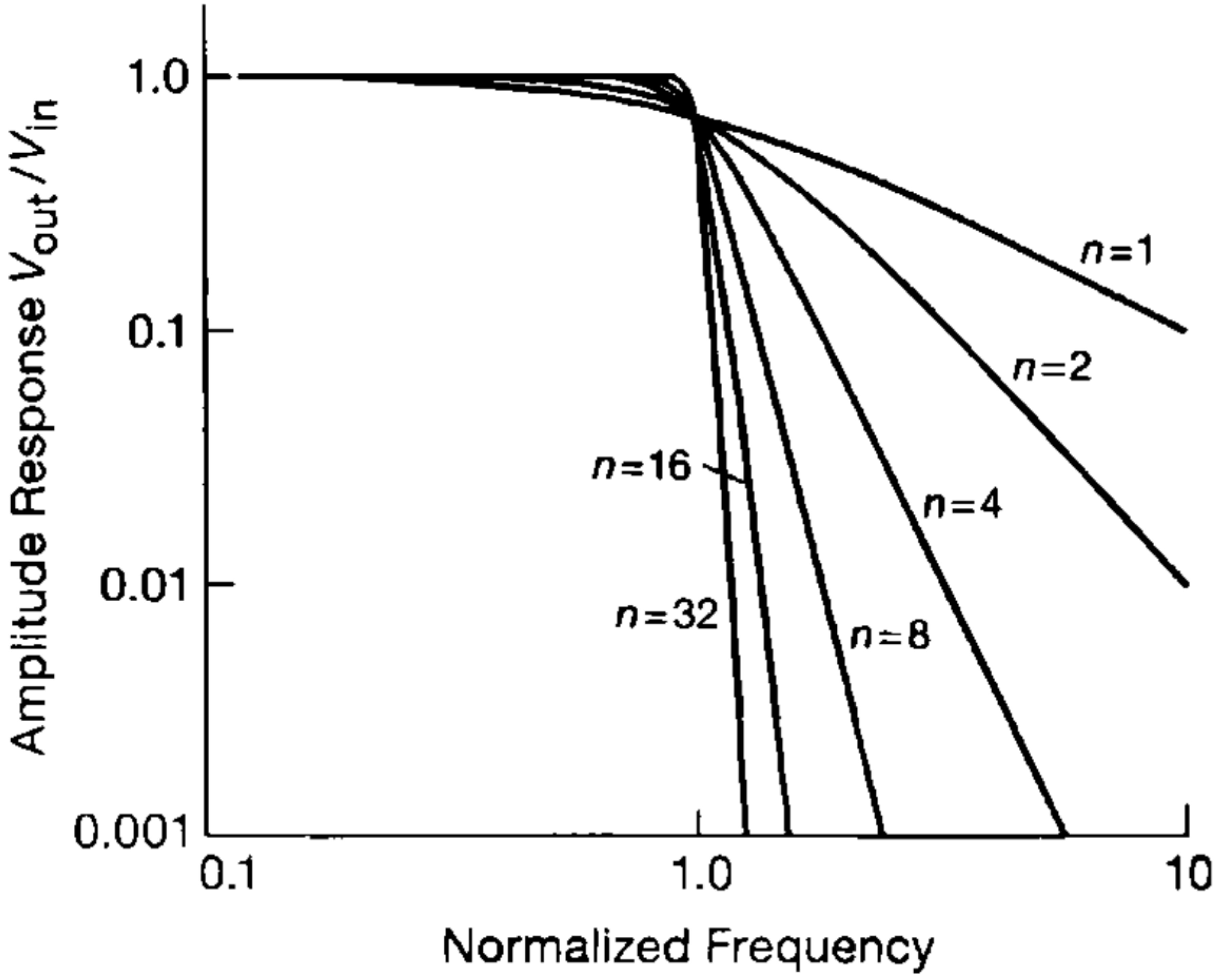
linear





Butterworth Filter

Impact of Order



from: Horrowitz, Hill "The art of electronics"

Mathematical description:

n	
1	$(1+s)$
2	$(1+1.414s+s^2)$
3	$(1+s)(1+s+s^2)$
4	$(1+0.765s+s^2)(1+1.848s+s^2)$
5	$(1+s)(1+0.618s+s^2)(1+1.618s+s^2)$
6	$(1+0.518s+s^2)(1+1.414s+s^2)(1+1.932s+s^2)$
7	$(1+s)(1+0.445s+s^2)(1+1.247s+s^2)(1+1.802s+s^2)$
8	$(1+0.390s+s^2)(1+1.111s+s^2)(1+1.663s+s^2)(1+1.962s+s^2)$
9	$(1+s)(1+0.347s+s^2)(1+s+s^2)(1+1.532s+s^2)(1+1.879s+s^2)$
10	$(1+0.313s+s^2)(1+0.908s+s^2)(1+1.414s+s^2)(1+1.782s+s^2)(1+1.975s+s^2)$

From this: Transfer function for 3rd order:

$$Z(s) = \frac{1}{(s+1)(s^2+s+1)}$$

In general:

Coefficients of the transfer function taken from tables, component properties derived from this.

for ex.: $a = C_1C_2R_1R_2$ $b = C_2 \cdot (R_1 + R_2)$ (for Sallen-Key LP 2nd order [S. 10])

Voltage Converters

In: Chapter 8 - Additional Topics

- Here: DC-DC Converters / Regulators

Voltage Converters

Overview

- Two basic types

switched-mode regulator *(Schaltregler / Schaltnetzteile)*

- most common type
- high efficiency
- Always a residual ripple on output
- Noise source
- Can increase and reduce voltages

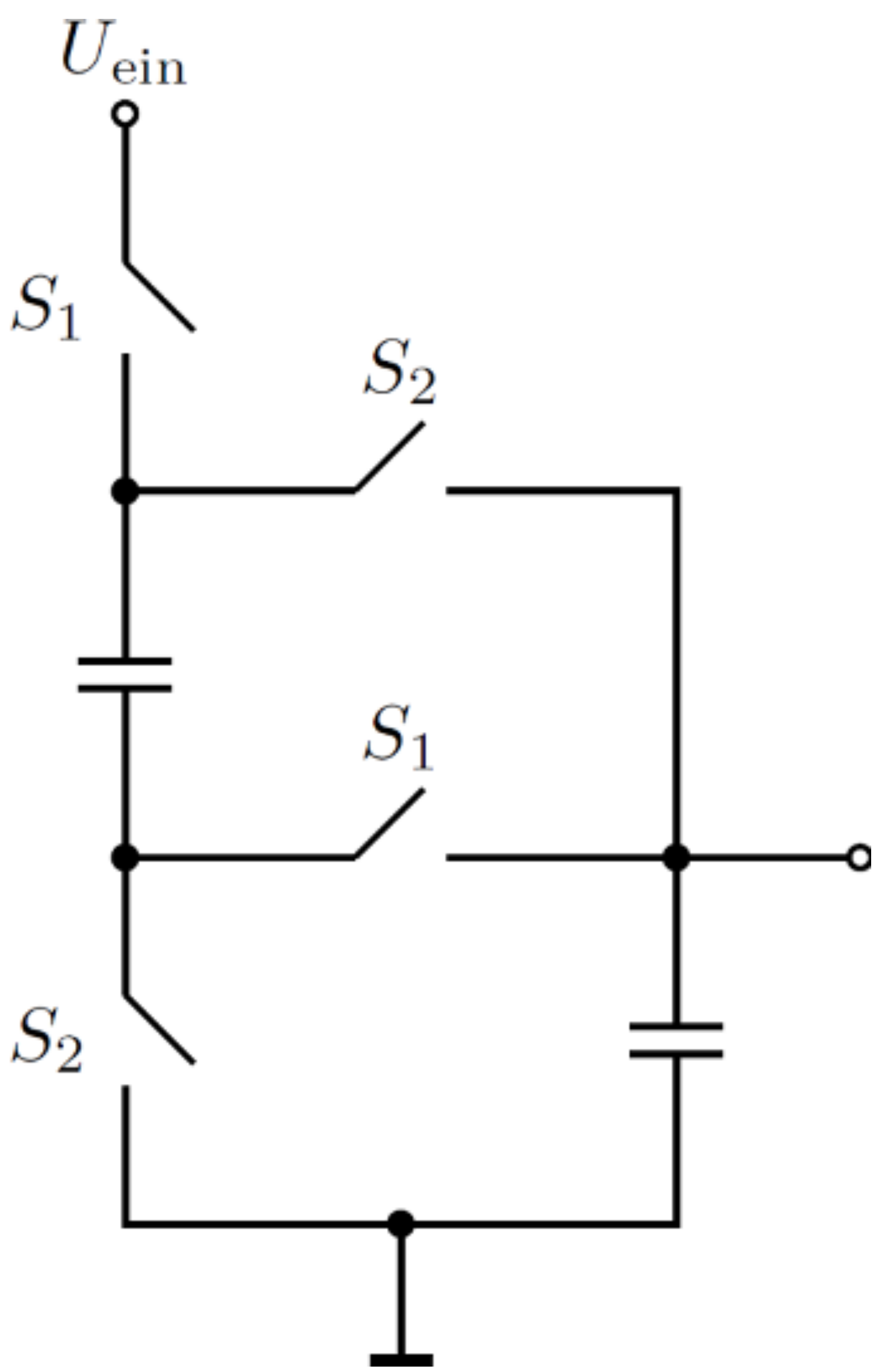
Linear regulator (*Linearregler)*

- Low efficiency
- Can only reduce voltages
- “Quiet” output

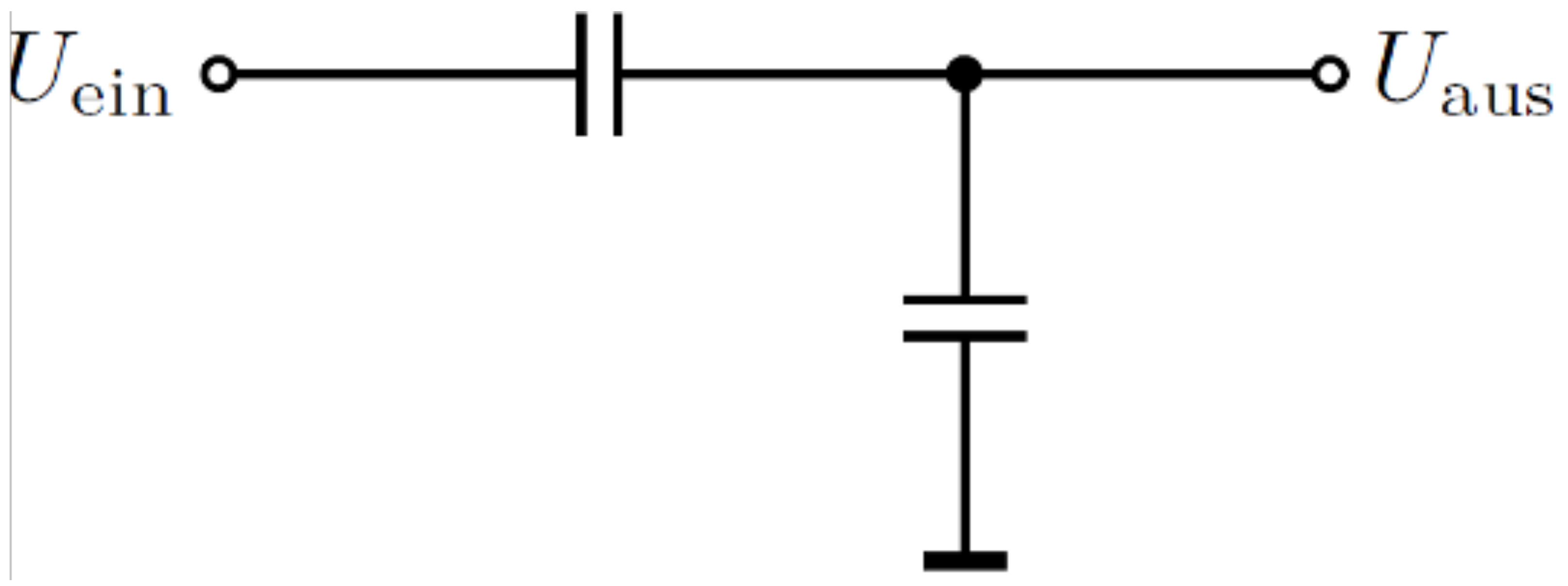
DC-DC Step-Down Converter

Simple voltage regulator

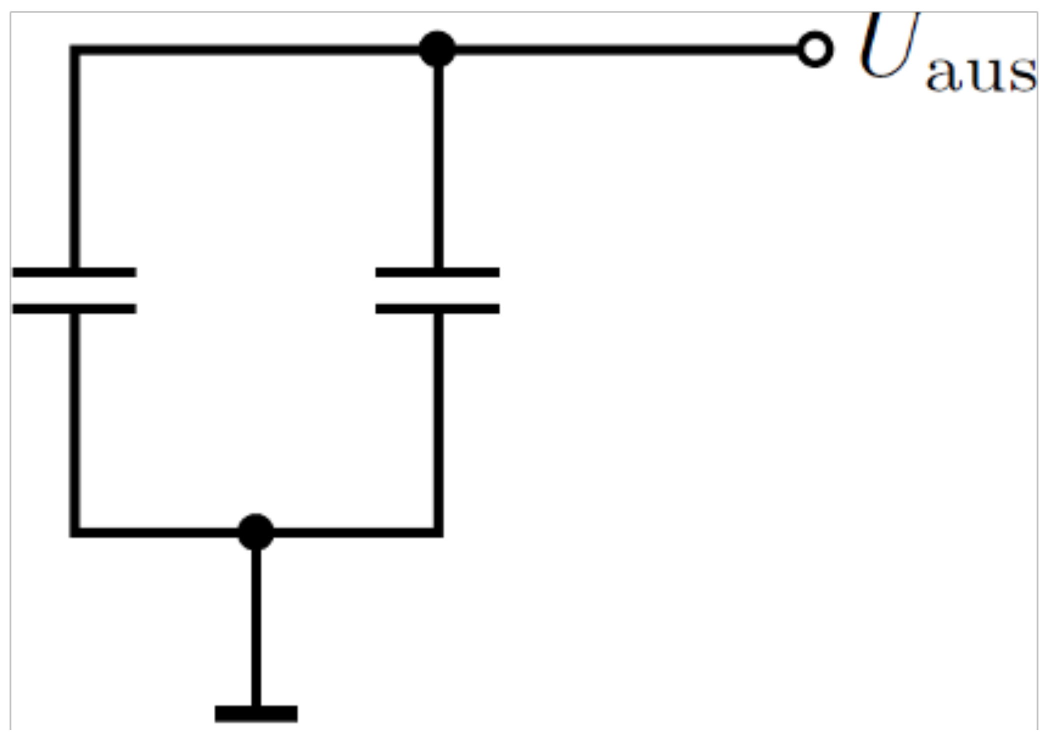
Switch S_i
in real life:
transistors



Charging:
S_1 closed,
S_2 open



Discharging:
S_1 open,
S_2 closed



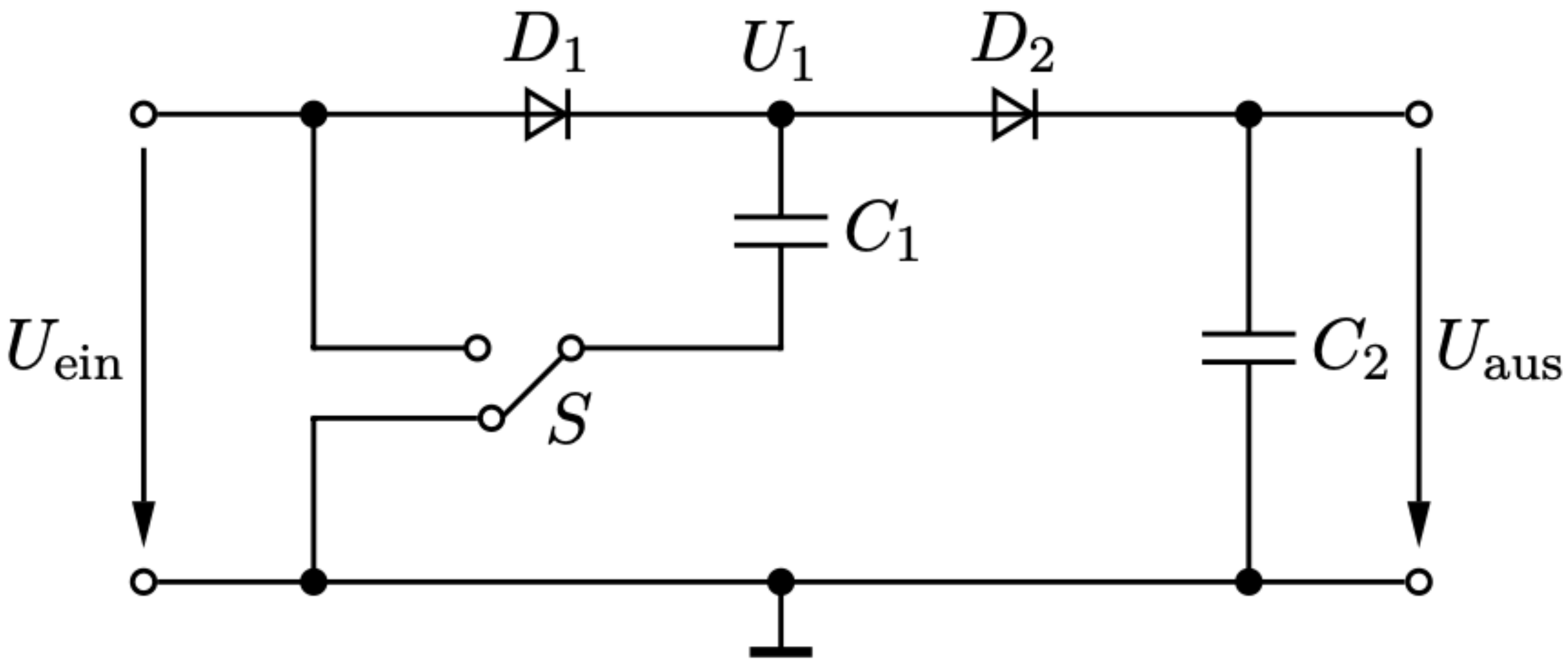
high switching frequency to achieve low fluctuations
in output voltage:
typically kHz - MHz

Here: $U_{aus} = 0.5 \times U_{ein}$
with more capacitors and switches other ratios can also be achieved.

Charge Pump

Voltage Multiplication

- Similar to Greinacher Circuit for AC input (chapter 03).



$U_{aus} = 2 \text{ x } U_{ein} - 2 \text{ x } U_D$

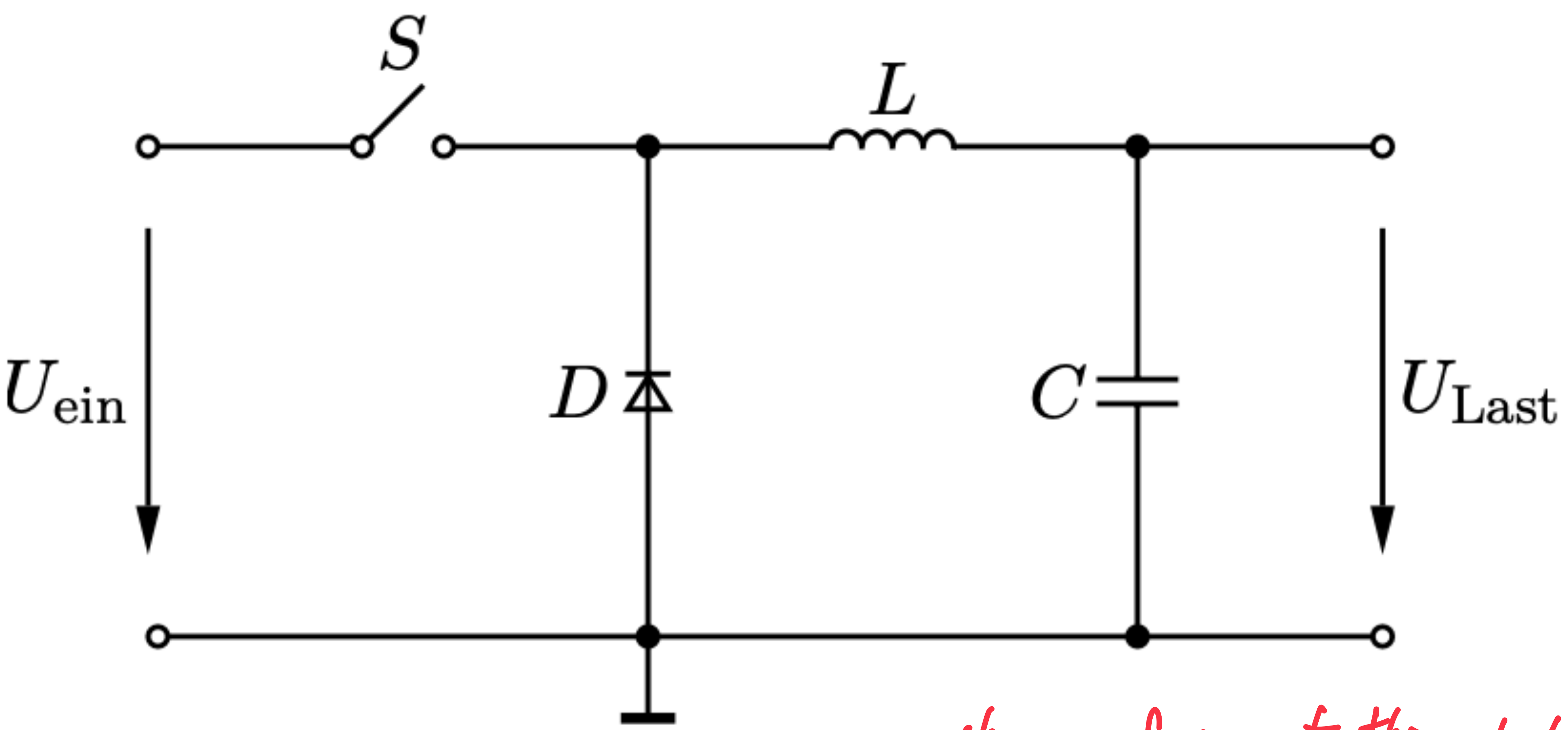
C_1: “pump capacitor”

C_2: “smoothing capacitor”

Buck Converter / Step-Down Converter

Down conversion - a classic circuit

- Inductor and capacitor



How it works

Switch *S* (in real life a transisor) is switched on and of with period *T*.

duty cycle $D = T_{an} / T$

Switch closed

$U_L = U_{ein} - U_{Last}$, diode in reverse bias

Switch open:

$U_L = - U_{Last}$, diode forward biased, inductor provides output

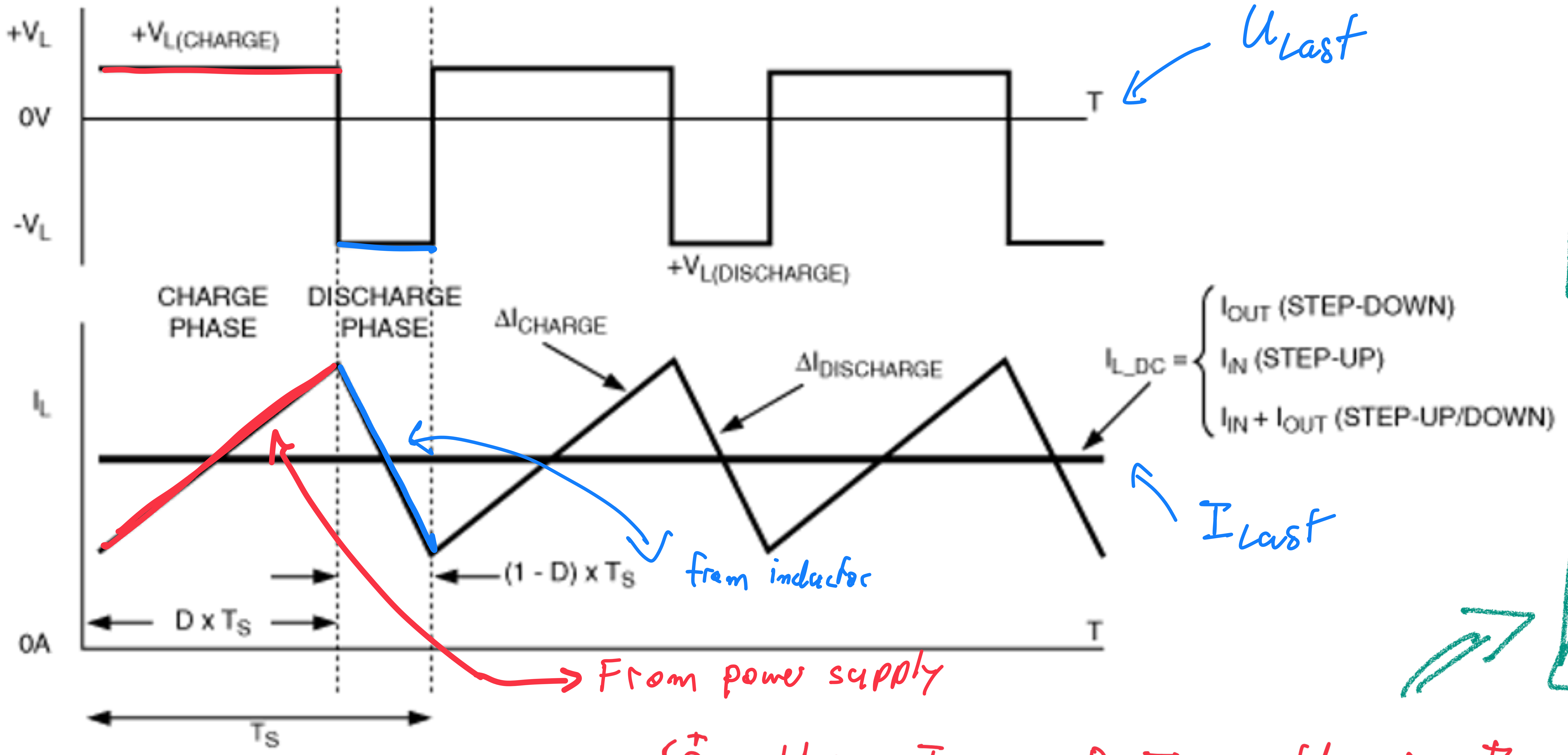
change of current through L over full cycle = 0 !

$$\int_0^{T_{an}} \frac{U_L}{L} dt + I_0 \approx \frac{U_{ein} - U_{Last}}{L} T_{an} + I_0 = \int_{T_{an}}^{T} \frac{U_L}{L} + I_0 \approx \frac{U_{Last}}{L} (T - T_{an}) + I_0 = \frac{U_{Last}}{L} T_{aus} + I_0$$

Results in:

$$(U_{ein} - U_{Last}) T_{an} = U_{Last} (T - T_{an}) \Rightarrow \frac{U_{Last}}{U_{ein}} = D$$

- Time dependence of voltages and currents



$$\frac{U_{Last}}{U_{ein}} = D$$

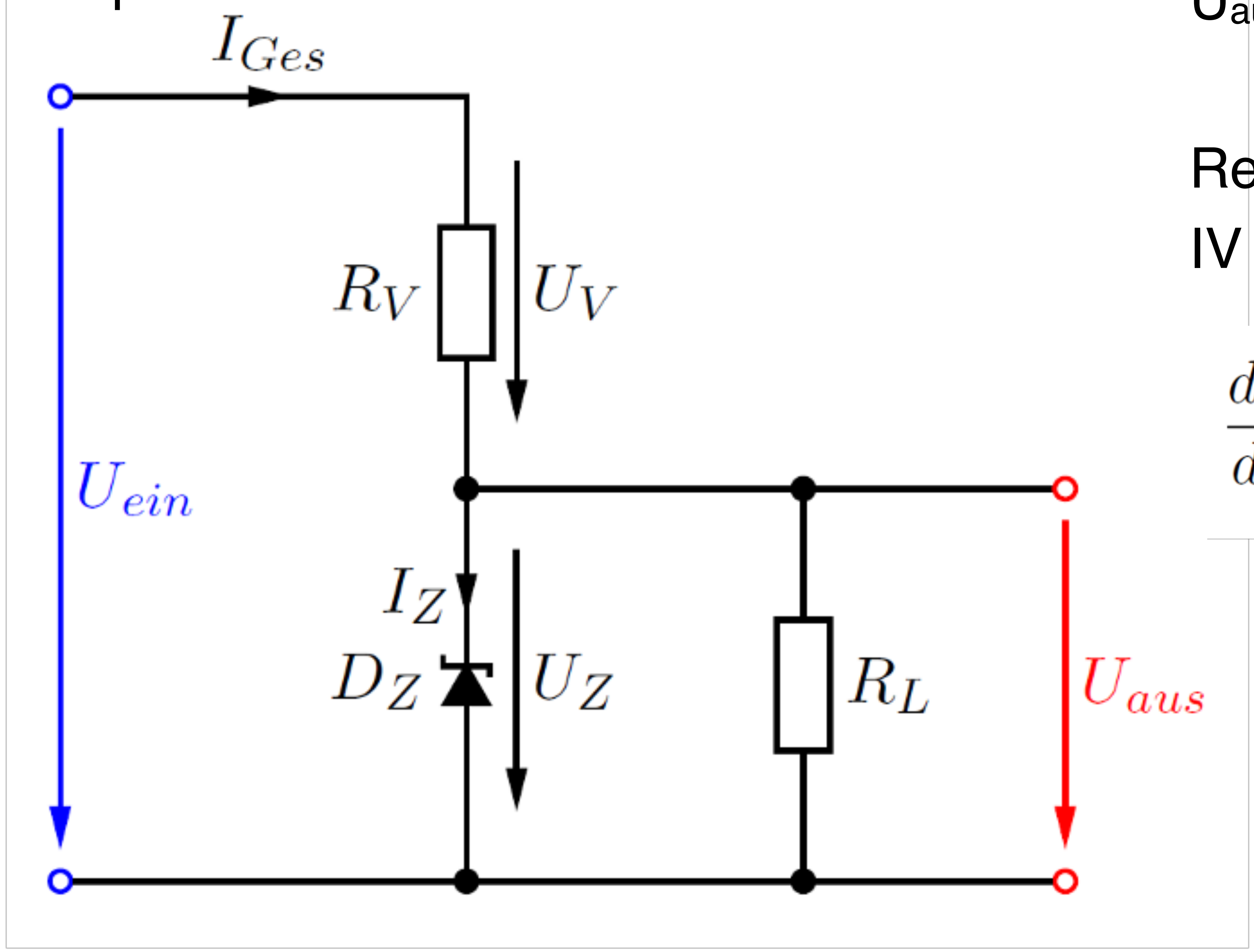
Efficiency ~100% !

$$\int P = U_{ein} \cdot I_{Last} \cdot D \cdot T = U_{Last} \cdot I_{Last} \cdot T$$

Voltage Regulation with Zener Diodes

Reminder

Chapter 03



For sufficiently high U_{ein}:

$U_{aus} = U_Z$; independent on U_{ein}.

Remains stable also for fluctuating load currents due to steep IV curve: U_Z reasonably independent from I_L.

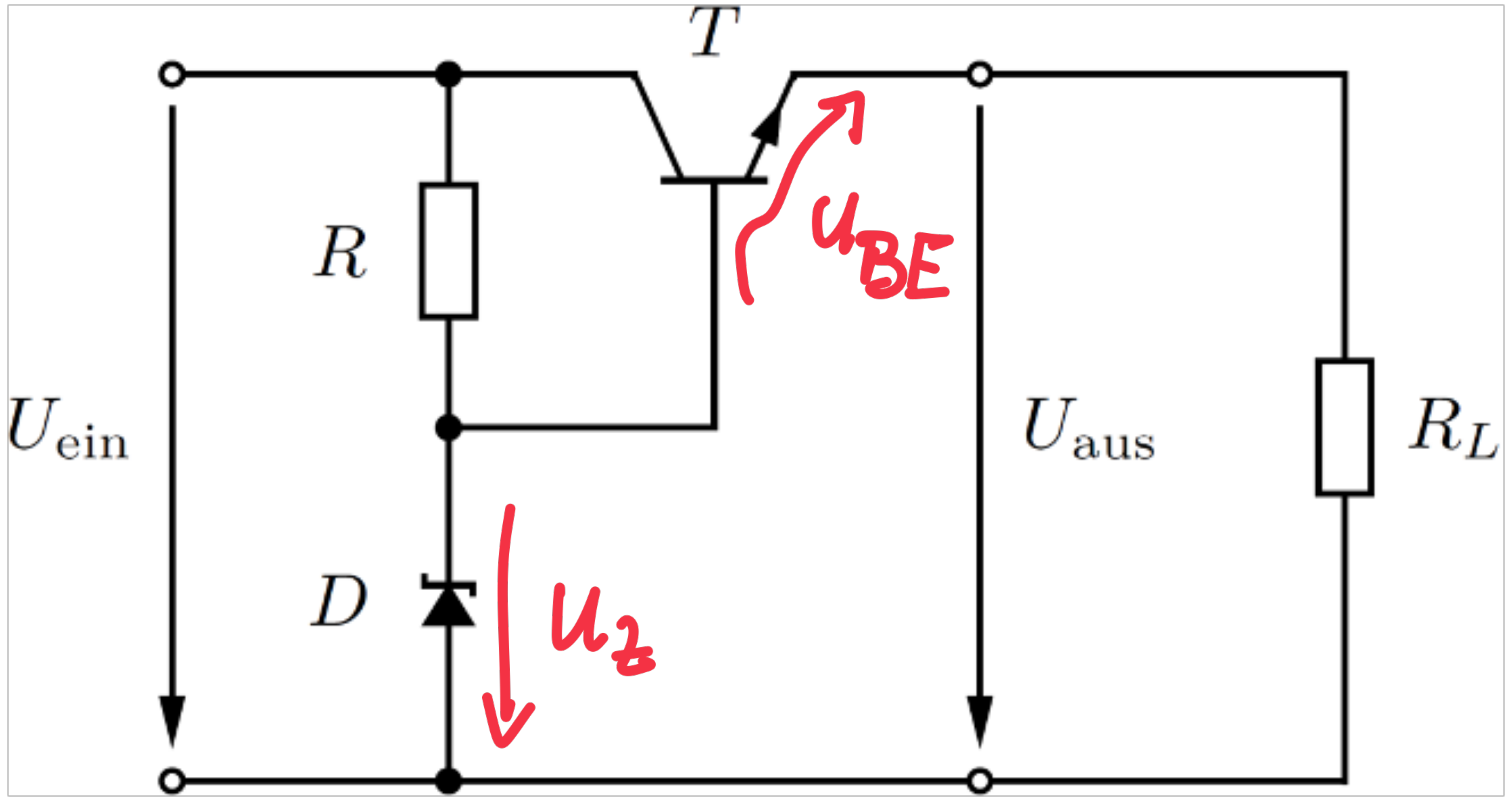
$$\frac{dU_{aus}}{dU_{ein}} = \frac{R_Z \parallel R_L}{(R_Z \parallel R_L) + R_V} \approx \frac{R_Z}{R_V}$$

(good approximation: $R_Z << R_L$)

Simple, but hits limits for high load currents. Strong temperature dependence due to diode.

Voltage Regulation with Zener Diodes

Extension for high Load Currents



- Additional transistor:
  Enables high load currents since current flows through T, not R.

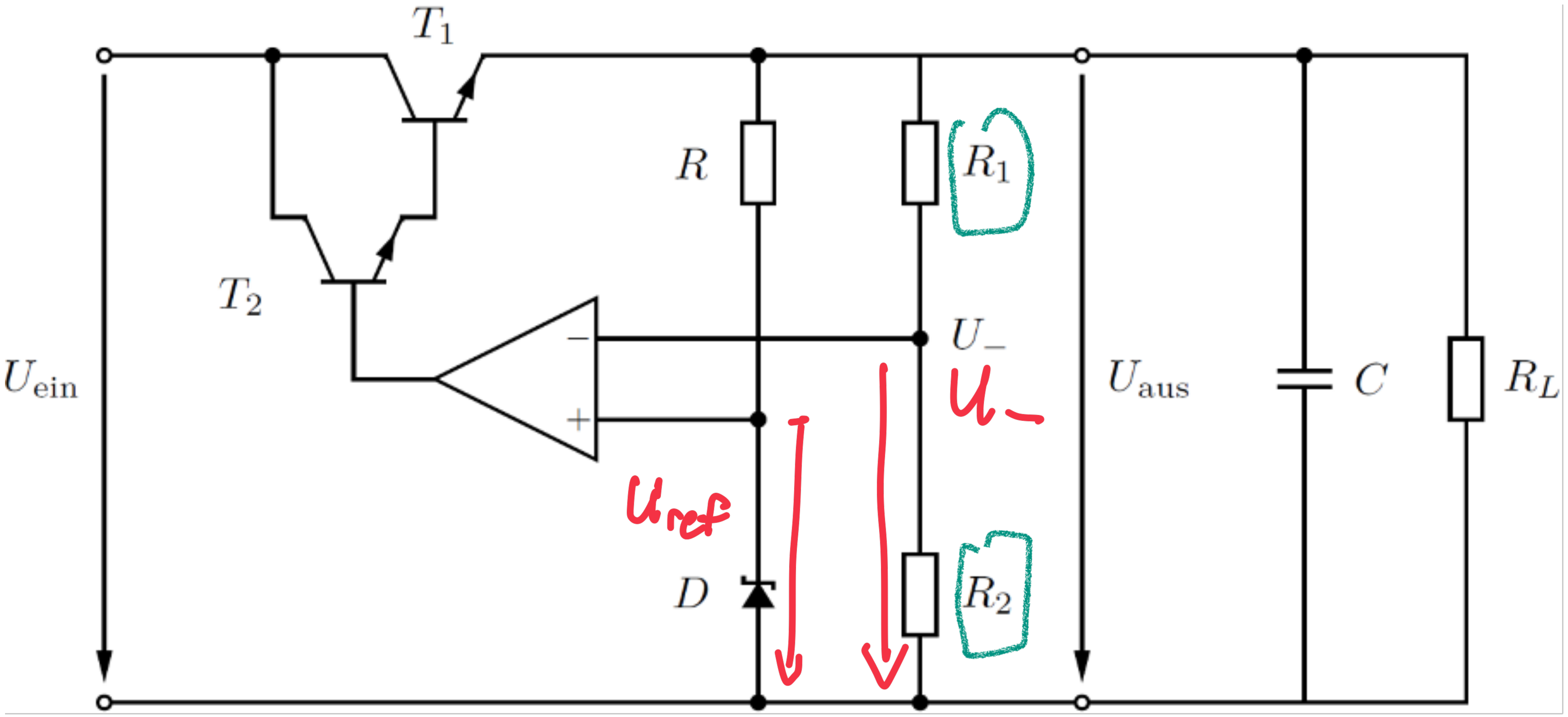
  $U_{aus} = U_Z - U_{BE}$

  Transistor configured as emitter follower (common collector).

But: U_{BE} depends on current: not constant - typically between 0.5 V and 0.8 V
U_{aus} only stable to within a few 100 mV!

Voltage Regulation

Extension with OpAmp



How does this work?

U_{aus} is given by R_1, R_2, the Zener-Diode serves as voltage reference:

Golden rule:

$$U_- = U_{ref}$$

$$\Rightarrow U_- = U_{aus} \cdot \frac{R_2}{R_1 + R_2}$$

$$\Rightarrow \boxed{U_{aus} = U_{ref} \left(\frac{R_1 + R_2}{R_2} \right)}$$

Darlington transistor T_1, T_2: Enables high load currents

Next (and final) Lecture:

Analog 15 - Chapter 08 - Tuesday, February 13, 2024

Time Plan for Next Lectures

A few Changes coming up!

Calender Week	Tuesday	Thursday
45	07.11. **Analog**	09.11. **Digital**
46	14.11. **Analog**	16.11. **Digital**
47	21.11. **Digital**	23.11. **Analog**
48	28.11. **Digital**	30.11. **Digital**
49	05.12. **Digital**	07.12. **Analog**
50	12.12. **Digital**	14.12. **Analog**
51	19.12. **Analog**	21.12. **Digital**
2	09.01. **Analog**	11.01. **Digital**
3	16.01. **Digital**	18.01. **Digital**
4	23.01. **Analog**	25.01. **Digital**
5	30.01. **Analog**	01.02. **Digital**
6	06.02. **Analog**	08.02. **Analog**
7	13.02. **Analog**	15.02. **-**

Electronics for Physicists

Analog Electronics

Frank Simon
Institute for Data Processing and Electronics

Karlsruhe Institute of Technology

KIT, Winter 2023/24

Institute for
Data Processing
and Electronics