Exercise 1 of the Lecture "Full waveform inversion"

2D Finite-Difference modelling of acoustic seismic waves

19.10.2021

Goal

Accurate numerical simulation is of fundamental importance to full-waveform inversion (FWI). In this exercise we solve the forward problem of 2D acoustic wave propagation by using the finite-difference (FD) method. We will simulate and analysis the waveform recorded in a crosshole model.

Introduction

In this exercise we use an FD method to solve the 2D acoustic wave equation numerically. The 2D acoustic wave equation with a constant density ρ is described by the following partial differential equation:

$$\left[\frac{\partial^2}{\partial t^2} - v^2(x,y)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right] p(x,y,t) \ = s(x,y,t), \tag{1}$$

where pressure p is a time- and space-dependent pressure field, $v(x,y)$ is the space-dependent sound velocity (P-wave velocity) and $s(x,y,t)$ is the source of disturbances. Setting the initial condition $p(x,y,t) = 0$ for all x and y at $t \leq 0$, we choose a causal solution. We can use the FD method to simulate the propagation of seismic waves via replacing all the time and spatial derivatives by an FD approximation.

We use an explicit time-domain FD method. We discretize time axis t^n with $n = 1 : N_t$ with an equally spaced time step dt. Similarly, space domain is discretized in a Cartesian spatial coordinates x_l, y_k ($l = 1 : N_x$, $k = 1 : N_y$) with a grid size of dx and dy. We denote the discretized pressure field as $p^n_{k,l}$, the source as $s^n_{k,l}$ and sound velocity as $v_{k,l}$, where the upper and lower indices denote the time step and the spatial grid, respectively (note, the order of lower indices correspond to y and x, respectively).

A simple central-difference formulas of the second order derivative with a second order accuracy (i.e., with a truncation error $\mathcal{O}(dt^2, dx^2, dy^2)$) is:

$$\frac{\partial^2 p}{\partial t^2} = \frac{p_{k,l}^{n+1} - 2p_{k,l}^{n} + p_{k,l}^{n-1}}{dt^2} + \mathcal{O}(dt^2). \tag{2}$$

Then, the central-difference approximation of equation (1) is written as:

$$\frac{p_{k,l}^{n+1} - 2p_{k,l}^{n} + p_{k,l}^{n-1}}{dt^2} - v_{k,l}^2 \left(\frac{p_{k+1,l}^{n} - 2p_{k,l}^{n} + p_{k-1,l}^{n}}{dy^2} + \frac{p_{k,l+1}^{n} - 2p_{k,l}^{n} + p_{k,l-1}^{n}}{dx^2} \right) = s_{k,l}^{n}, \tag{3}$$

and its explicit formula can be written as:

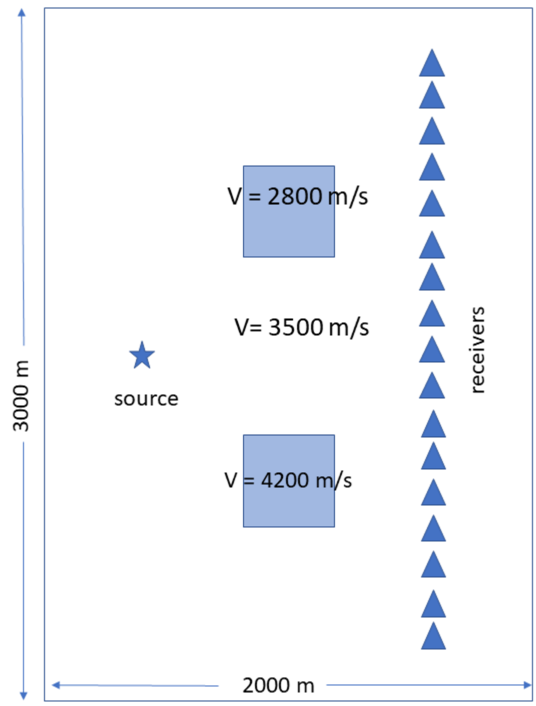
$$p_{k,l}^{n+1} = 2p_{k,l}^{n} - p_{k,l}^{n-1} + dt^2 \left[v_{k,l}^2 \left(\frac{p_{k+1,l}^{n} - 2p_{k,l}^{n} + p_{k-1,l}^{n}}{dy^2} + \frac{p_{k,l+1}^{n} - 2p_{k,l}^{n} + p_{k,l-1}^{n}}{dx^2} \right) + s_{k,l}^{n} \right], \tag{4}$$

We will call the pressure field $p_{k,l}^{n}$ at the whole spatial grid for a fixed time step n as a *snapshot*. The recursive algorithm (4) allows us to compute pressure snapshots at all time steps $1 : N_t$ by using the initial conditions that $p_{k,l}^{-1}$ and $p_{k,l}^{0}$ are zeros.

The matlab script "forward.m" is provided to solve the 2D acoustic-wave equation by using FD method with 2nd-order accuracy in both space and time.

Model 1 (homogeneous half-space model)
Size of the model : 3000 (y) * 2000 (x) m
Source location: (1500,500) m
Receivers' location: (200, 1500), (250, 1500), (300, 1500), ..., (2800, 1500) m
Velocity of the model: 3500 m/s
Density of the model: 2500 kg/m^3
Source time function: 15 Hz Ricker wavelet
Total recording time: 1.000 s

Model 2 (crosshole model)
Every parameter is the same as model 1, then adding two rectangular inclusions:
Inclusions 1: v= 2800 m/s (800 m <= y <= 1200 m, 800 m <= x <= 1200 m),
Inclusions 2: v= 4200 m/s (1800 m <= y <= 2200 m, 800 m <= x <= 1200 m),



Exercise

1. Choose appropriate grid spacing and time step to simulate the synthetic waveform of Model 1. Plot the synthetic waveform as a shot gather and explain the wavetype of (each) seismic event in the shot gather. (5 points)
2. Change the velocity V of Model 1 from 2500 m/s to 4500 m/s with an interval of 50 m/s and simulate the shot gather of each new model. Calculate the l_2-norm difference between the shot gathers of each new model and Model 1 (task 1). Plot the l_2-norm difference as a function of V. (10 points)
3. Simulate the synthetic waveform of Model 2. Plot the *snapshot* of Model 2 at 0.2 s, 0.4 s, 0.6 s, and 0.8 s by using the function *imagesc* in matlab. Plot the synthetic waveform as a shot gather. Plot the difference between the synthetic waveforms of Model 2 and Model 1 (task 1) and explain the wavetype in it. (15 points)

Reports and scripts

Name your report (in pdf format) as "exercise1_YN.pdf" where YN is your name. Send it to thomas.bohlen@kit.edu before the end of **07. Nov. 2021**.