

# Full Waveform Inversion

## Applications to shallow seismic surface waves

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# Outline

1 Introduction

2 Methodology

- FD Simulation
- Elastic FWI
- Geometrical spreading correction
- Attenuation

3 2D Field data applications

- Layered subsurface
- 2-D local trench
- Fault Zone

4 Towards 3D viscoelastic FWI of 3D 9-C field data

5 Summary

# Introduction

Near Surface in this context refers to

- “Critical Zone”
- Up to 20-30 m depth
- Very strong vertical and sometimes lateral variations of visco-elastic material properties (a few 100% per cent)
- Weathering zone – transition zone between earth and atmosphere

Imaging of the “critical zone” is important for

- Geotechnical site characterization, e.g. stability of buildings
- Hazard analysis
  - Detection of cavities
  - Vs30: local site amplification due to surface waves
- Prospecting archeological objects
- etc.

# Introduction

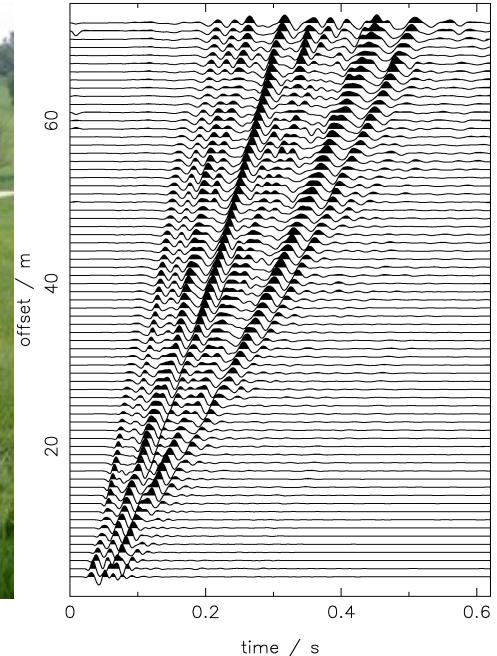
**Shallow seismic surface waves** are

- easily excited
- strong signals
- highly sensitive for Vs (depth)
- penetrating up to 30-40 m depth

useful for geotechnical site characterization

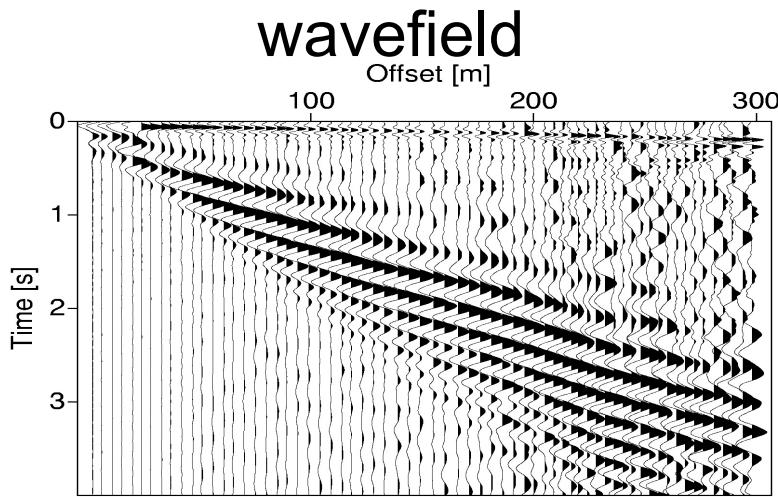


**Field data acquisition**



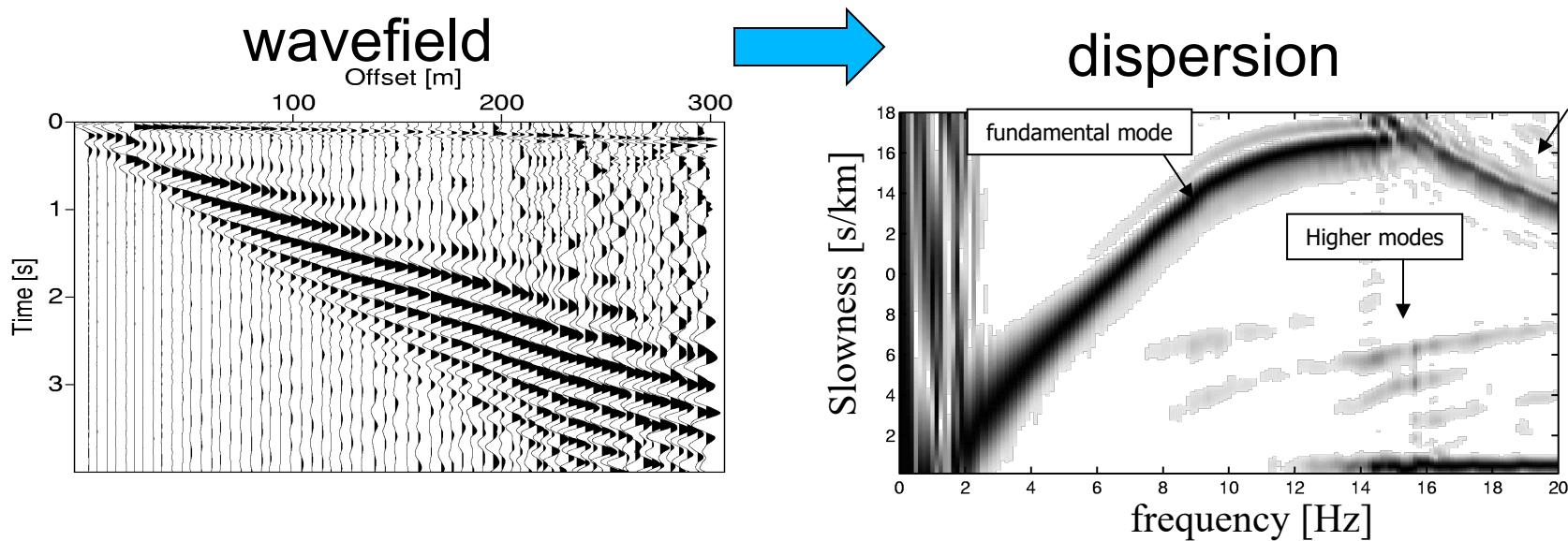
# Introduction

Classical approach: 1-D inversion of (local) dispersion curves



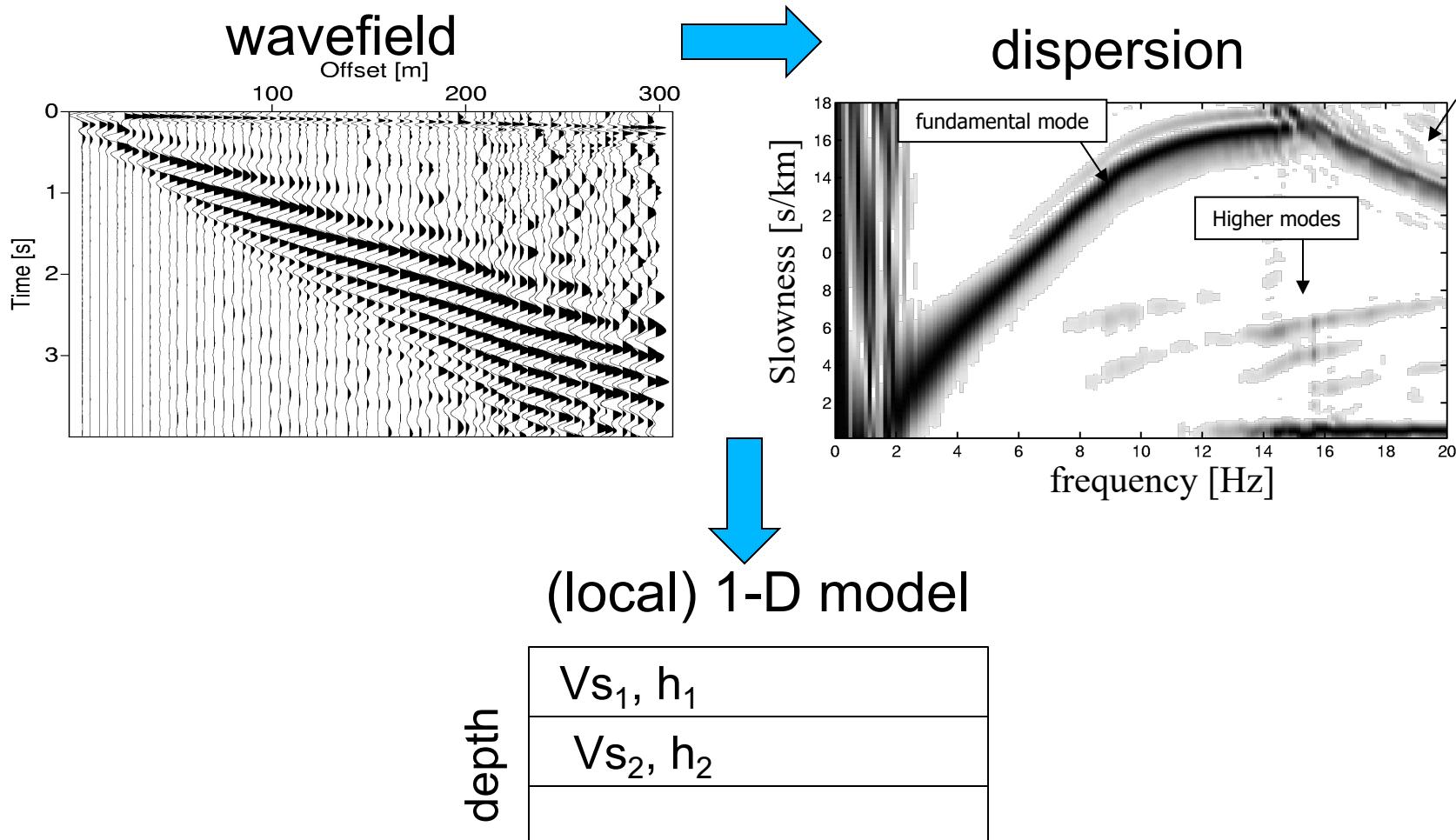
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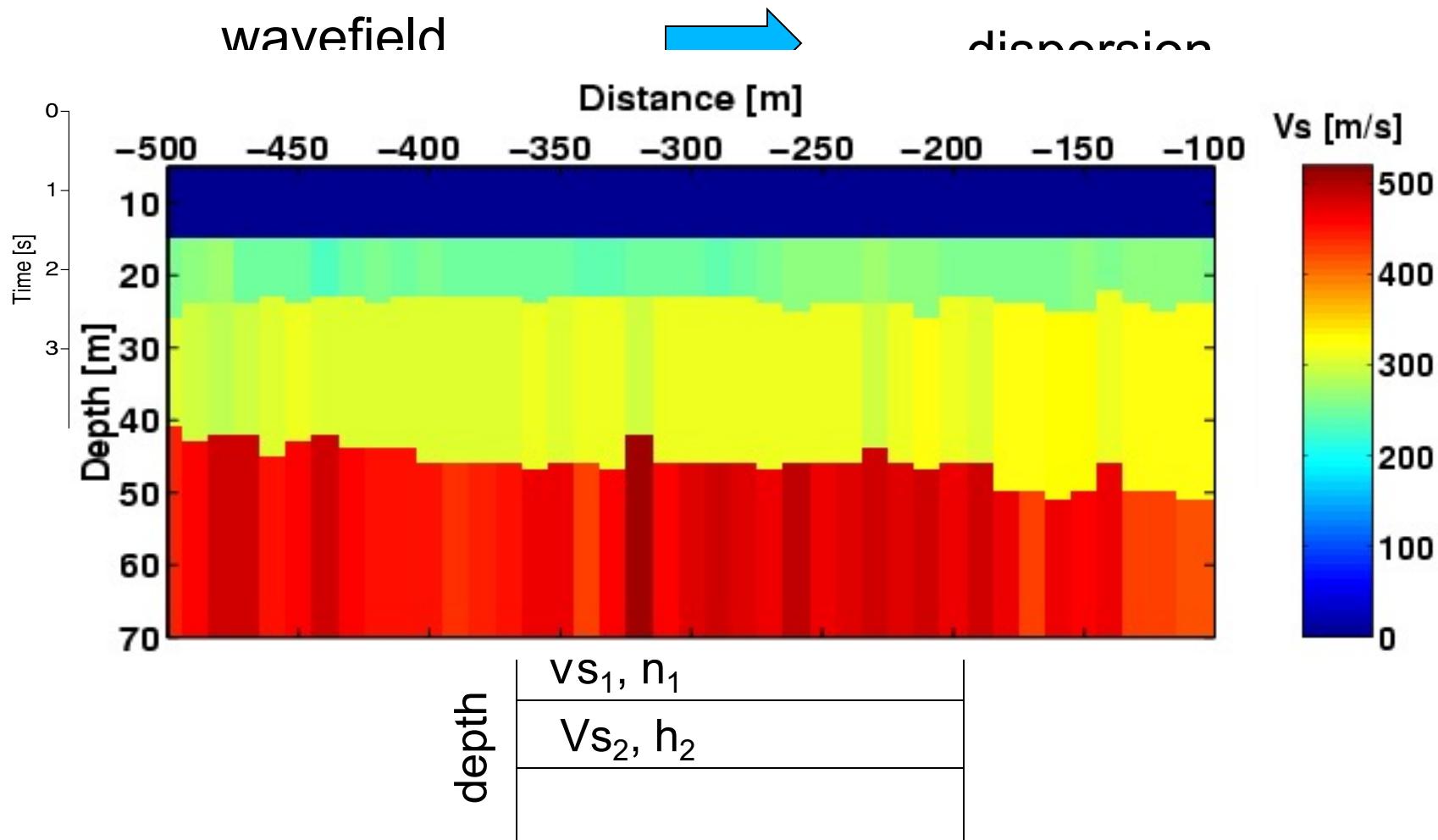
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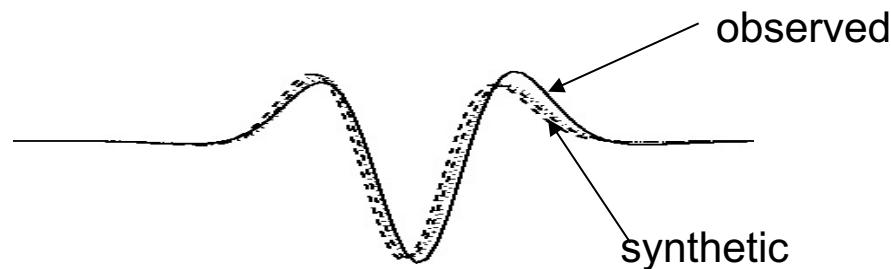
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# Introduction

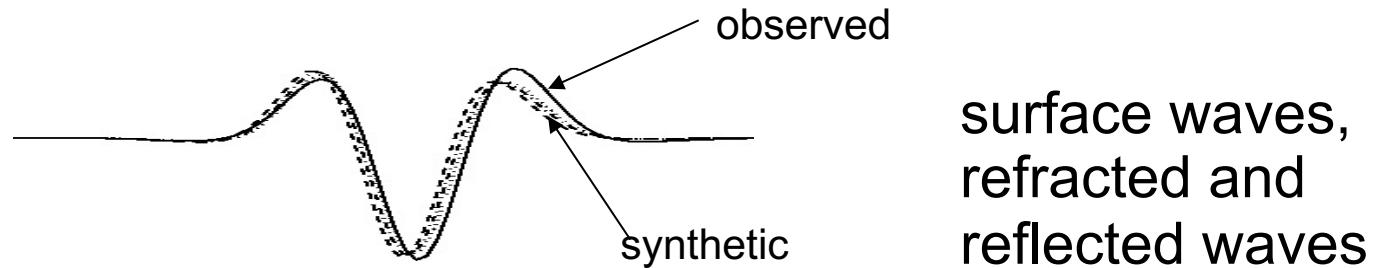
**FWI exploits the full information content of seismic signals !**



surface waves,  
refracted and  
reflected waves

# Introduction

**FWI exploits the full information content of seismic signals !**



**Challenges for 2-D/3-D elastic FWI of surface waves**

- 1. Robust workflow ?**
- 2. Can we infer lateral variations of  $V_s$  ?**
- 3. Can we derive multi-parameter models of  $V_s$ ,  $V_p$ ,  $Q_p$ ,  $Q_s$ , density ?**

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# 2D elastic Full Waveform Inversion (FWI)

- 2D elastic FWI: joint inversion of  $V_p$ ,  $V_s$ , density

# 2D elastic Full Waveform Inversion (FWI)

- 2D elastic FWI: joint inversion of Vp, Vs, density
- L2-Misfit of normalized seismograms

$$E = \frac{\sum_i^{N_s} \sum_j^{N_r} |\hat{s}_{i,j} - \hat{d}_{i,j}|^2}{N_s N_r}$$

$\hat{d}_{i,j} = \mathbf{d}_{i,j} / |\mathbf{d}_{i,j}|$  : normalized observed seismograms  
 $\hat{s}_{i,j} = \mathbf{s}_{i,j} / |\mathbf{s}_{i,j}|$  : normalized synthetic seismograms

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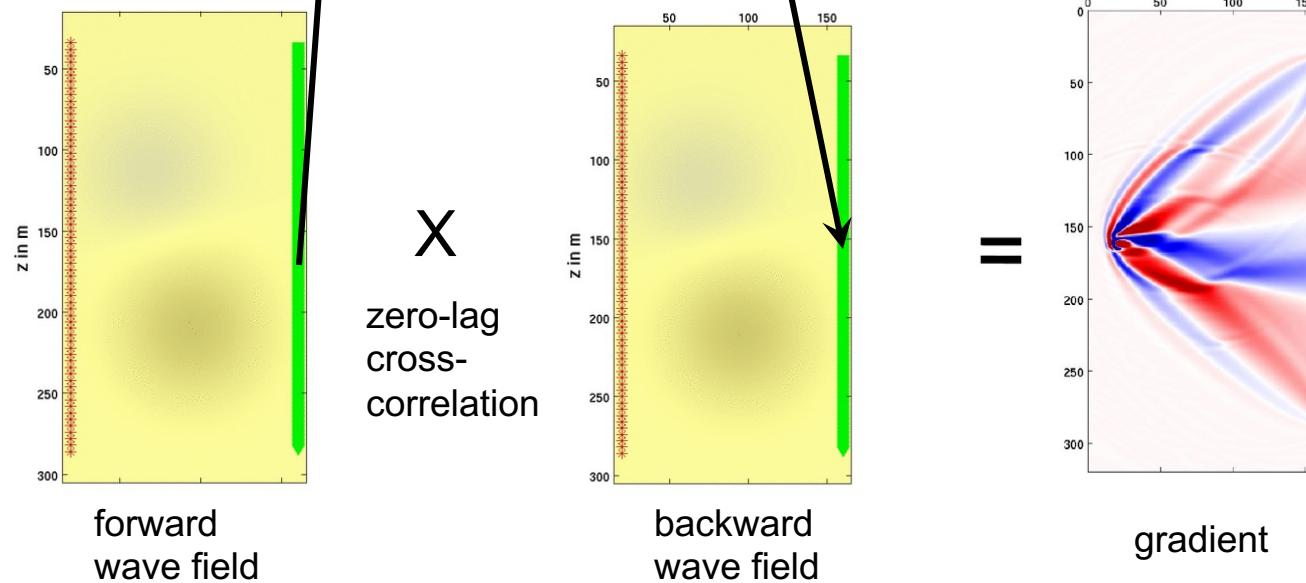
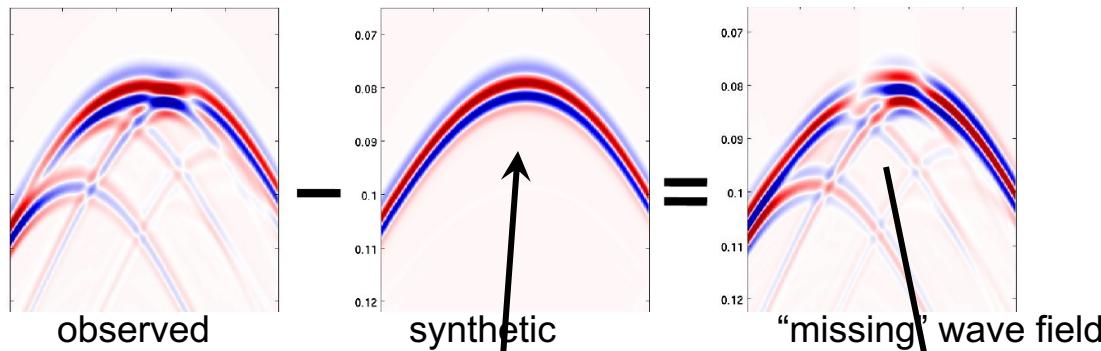
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- Conjugate gradient method or L-BFGS method for model update
- Gradient calculation with the adjoint state method
- 2D viscoelastic Finite Difference time domain modeling
- Implementation of viscoelastic damping by a generalized Standard Linear Solid (GSLS)

# Adjoint State Method



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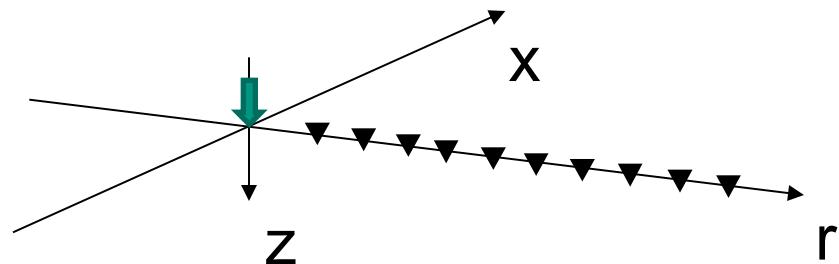
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# Geometrical spreading in homogenous acoustic medium

Point source (field data)

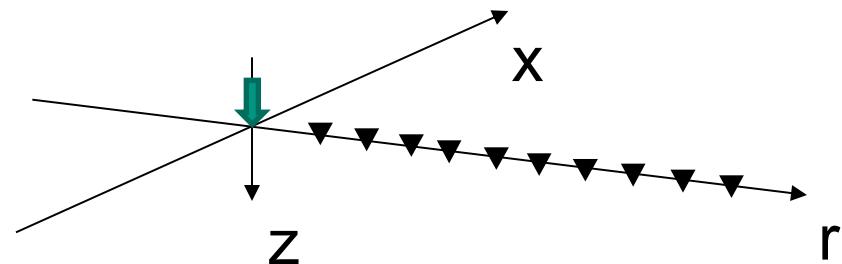
$$G^{3D} = \frac{1}{r} e^{ikr}$$



# Geometrical spreading in homogenous acoustic medium

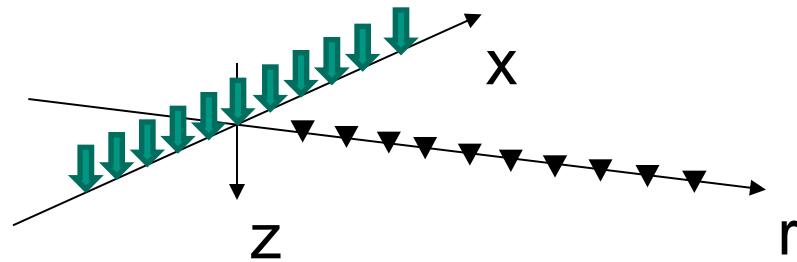
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Line source (2D FWI)

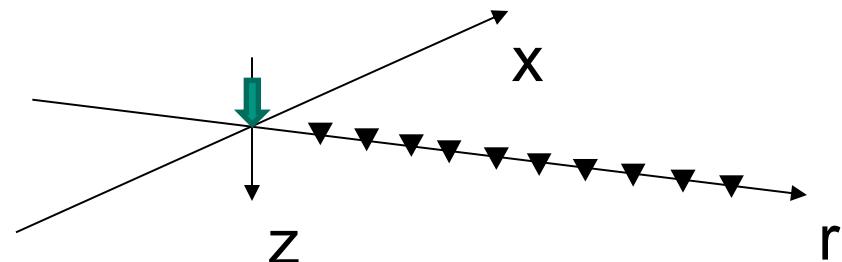
$$G^{2D} \simeq \sqrt{\frac{2\pi}{kr}} e^{ikr} e^{i\pi/4}$$



# Geometrical spreading in homogenous acoustic medium

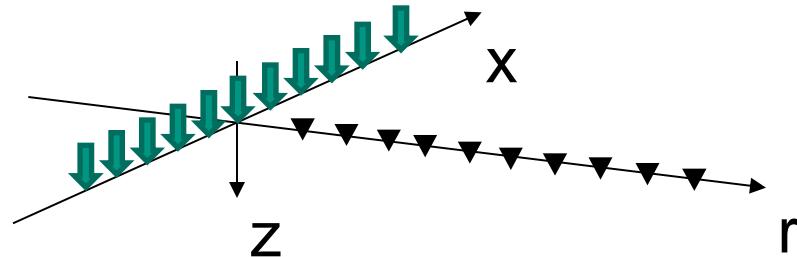
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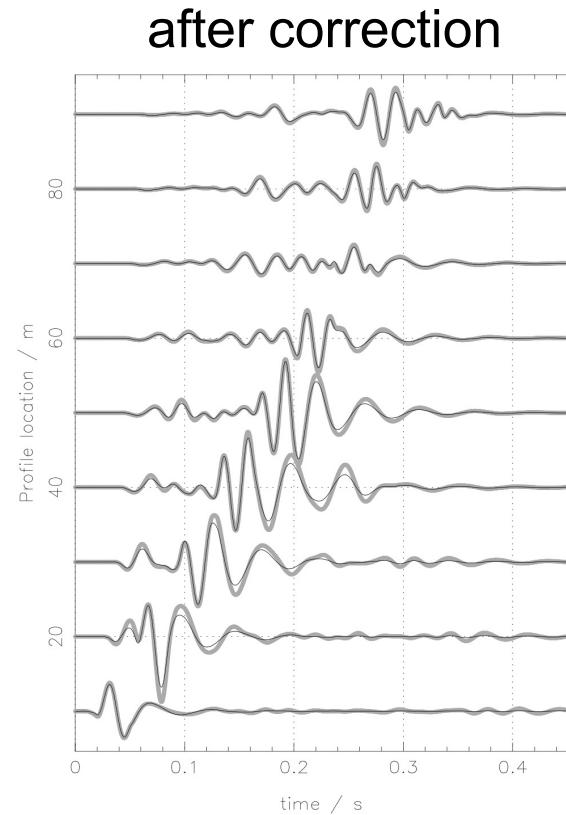
Correction

(Forbriger et al., 2014)

$$\frac{G^{2D}}{G^{3D}} \simeq \sqrt{\frac{2\pi r}{k}} e^{i\pi/4} \quad k = \frac{\omega}{c}$$
$$F_{amp} = r \sqrt{2} \sqrt{t^{-1}}$$

$\underbrace{\sqrt{2rc} \sqrt{\frac{\pi}{\omega}} e^{i\pi/4}}_{F_{amp} \text{ near field}} \quad \tilde{F} \left( \sqrt{t^{-1}} \right)$

# Synthetic Test of 2D/3D-Transformation



- line source seismograms
- corrected point source seismograms

- works surprisingly well for shallow seismic wave fields
- single-trace transformation
- applicable also in case of lateral heterogeneity

(Schäfer et al., 2014)

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- **Attenuation**

3 2D Field data applications

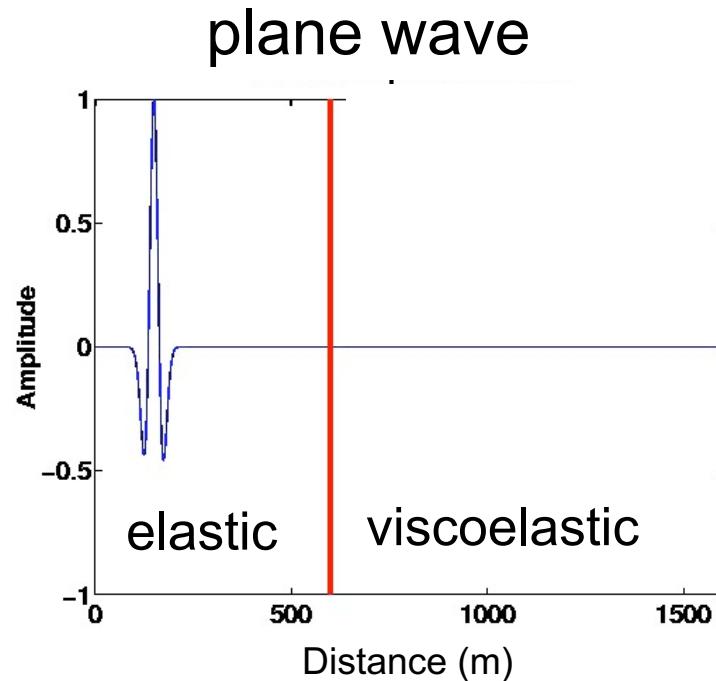
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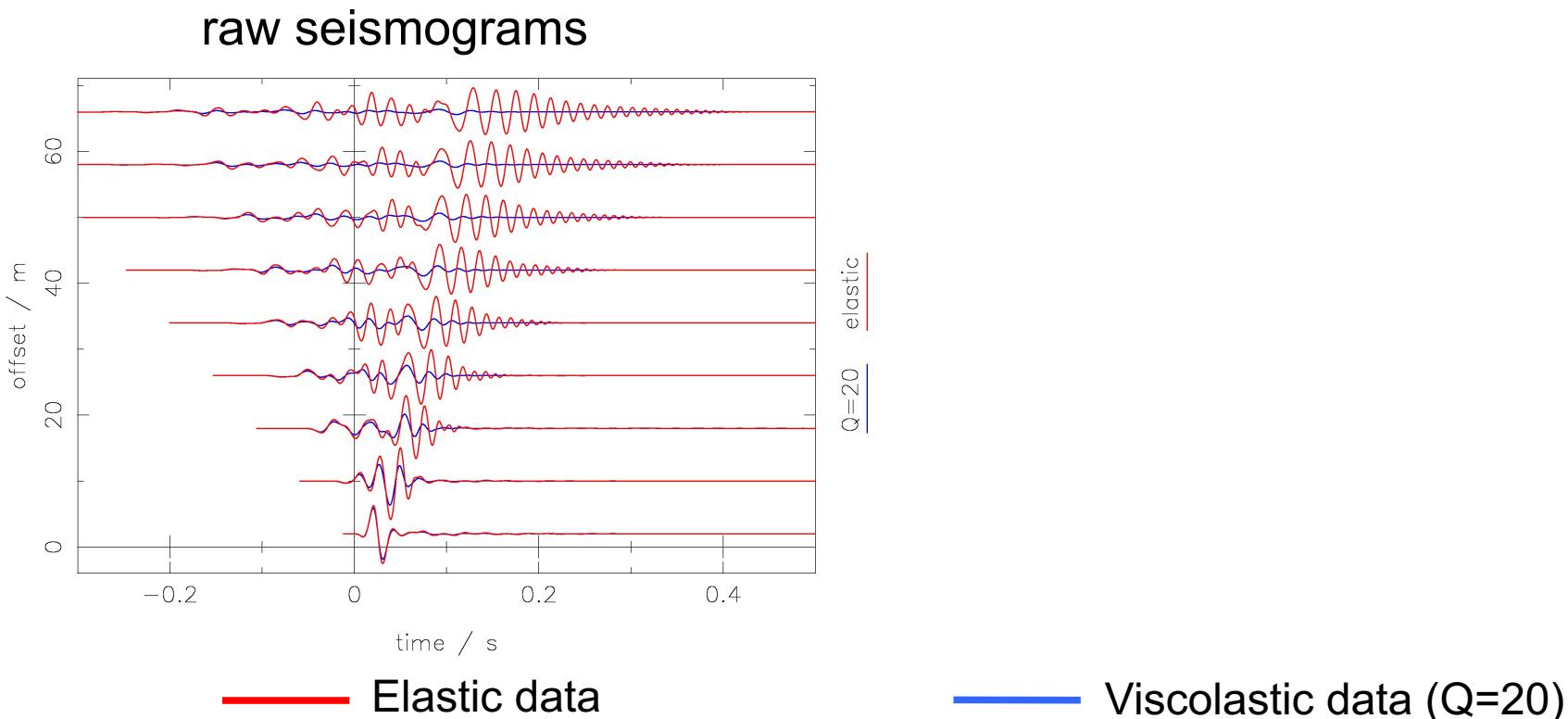
# Effects of Attenuation

1. Amplitude decay with distance
2. Loss of high frequencies with distance
3. Dispersion



# Challenge II: Attenuation

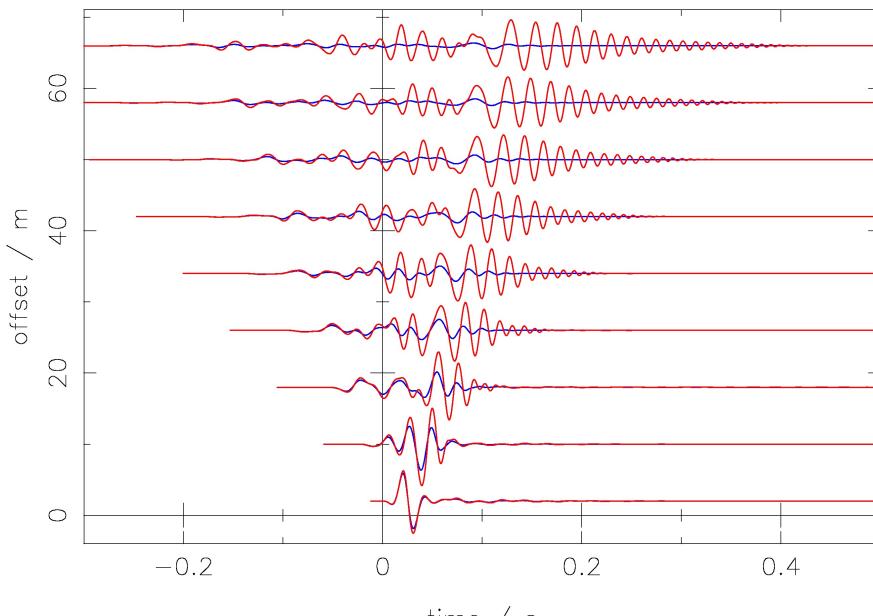
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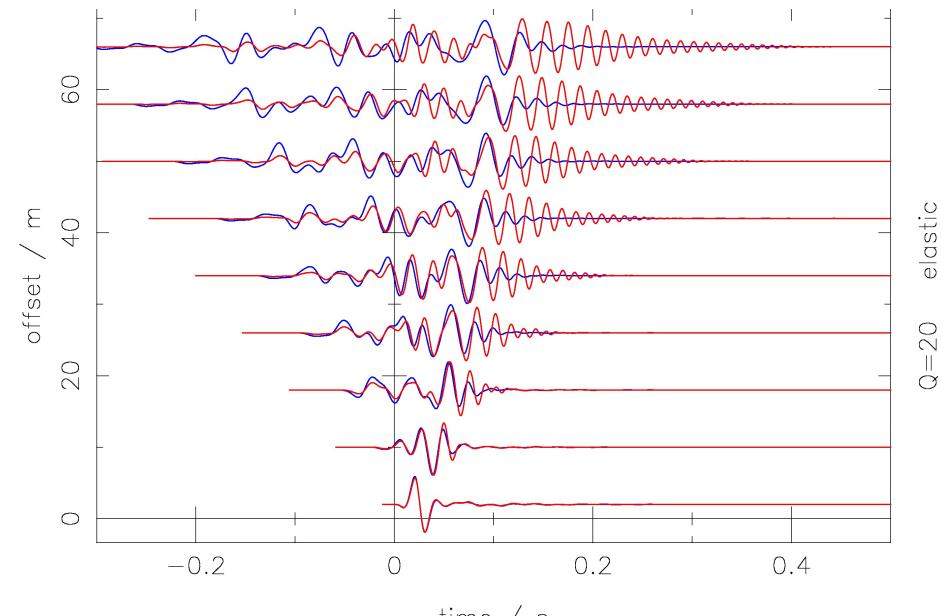
1. Amplitude decay with distance → misfit definition
2. Loss of high frequencies with distance
3. Dispersion

raw seismograms



Elastic data

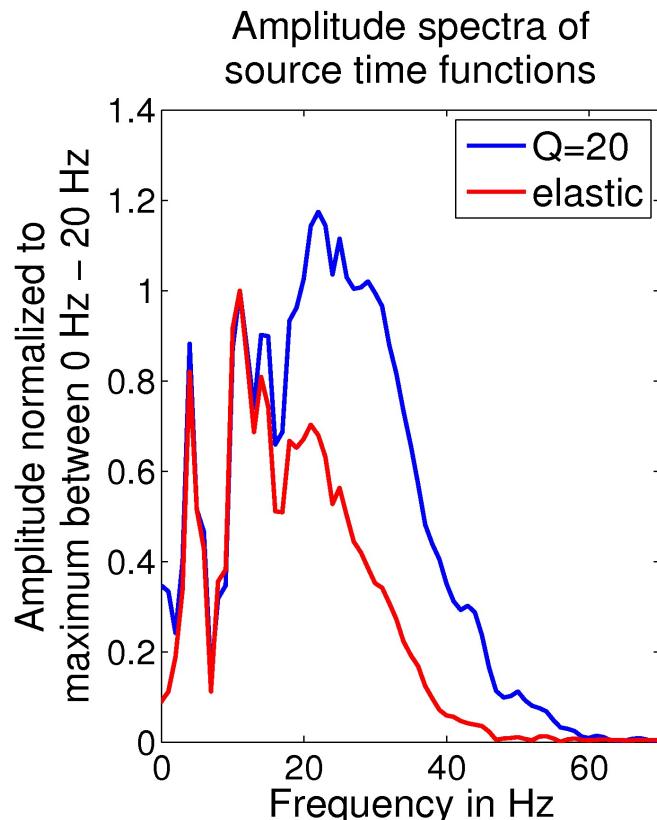
Misfit definition using normalized seismograms



Viscoelastic data ( $Q=20$ )

# Challenge II: Attenuation

1. Amplitude decay with distance
2. **Loss of high frequencies with distance → source signal inversion**



Stabilized deconvolution in the frequency domain

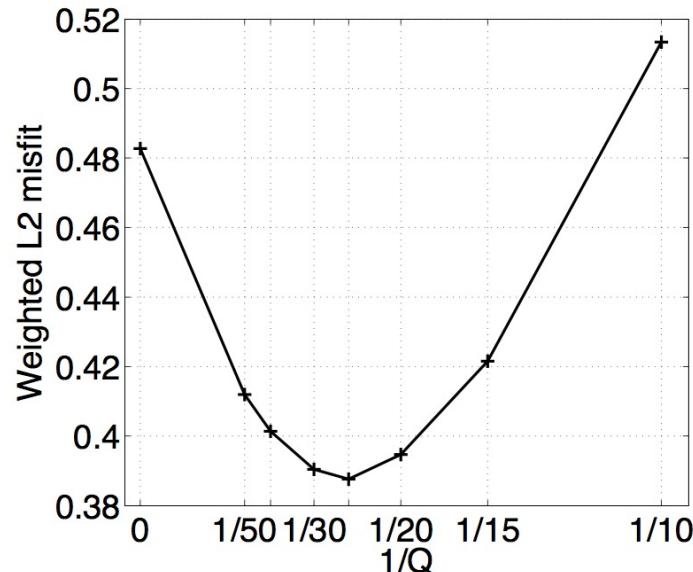
The source wavelet correction filter acts as a low-pass-filter

(Groos et al., 2014)

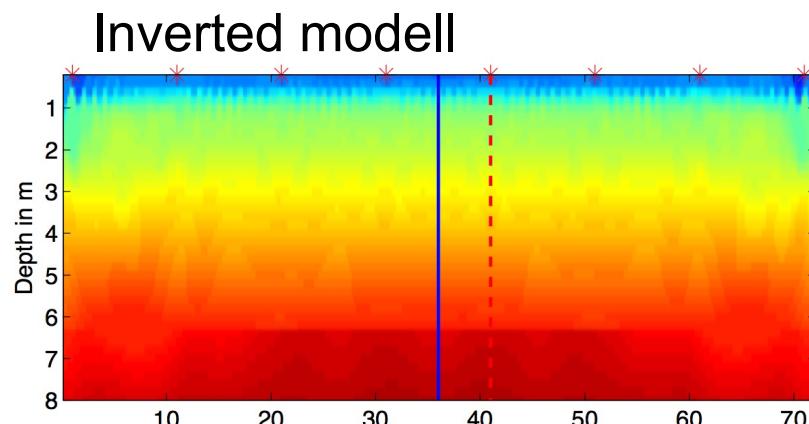
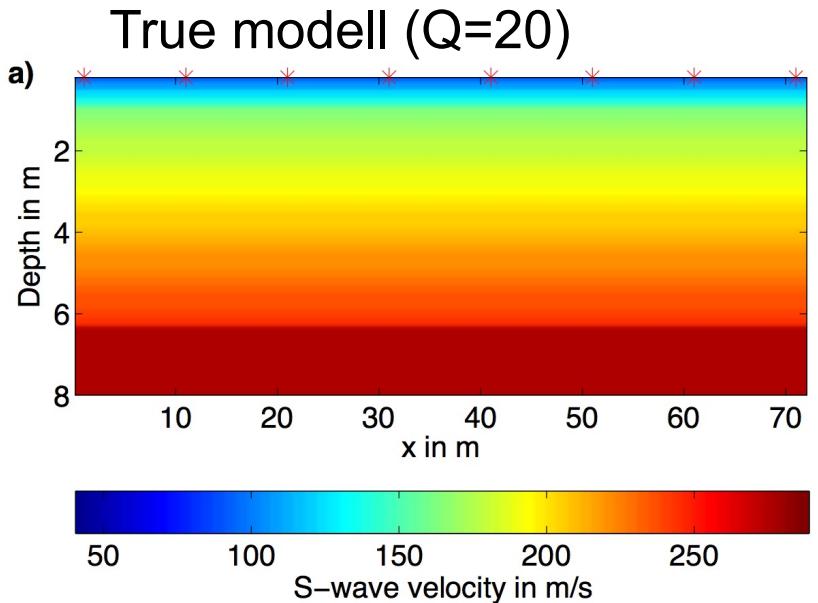
# Challenge II: Attenuation

1. Amplitude decay with distance
  2. **Loss of high frequencies with distance**
  3. **Dispersion**
- } Viscoelastic forward modelling

A priori Q-estimation



# Synthetic reconstruction tests are successful



## Pre-processing

1. Geometrical spreading correction by single-trace transformation
1. Q-estimation

## During the elastic FWI

1. Use L2-misfit of normalized seismograms
2. Source wavelet inversion
3. Viscoelastic forward modelling

(Groos et al., 2014)

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3 **2D Field data applications**

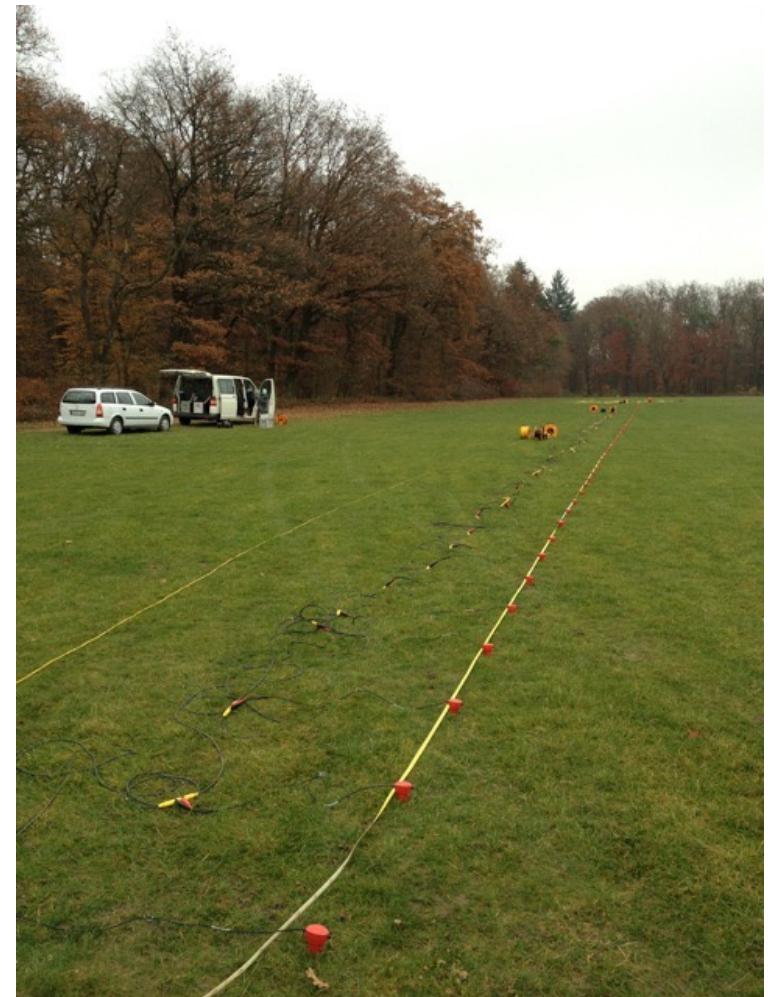
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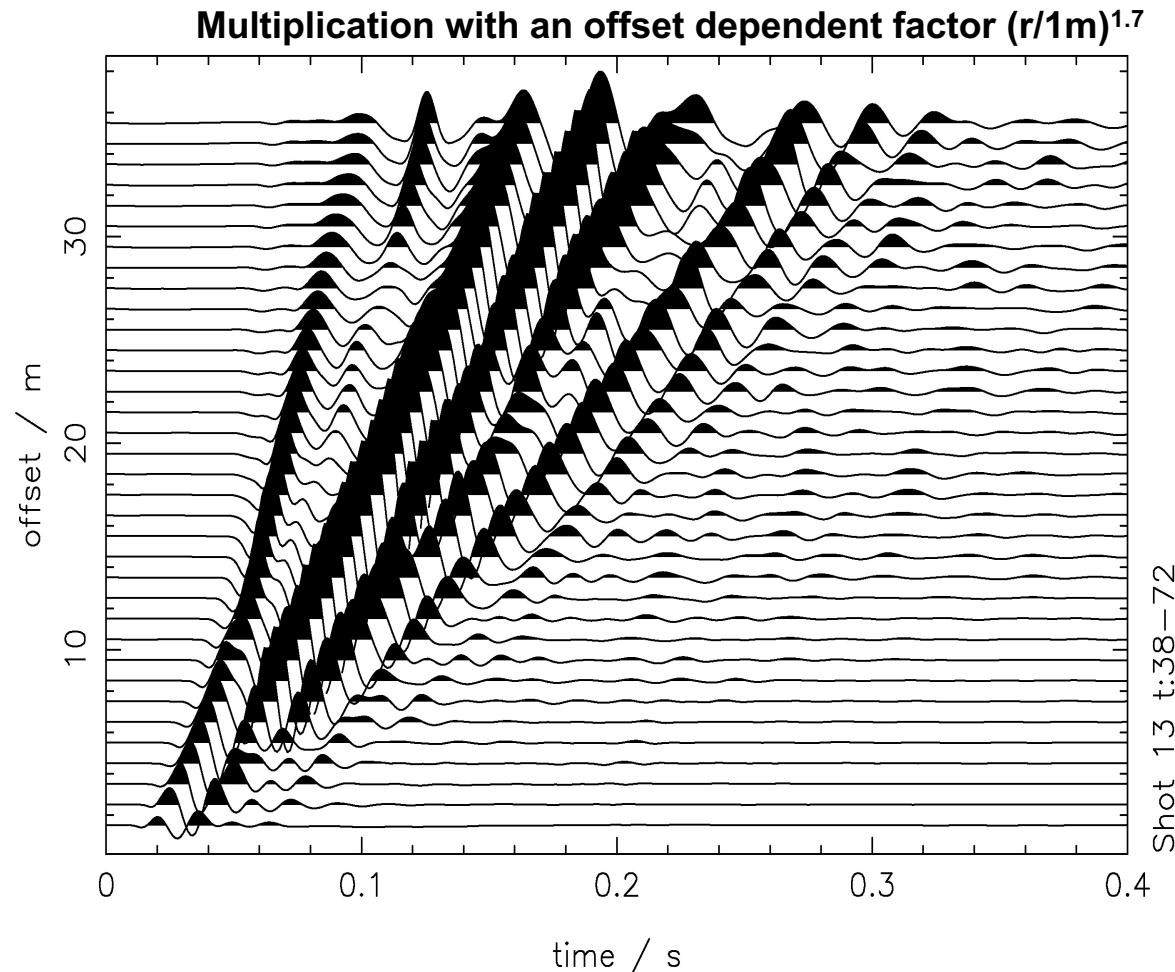
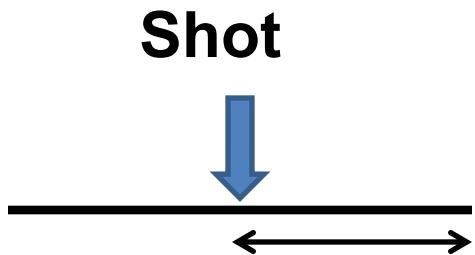
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# Field data: 1-D case

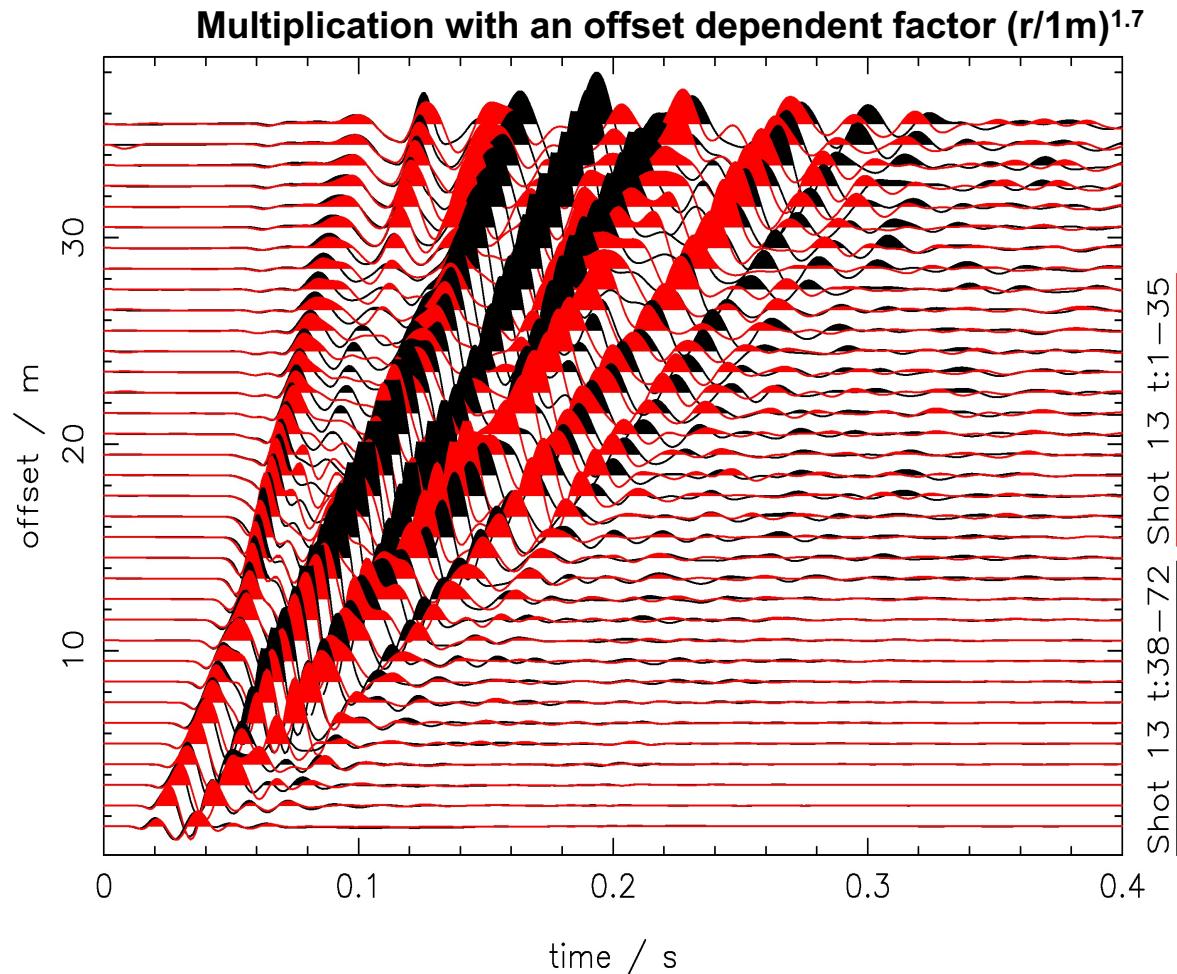
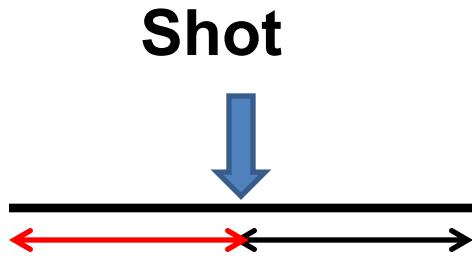
- Glider airfield near Karlsruhe (Germany)
- Acquisition
  - linear profile
  - vertical geophones every 1m
  - hammer blows every 2m



# 1-D case: Field data

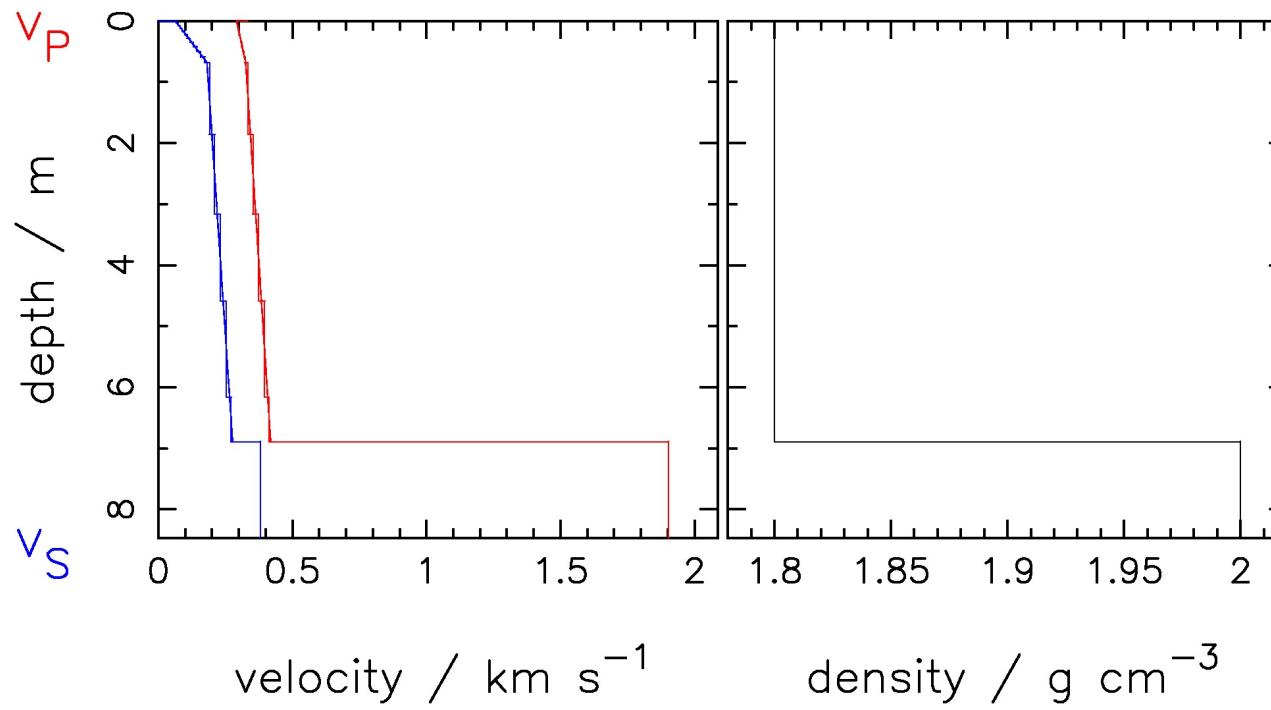


# 1-D case: Field data



# Initial 1D-model

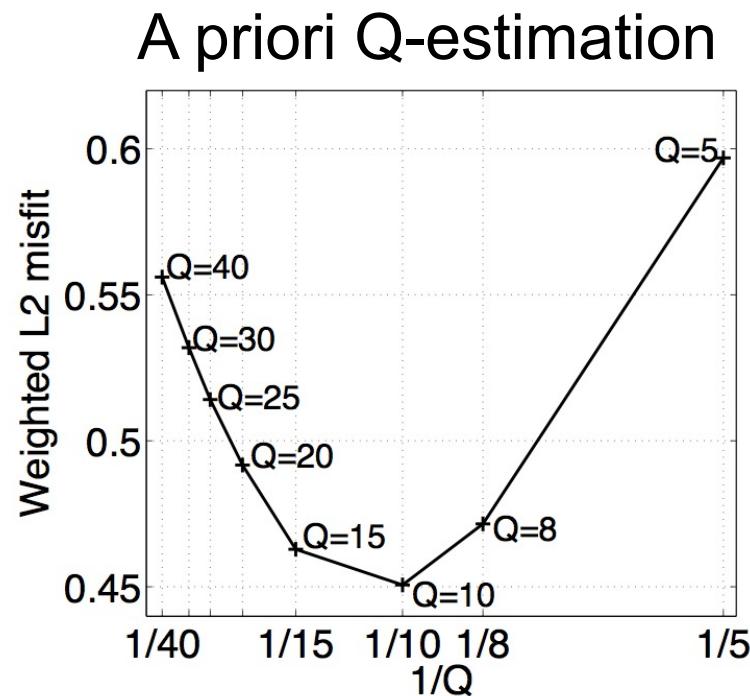
- derived by an inversion of **first arrival travel times ( $v_p$ )** and **Fourier-Bessel expansion coefficients ( $v_s$ )**
  - fluvial sediments (gravel and sand)
  - ground water table in 6.9 m depth



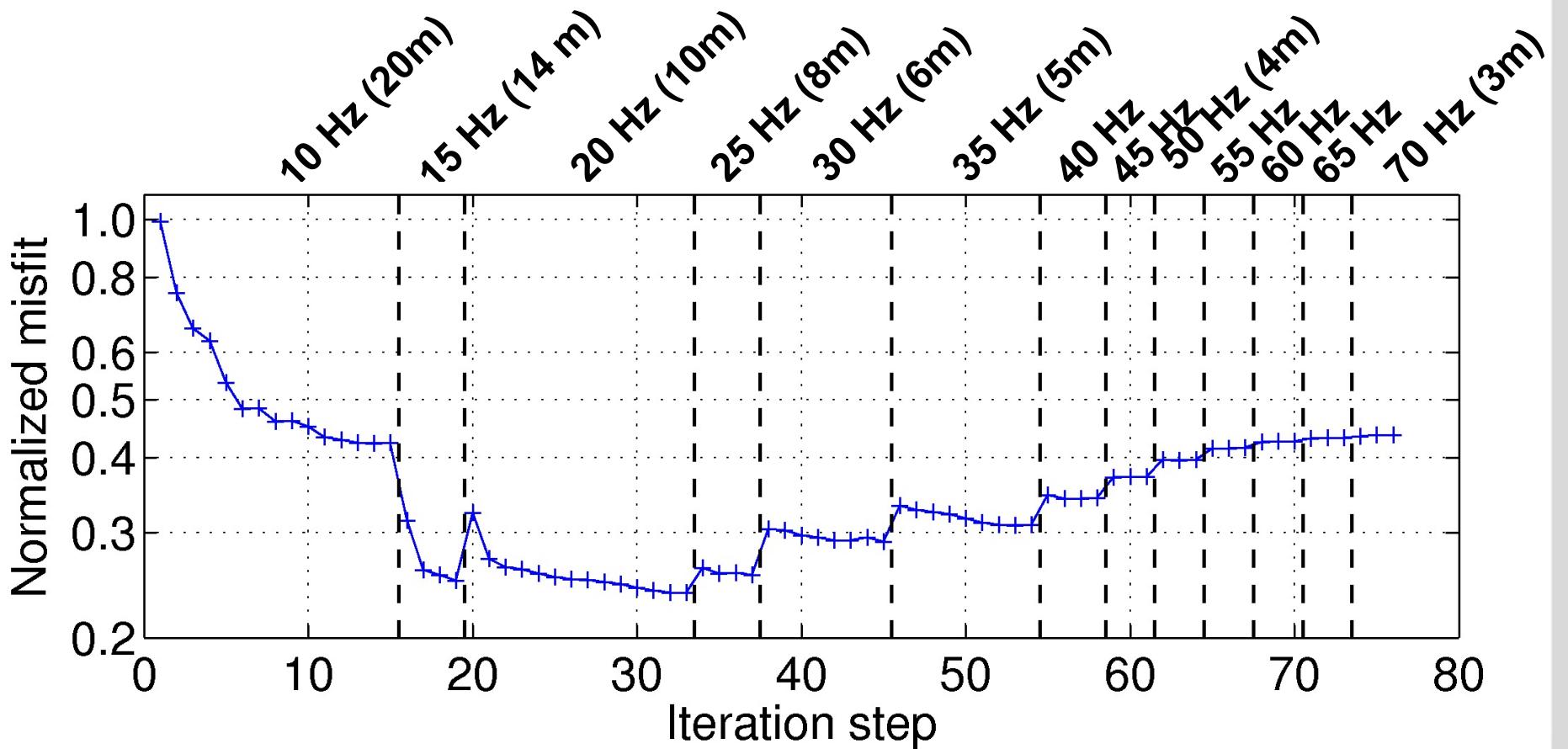
(Groos, 2013; Groos et al., 2017)

# 1-D case: Preprocessing

1. 2D/3D geometrical spreading correction
2. Estimation of quality factor:  $Q_s=Q_p=15$



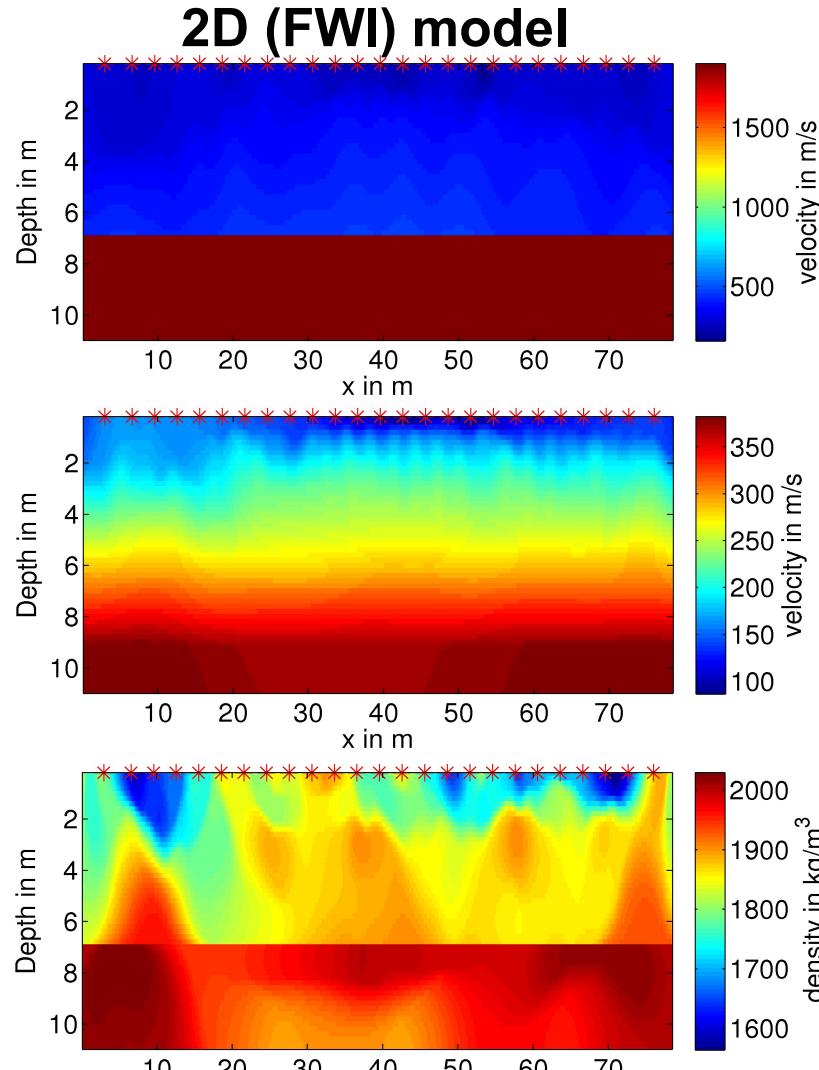
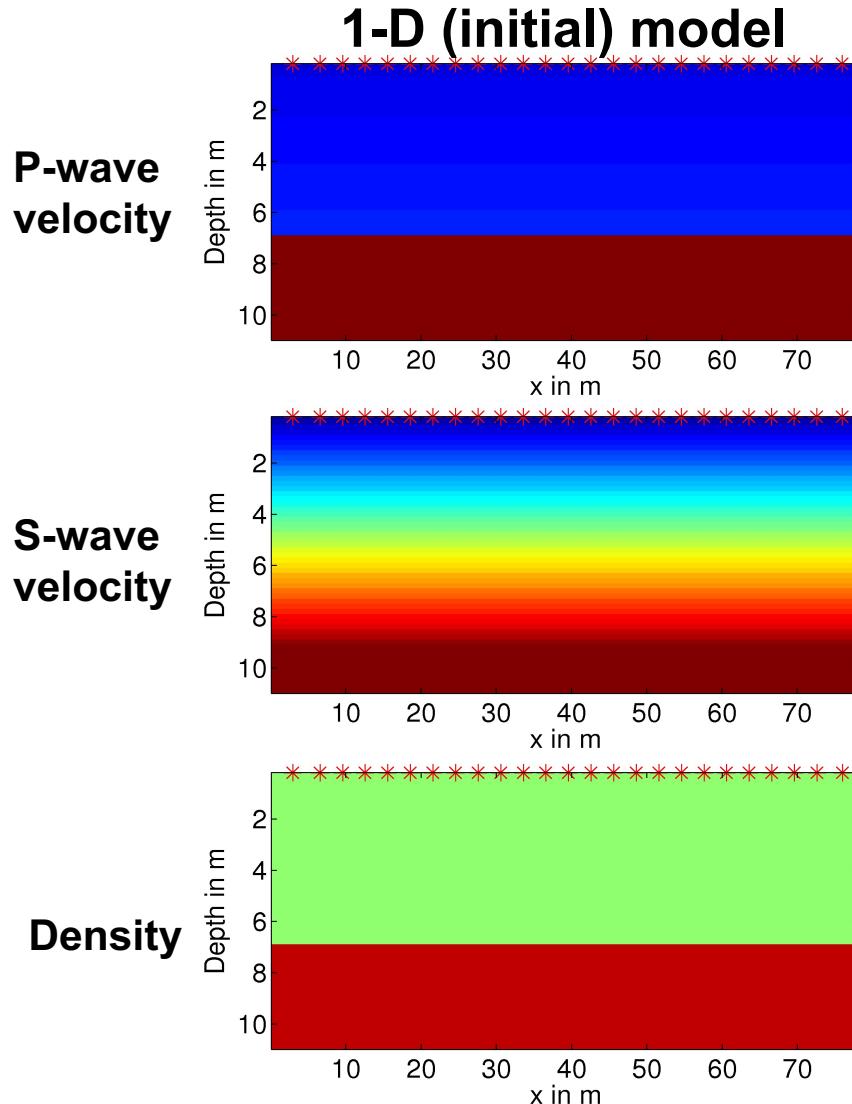
# 1-D case: Evolution of data misfit



Total computation time: 9 h on 16 CPUs

(Groos, 2013)

# 1-D case: inversion result



(Groos et al., 2017)

# 1-D case: inversion result

1-D (initial) model

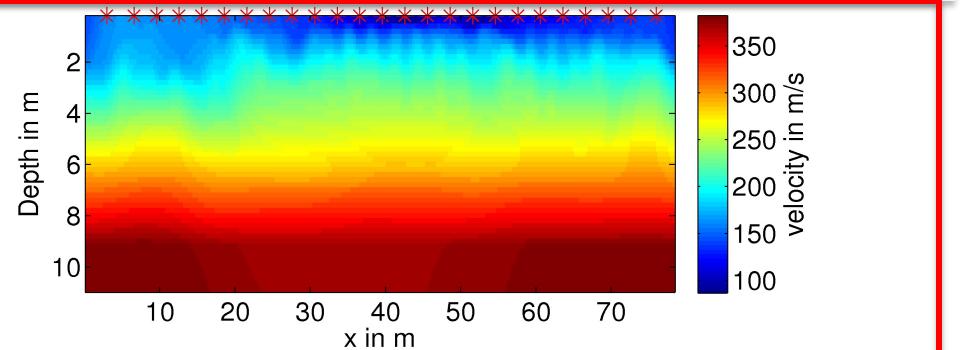
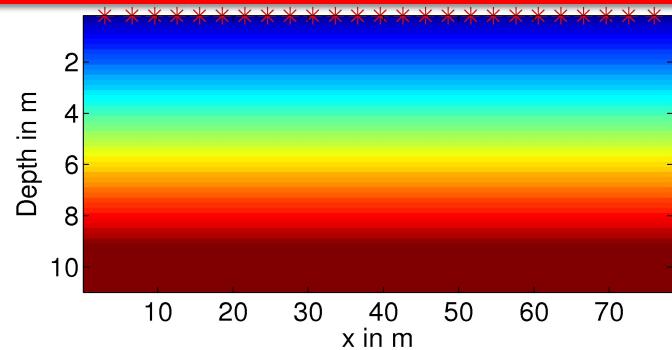
P-wave  
velocity



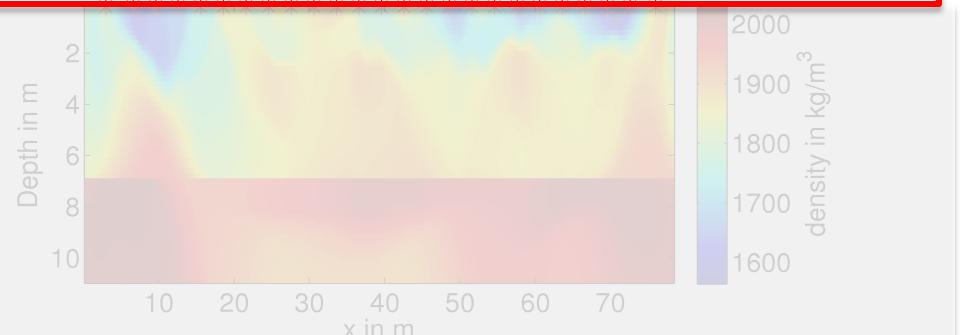
2D (FWI) model



S-wave  
velocity

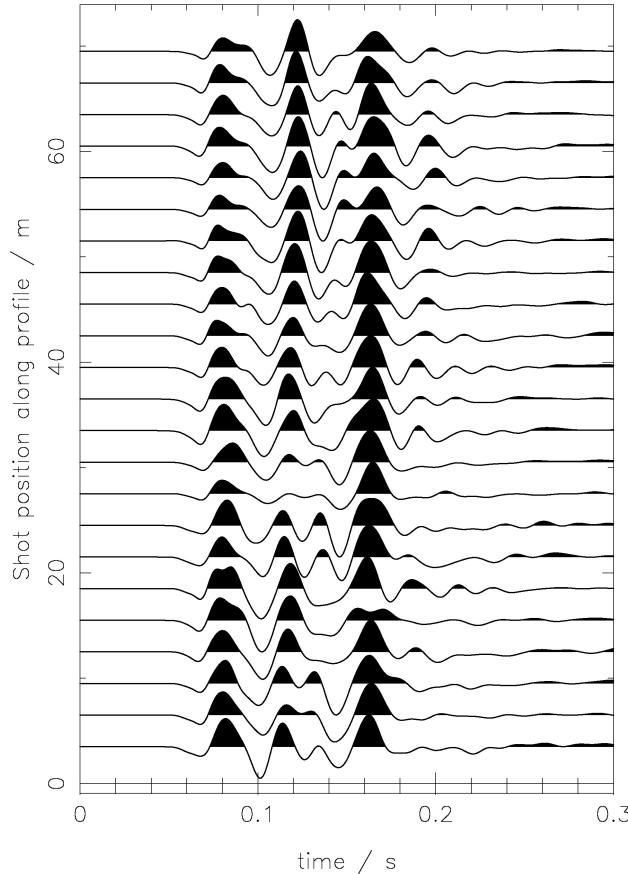


Density



# 1-D case: common offset gather (20.5 m)

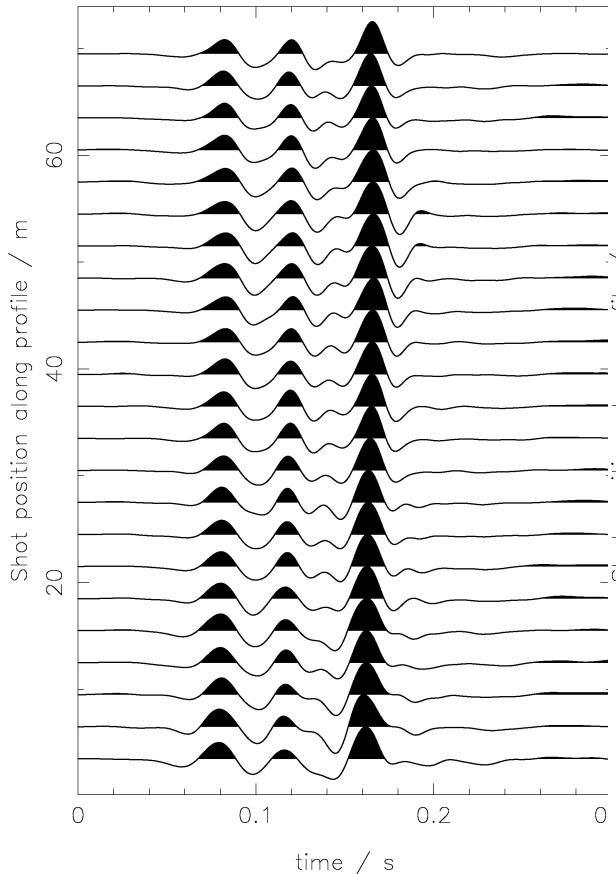
Field data



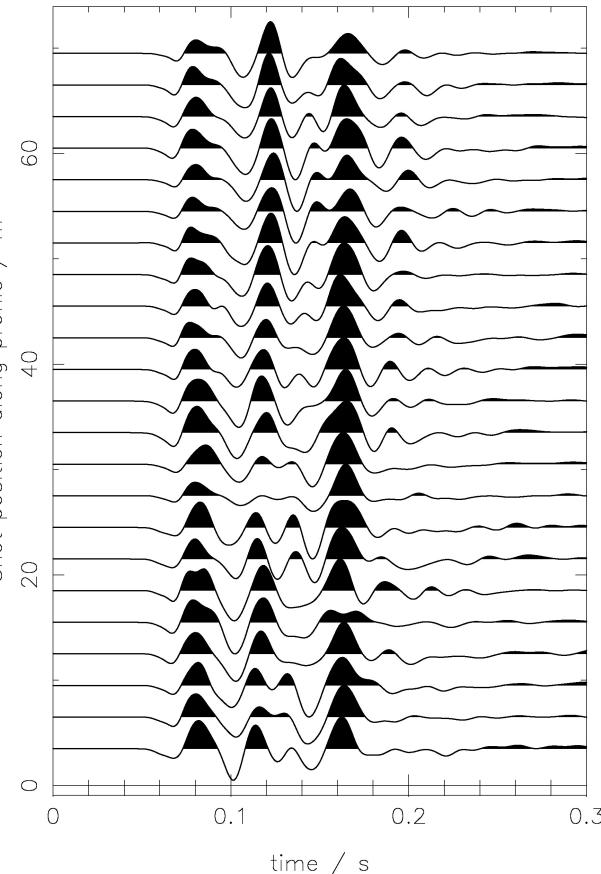
(Groos et al., 2017)

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1D (initial) model



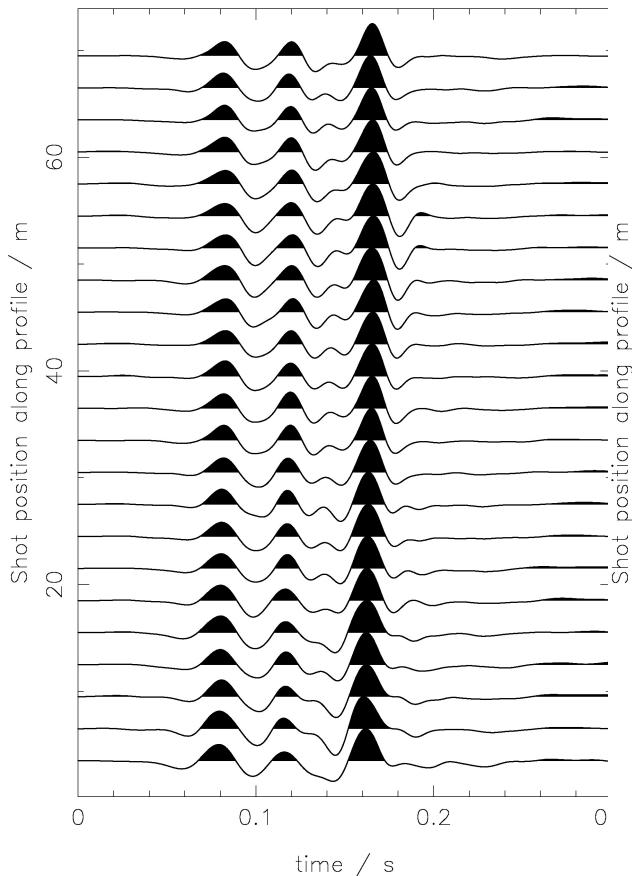
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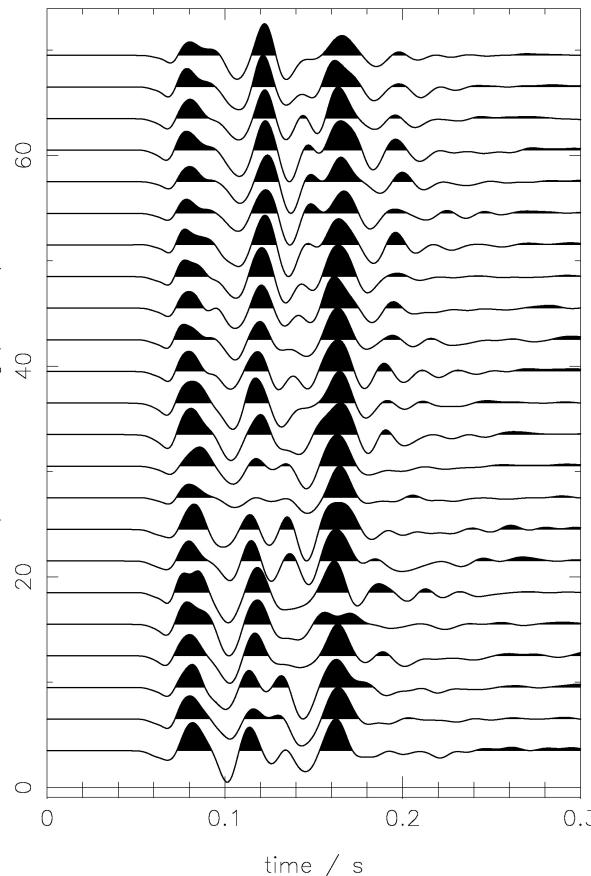
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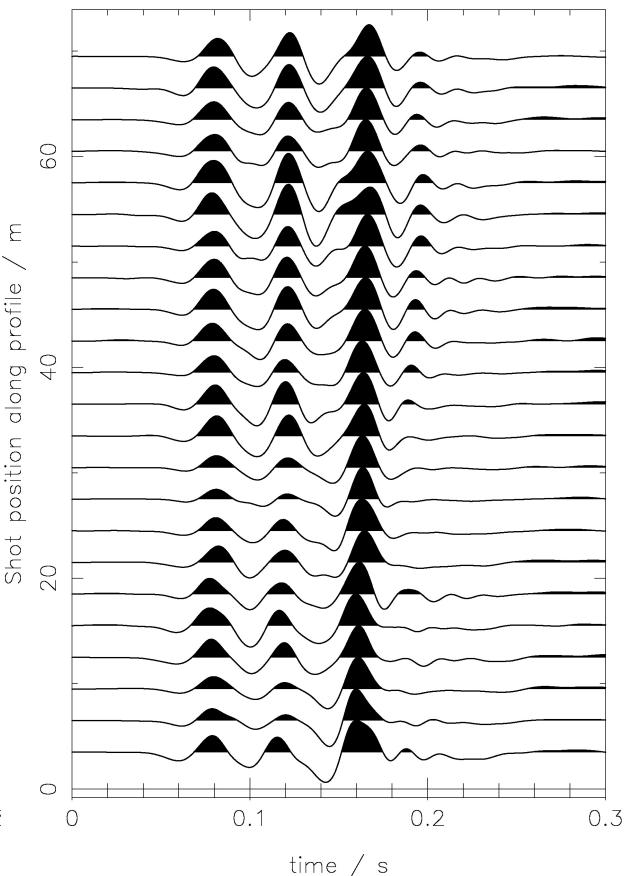
1D (initial) model



Field data



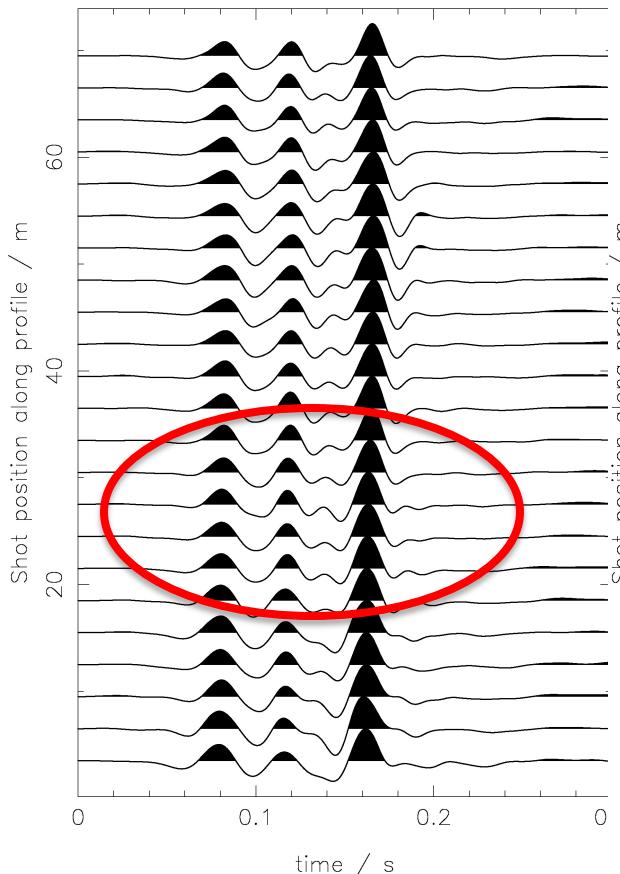
2D (FWI) model



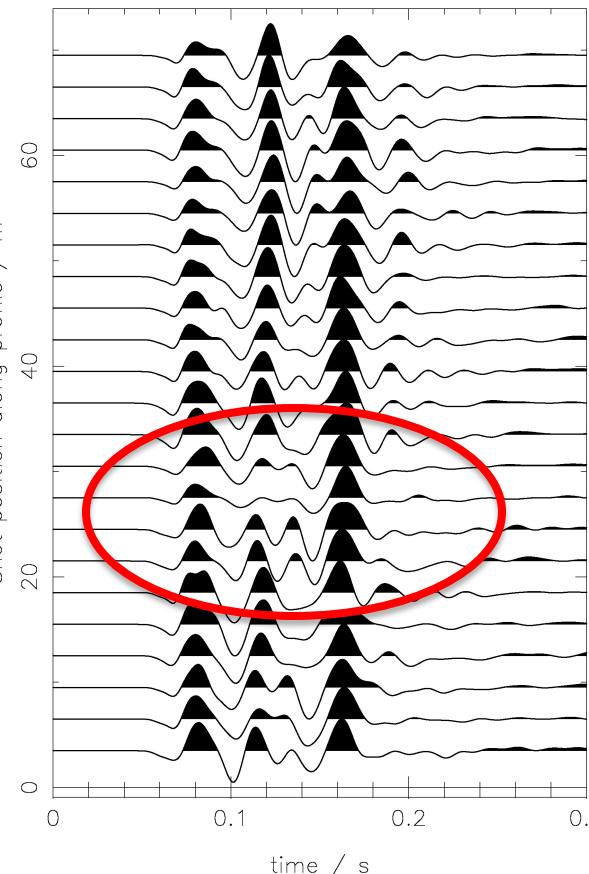
(Groos et al., 2017)

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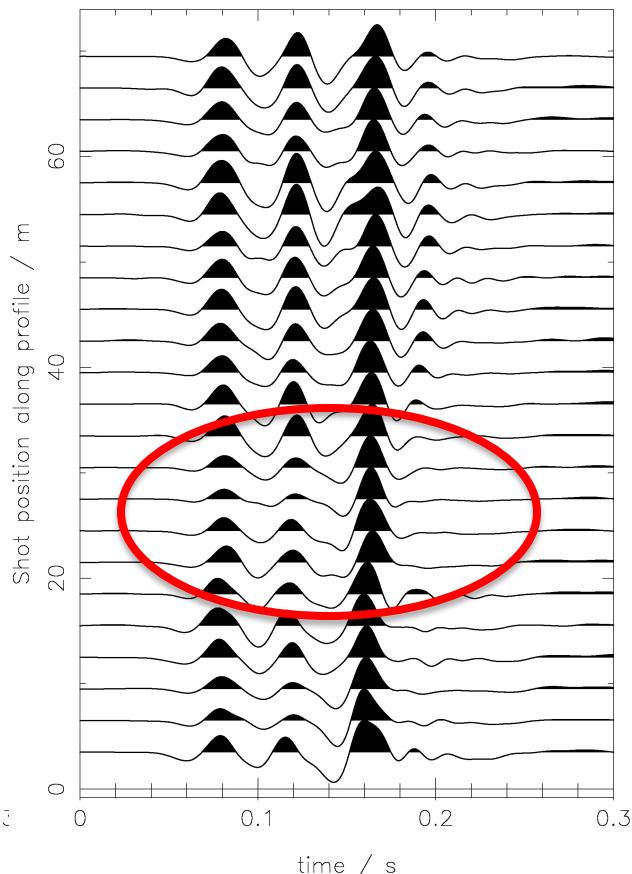
1D (initial) model



Field data



2D (FWI) model



2-D FWI model explains lateral variations

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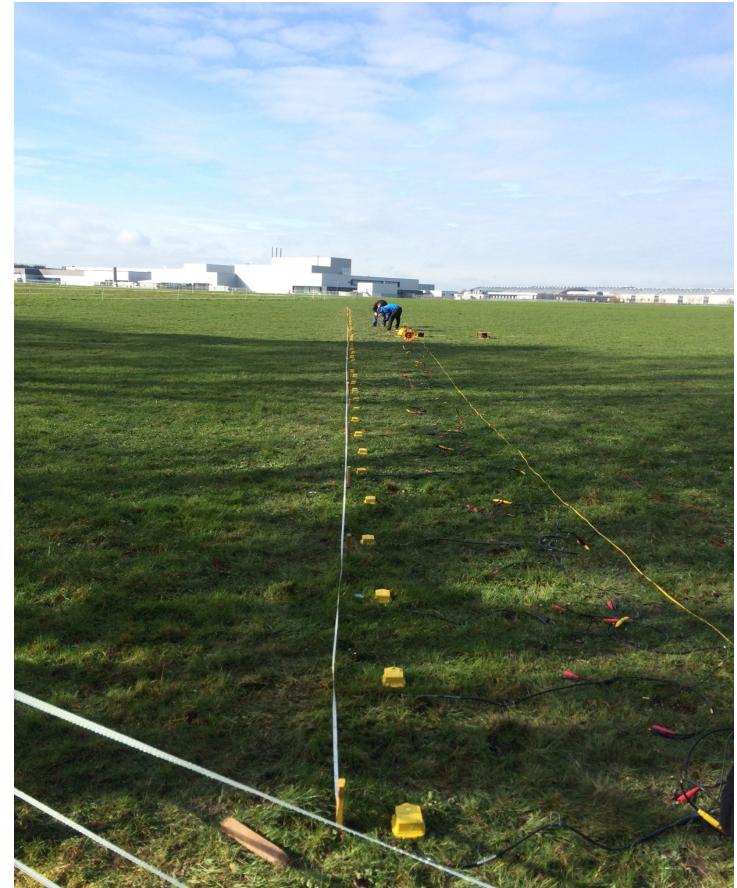
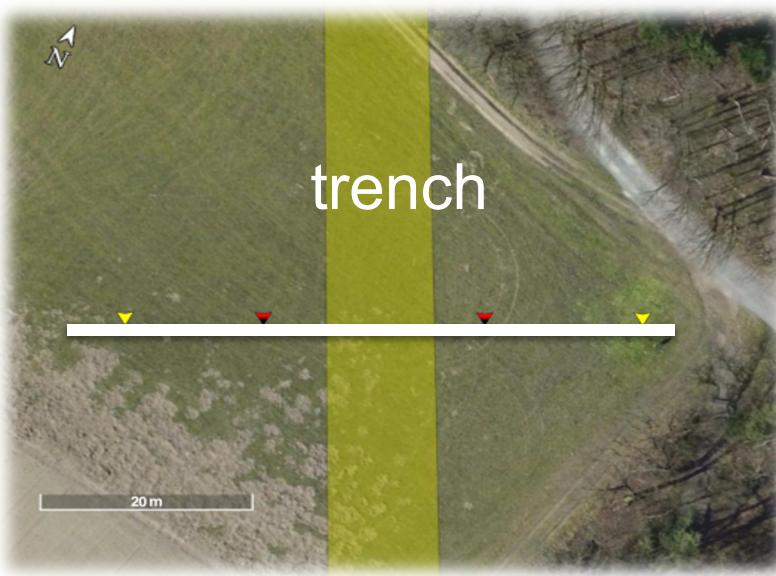
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## 2. Example: Small-scale trench

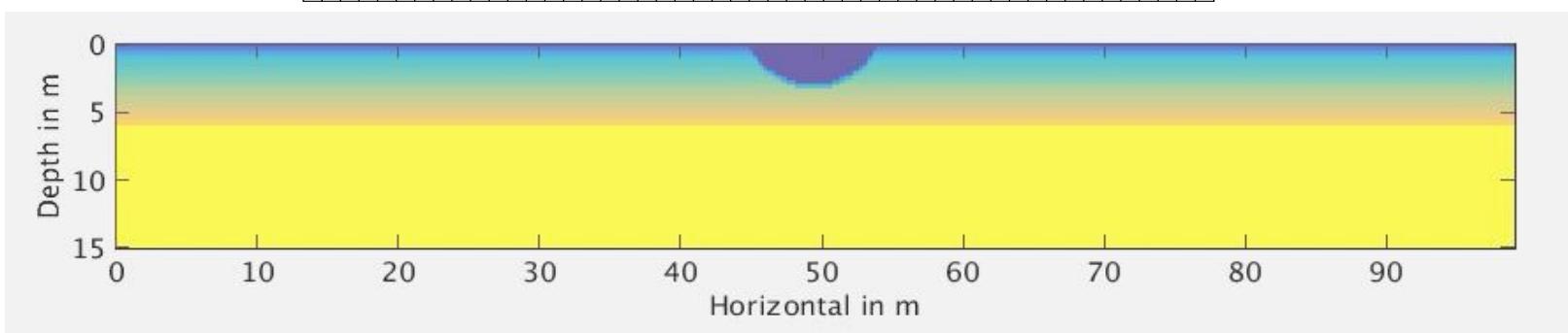
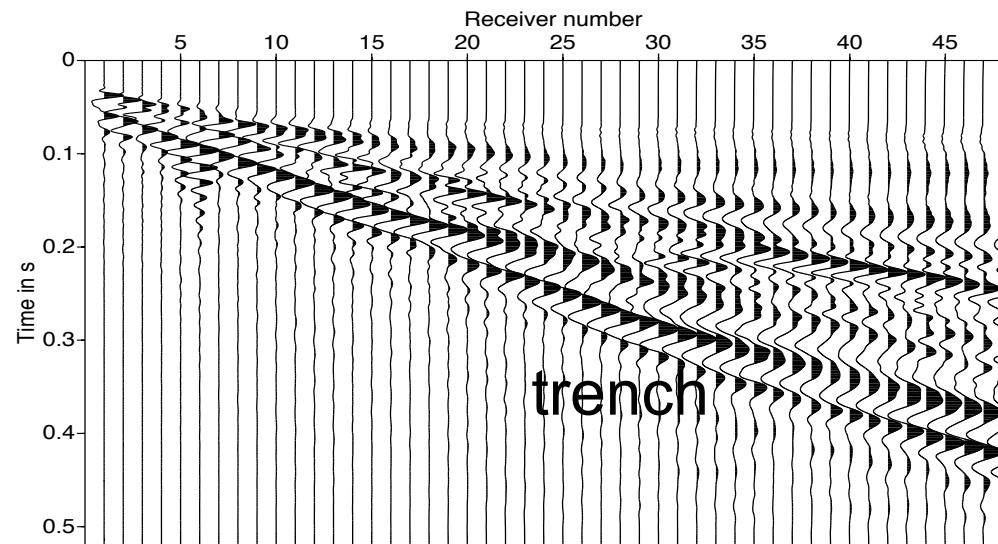
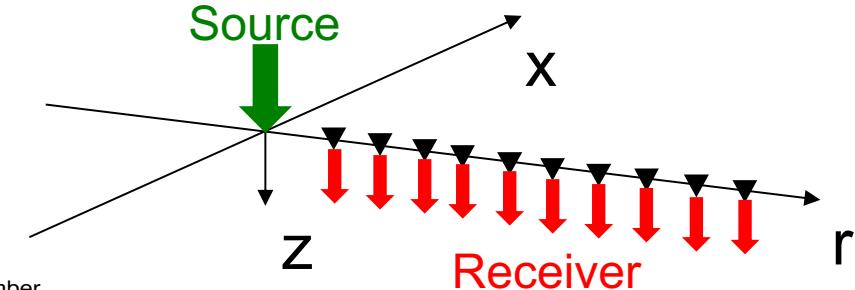
- Profile crosses known trench  
„Ettlinger Linie“ excavated in  
the 18<sup>th</sup> century



(Wittkamp & Bohlen, 2016)

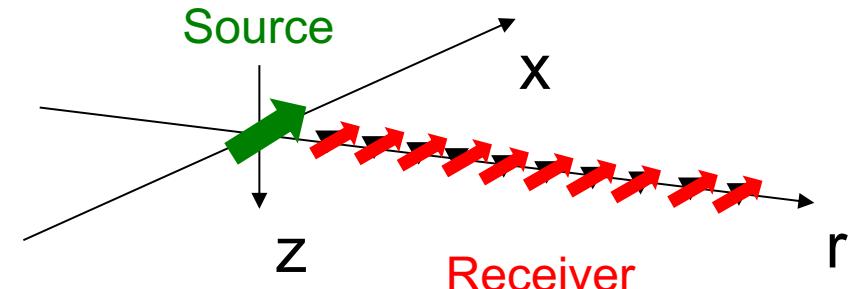
# Rayleigh waves

Rheinstetten  
field data

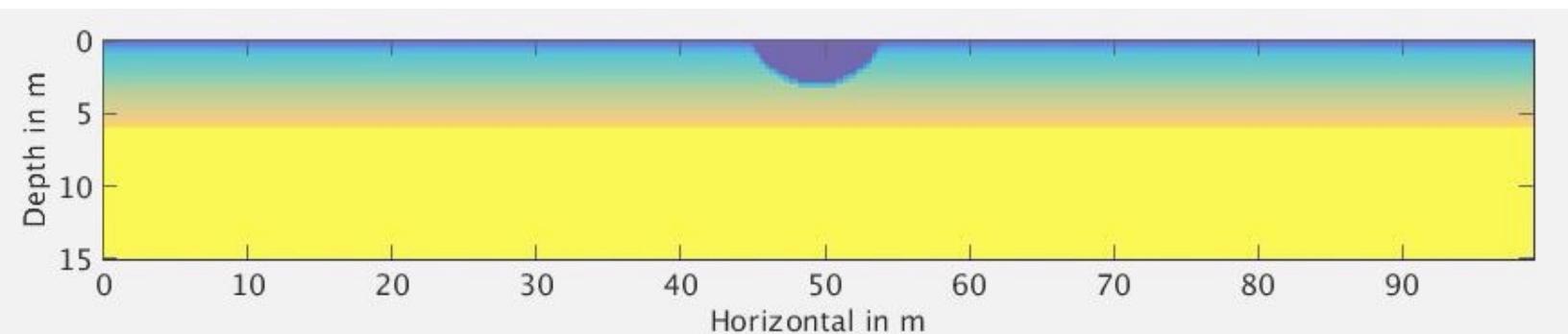
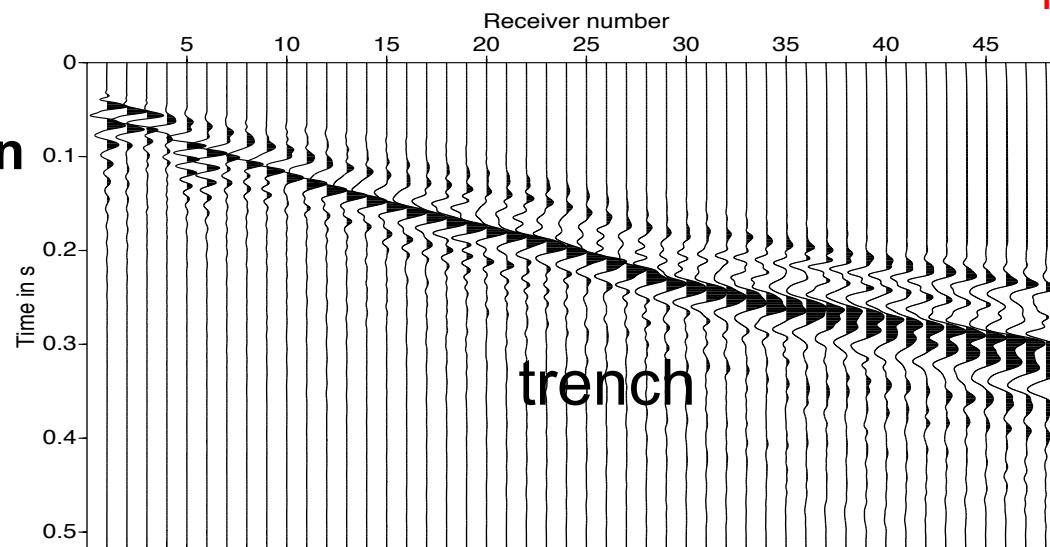


(Wittkamp & Bohlen, 2016)

# Love waves



Rheinstetten  
field data



# FWI configuration

- **Multi-stage approach:**

- Gradually increasing frequency content: 4 Hz – 130 Hz
  - Parameter classes: 1.)  $v_S$  &  $v_P$ , 2.)  $v_S$  &  $v_P$  &  $\rho$

- **Source time function estimation:**

- Stabilized deconvolution (Pratt, 1999; Groos et al., 2014)
  - Offset range: 5 m – 10 m

- **Optimization:** L-BFGS and subsequently CG

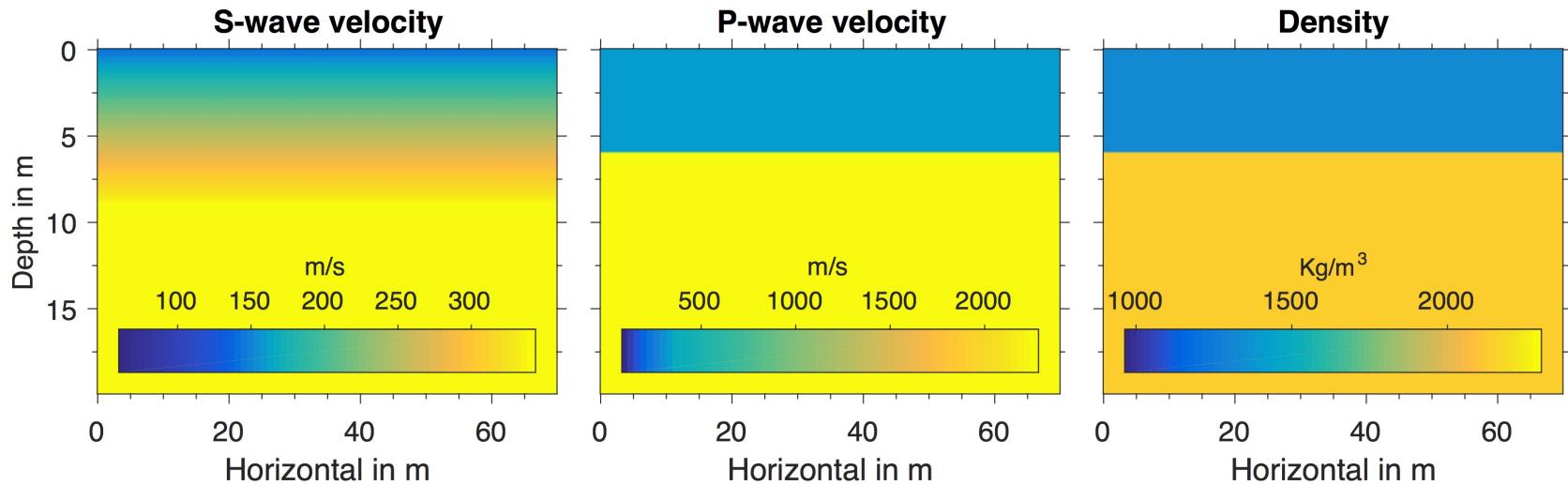
## FWI studies:

- A) Individual Love wave FWI
- B) Individual Rayleigh wave FWI
- C) Simultaneous joint FWI

(Wittkamp & Bohlen, 2016)

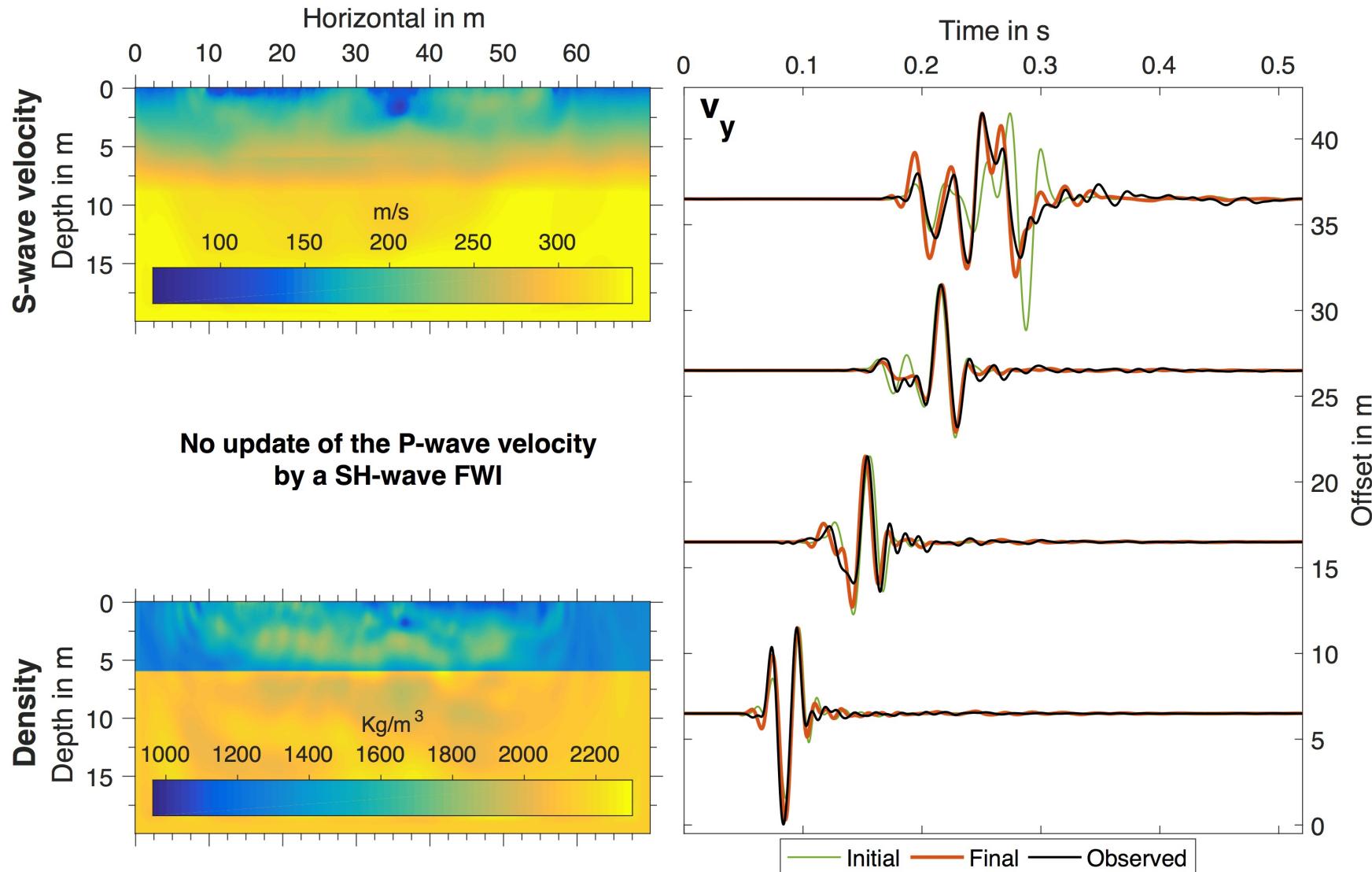
# Initial model

- **S-wave velocity:** Educated guess by dispersion spectra
- **P-wave velocity:** P-wave travel time analysis
- **Density:** Gardner's relation from the P-wave velocity
- **Attenuation:**  $Q = 15$  by grid-search

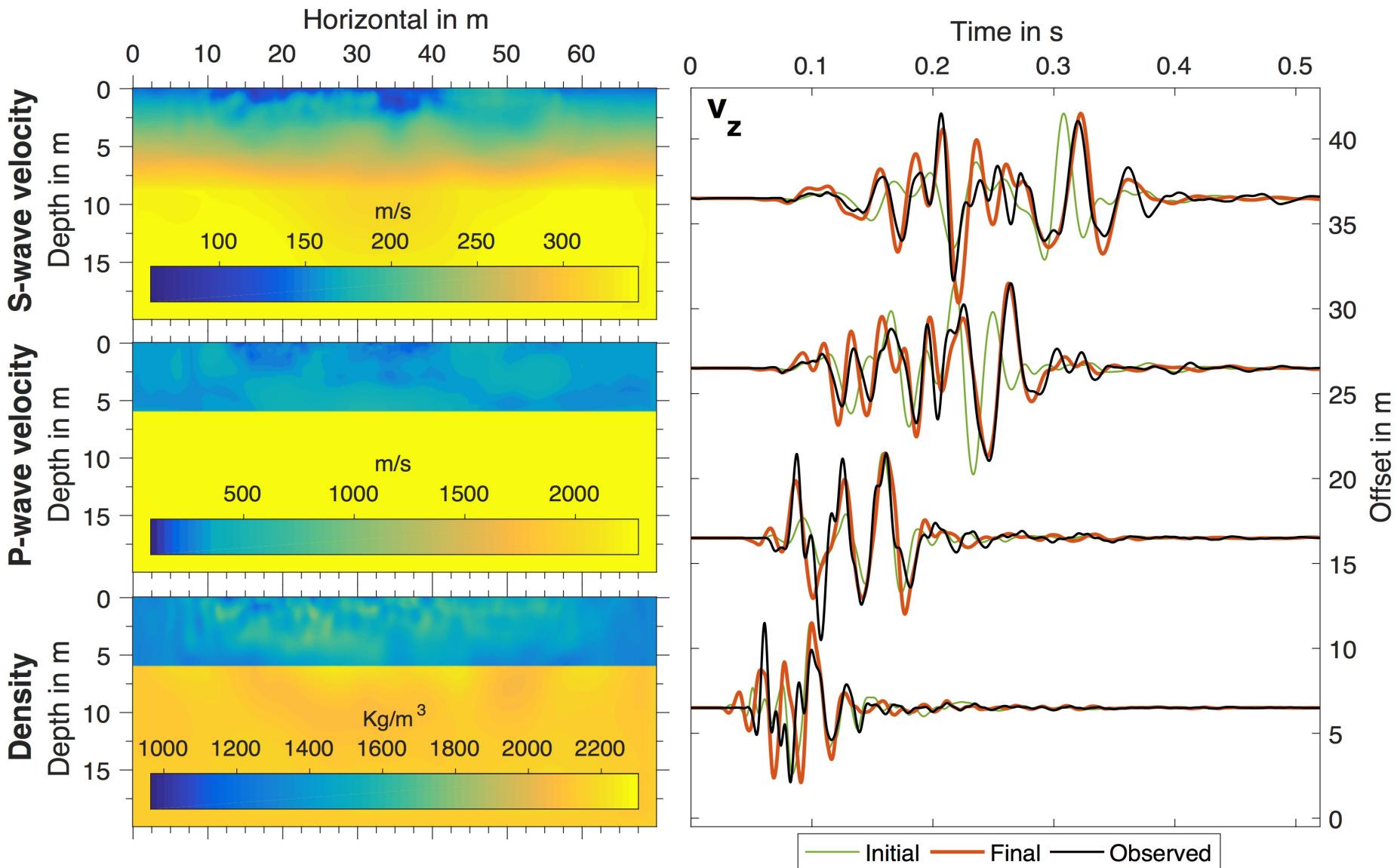


⇒ Predicts all main phases of the recorded wave field (*Wittkamp & Bohlen, 2016*)

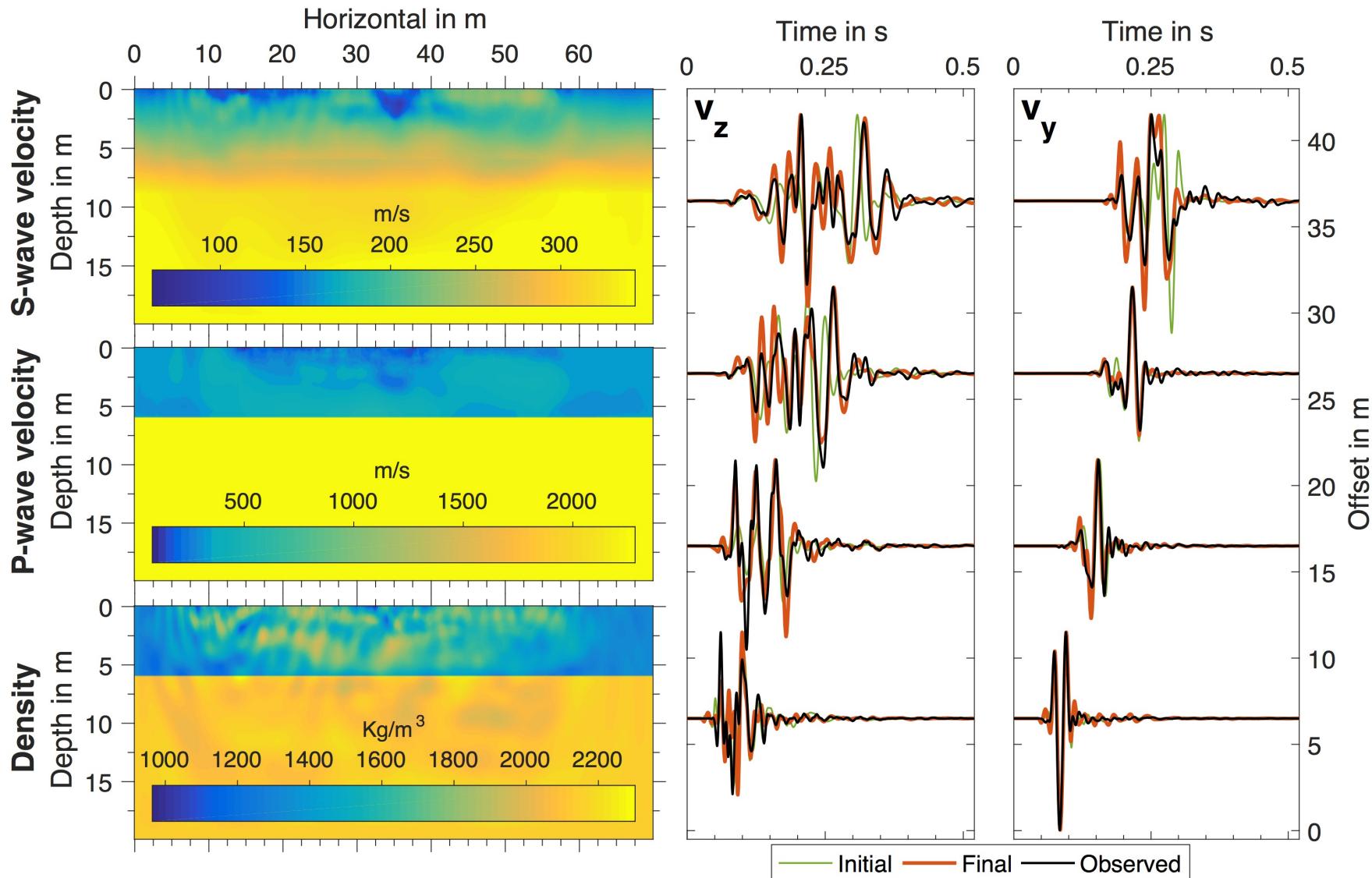
# FWI of Love waves



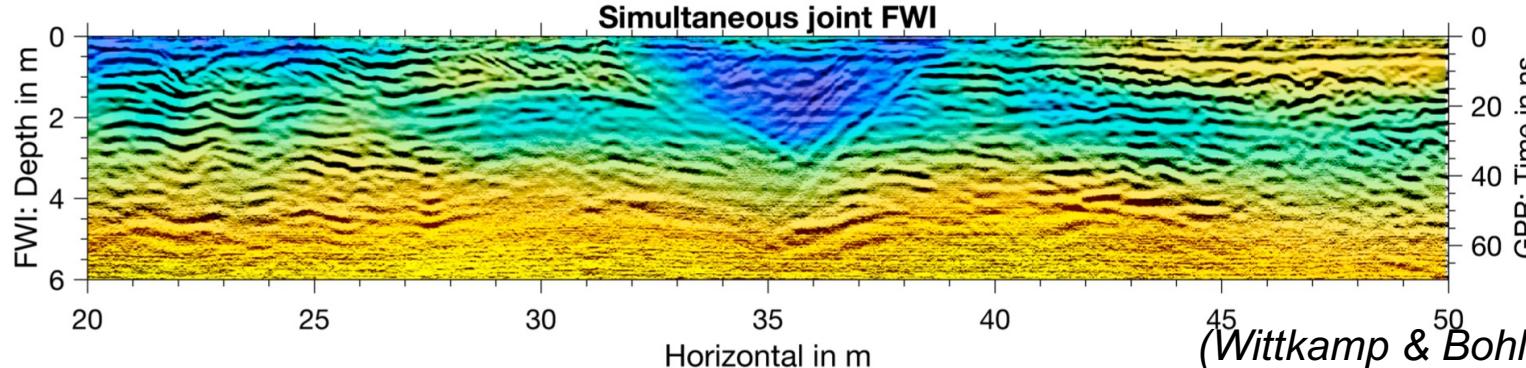
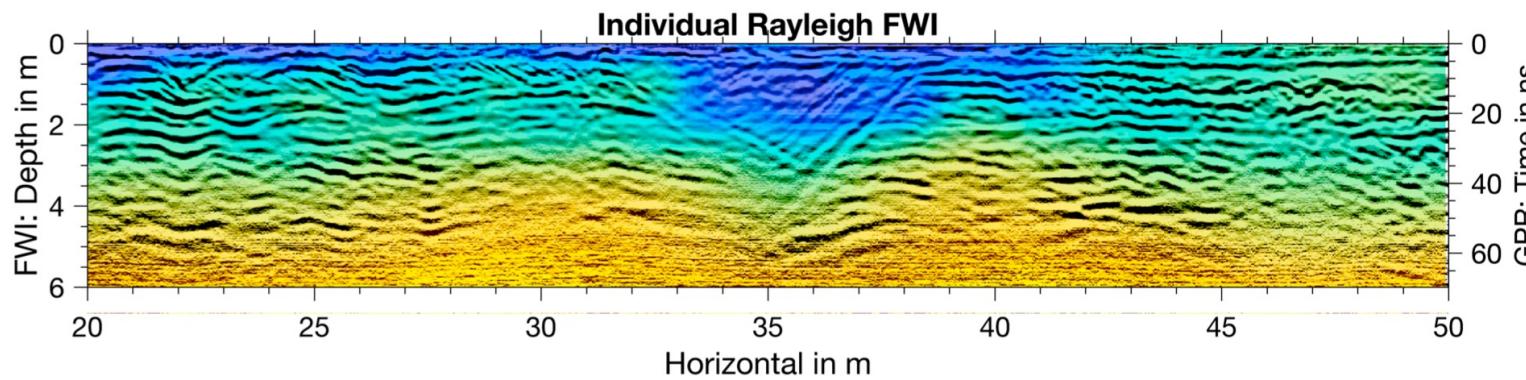
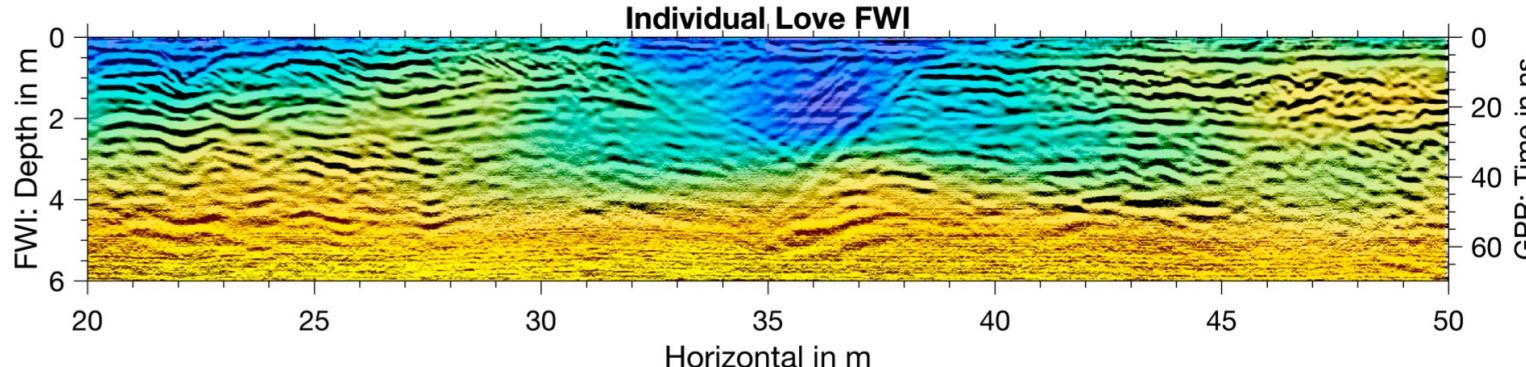
# FWI of Rayleigh waves



# Joint FWI of Rayleigh and Love waves



# Comparison of S-wave velocity models



(Wittkamp & Bohlen, 2016)

# Outline

1 Introduction

2 Methodology

- FD Simulation
- Elastic FWI
- Geometrical spreading correction
- Attenuation

3 2D Field data applications

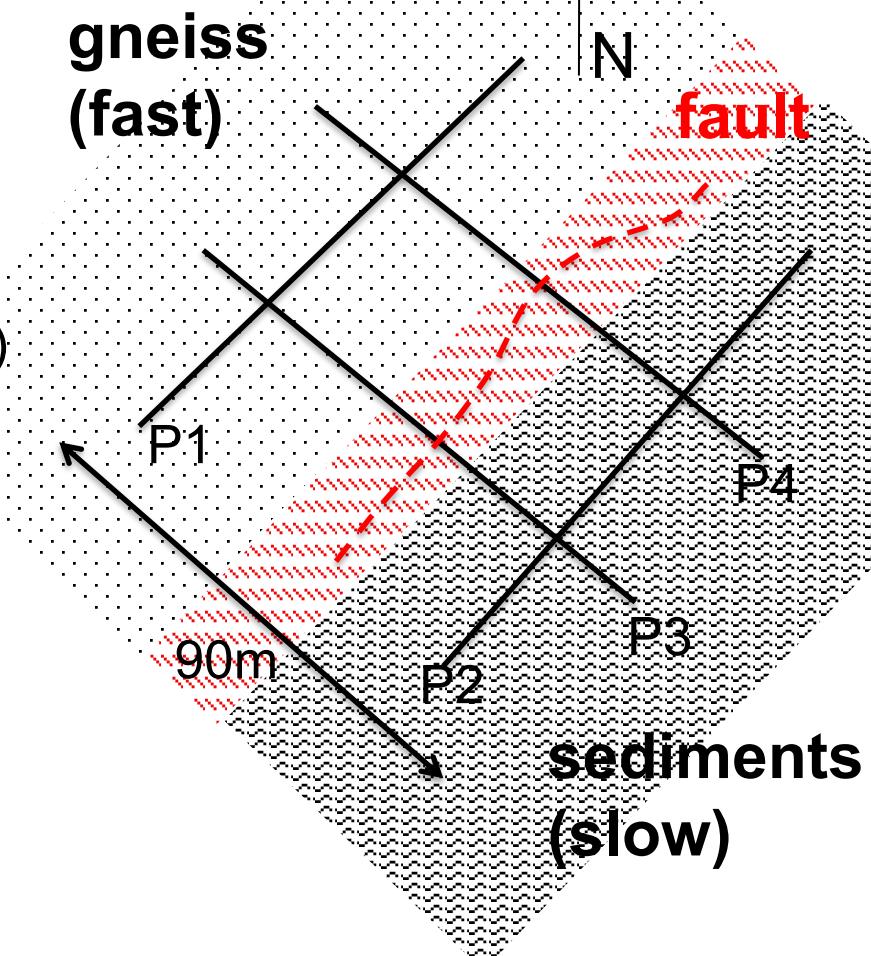
- Layered subsurface
- 2-D local trench
- **Fault Zone**

4 Towards 3D viscoelastic FWI of 3D 9-C field data

5 Summary

### 3. Example: Fault Zone

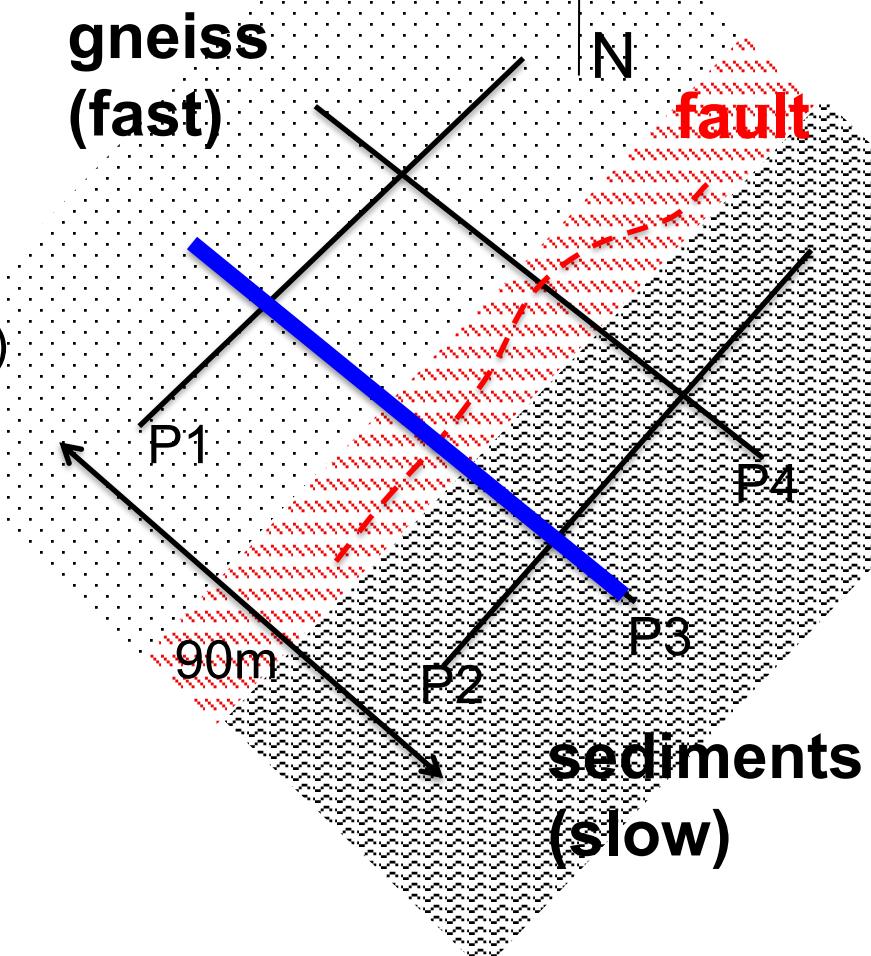
- 2D structure across fault at the southern rim of the Taunus near Frankfurt (Main), Germany
- NW: sericite-gneiss (fast > 300m/s)
- SE: sediments (slow < 300 m/s)
- 4 linear profiles
  - Vertical geophones every 1m
  - Hammer blows every 4m
- Geophysical measurements show small lateral variations **parallel** to the fault



(Schäfer, 2014)

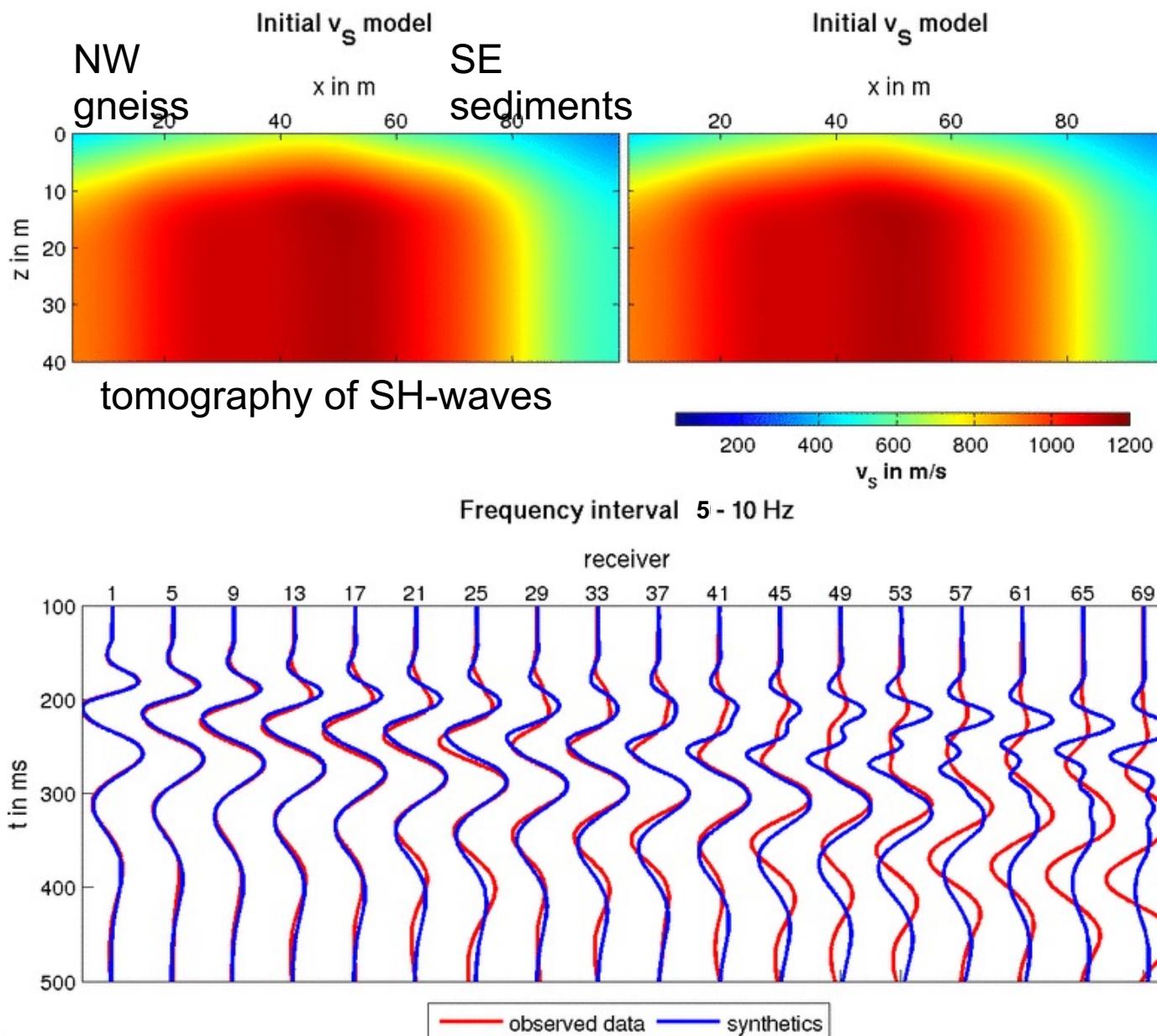
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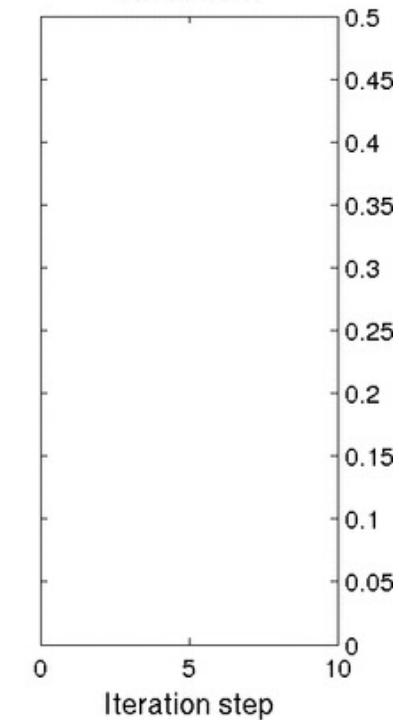


(Schäfer, 2014)

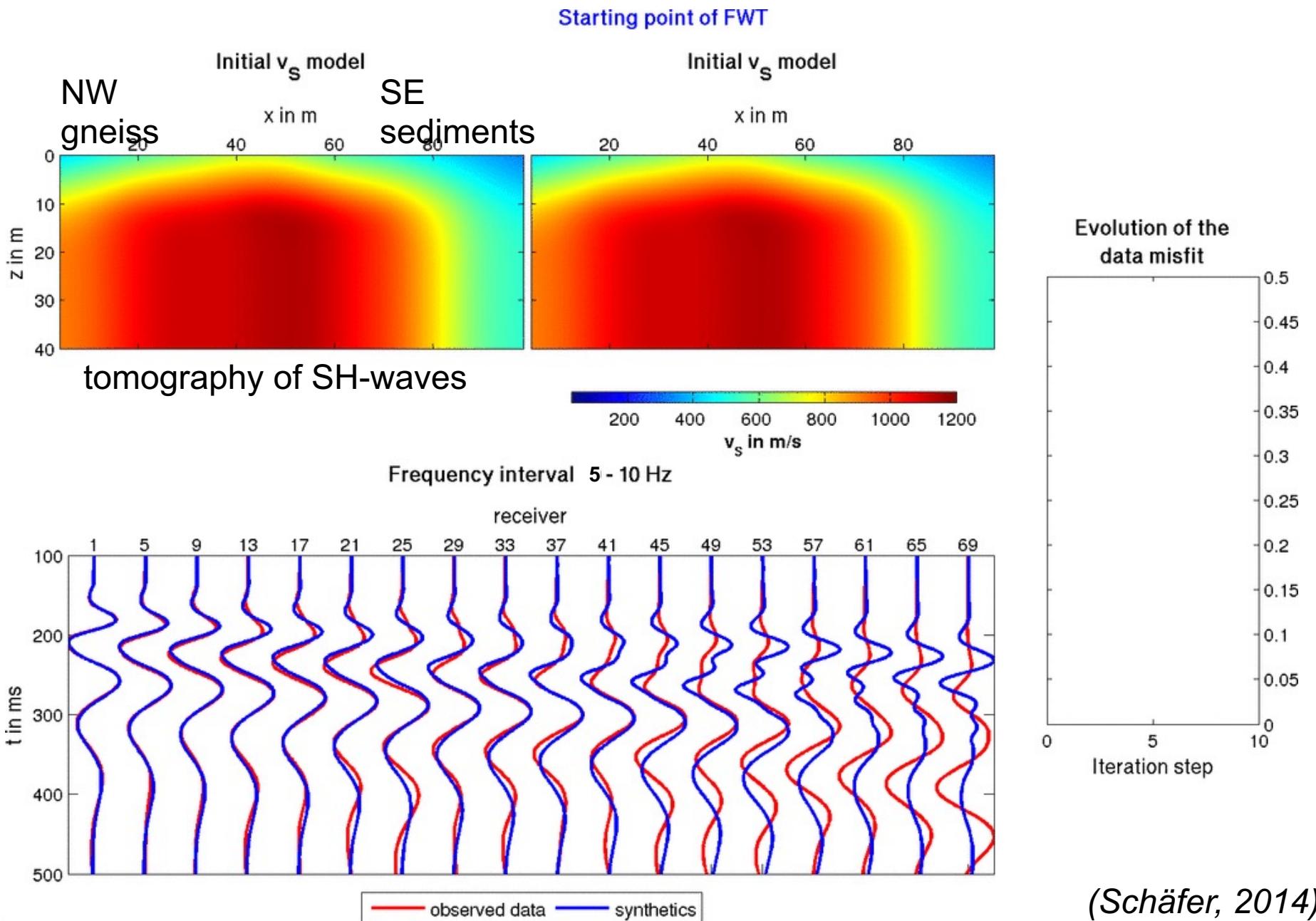
### Starting point of FWI



**Evolution of the data misfit**

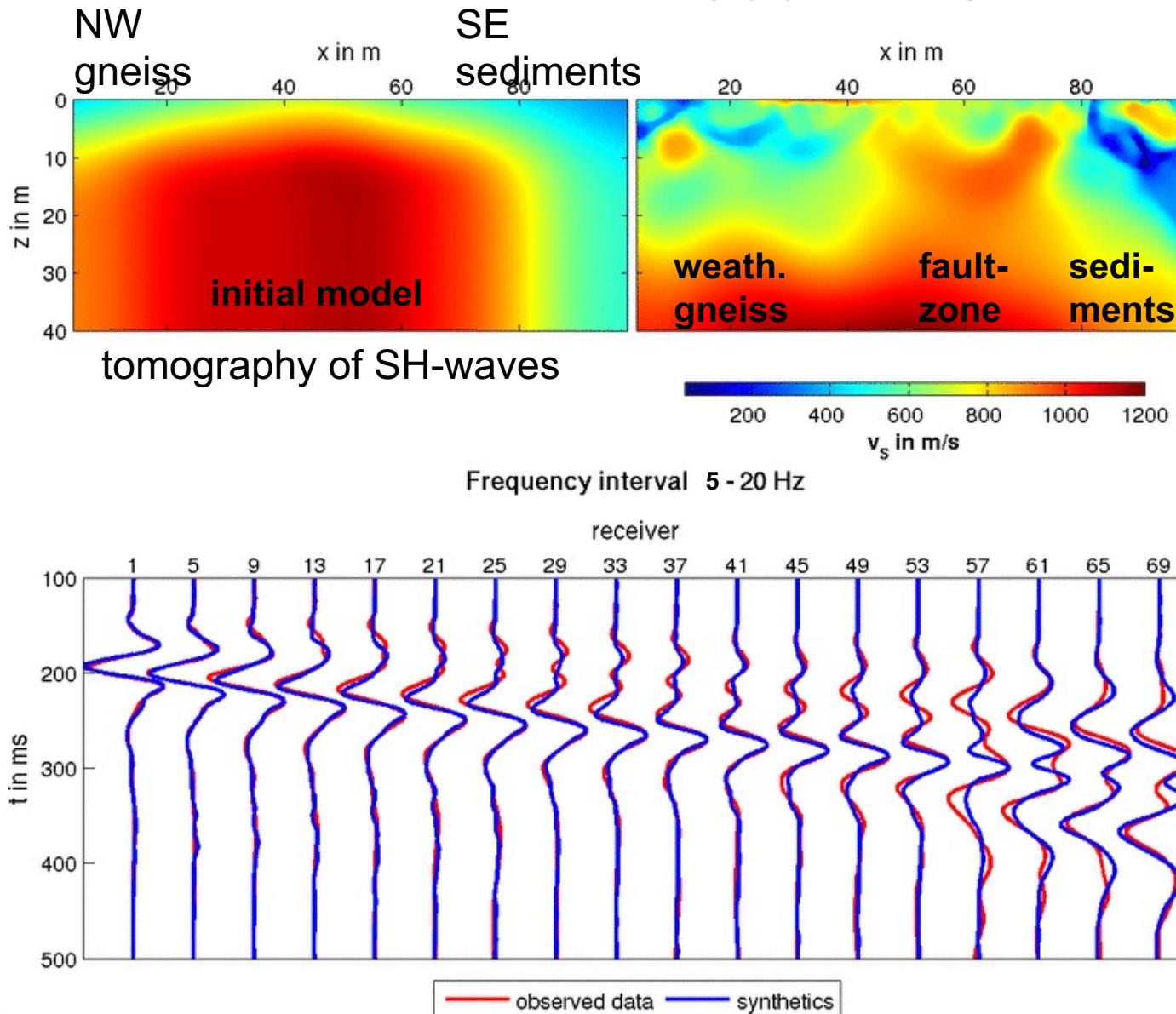


(Schäfer, 2014)

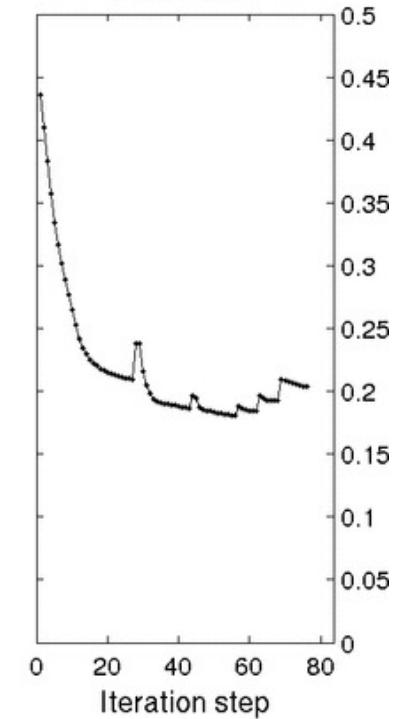


(Schäfer, 2014)

# FWI Result <20 Hz



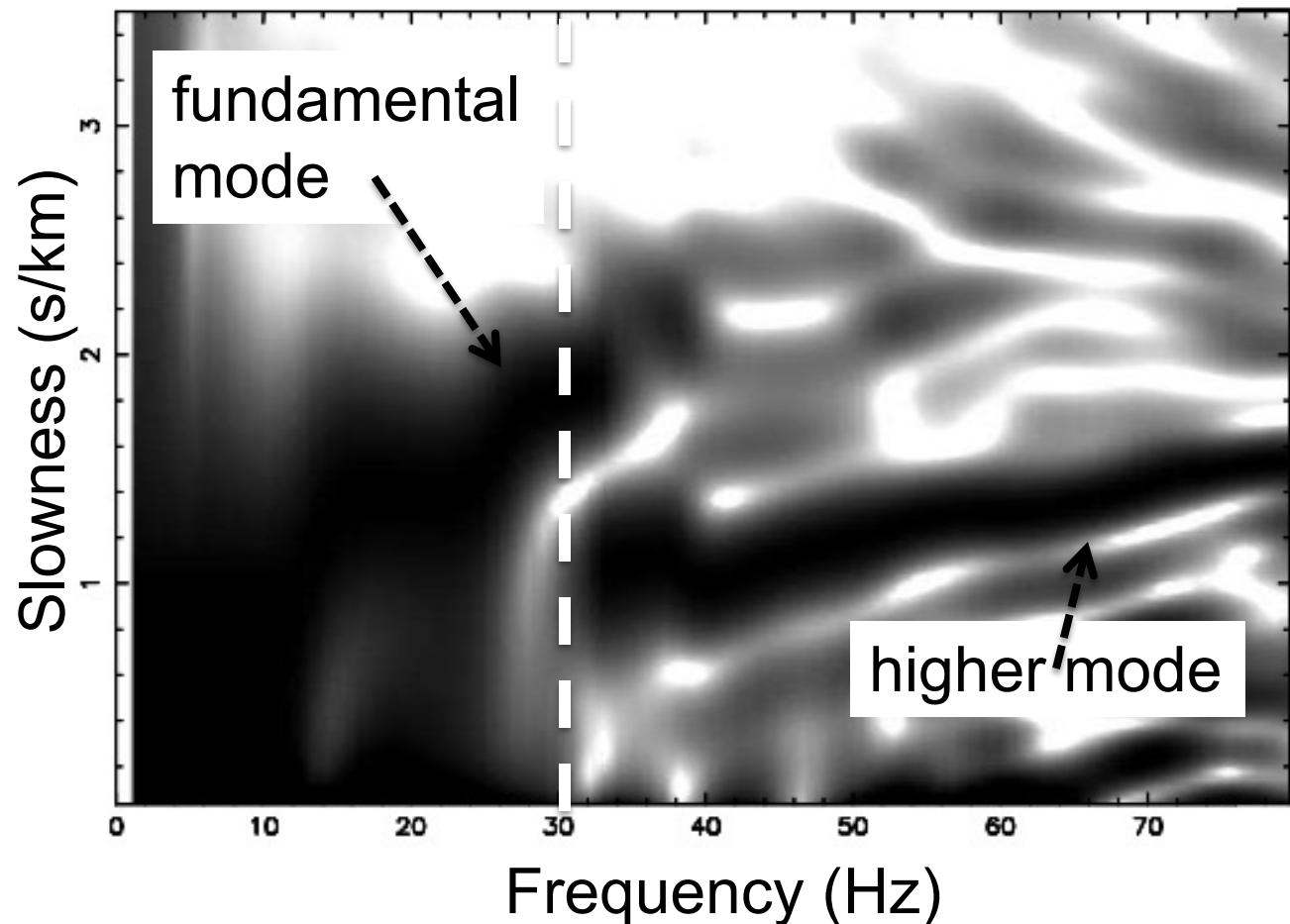
Evolution of the data misfit



(Schäfer, 2014)

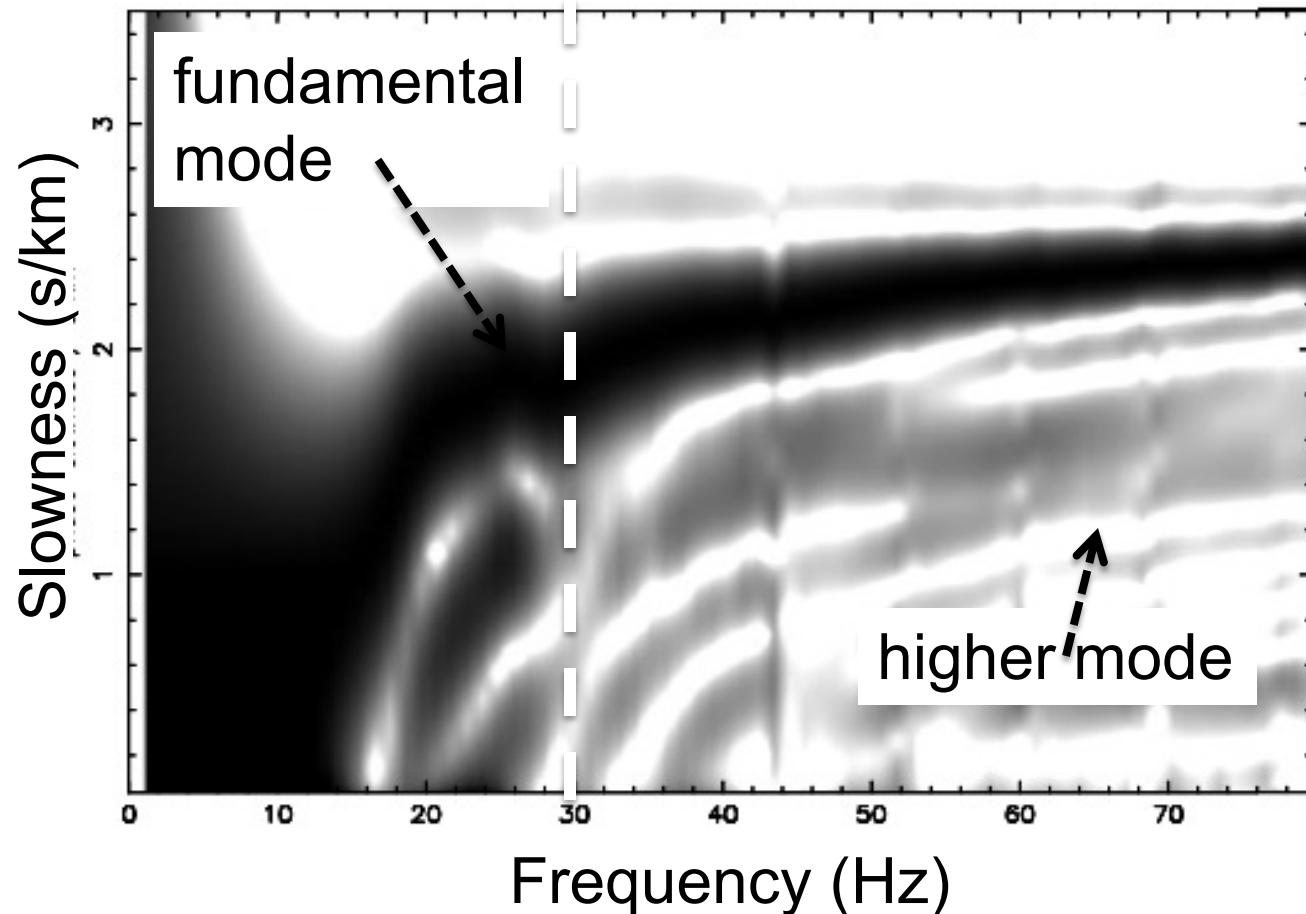
# Slowness-frequency spectra

Field data



# Slowness-frequency spectrum

Starting model



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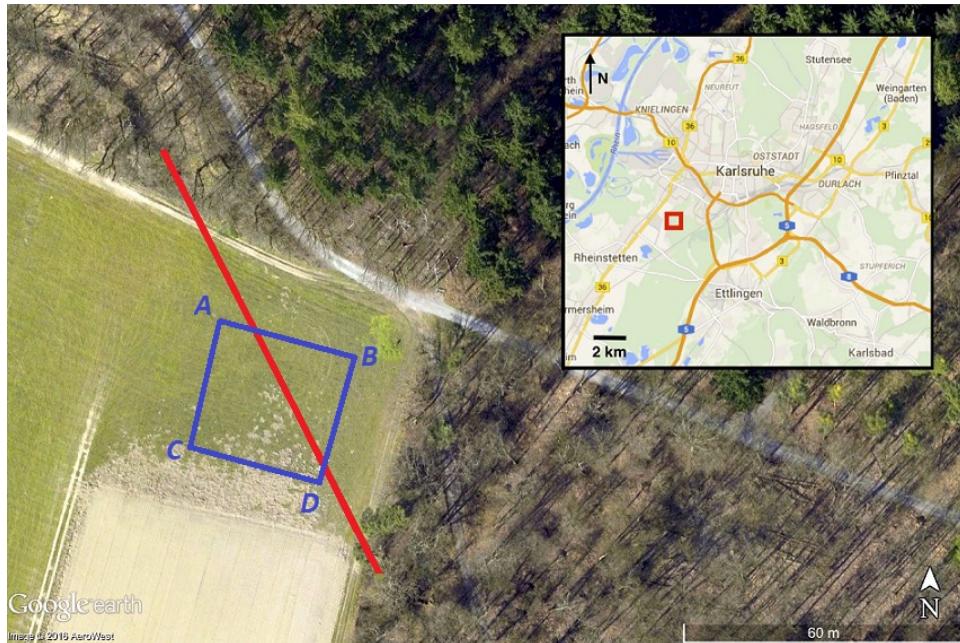
- Layered subsurface
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4 **Towards 3D viscoelastic FWI of 3D 9-C field data**

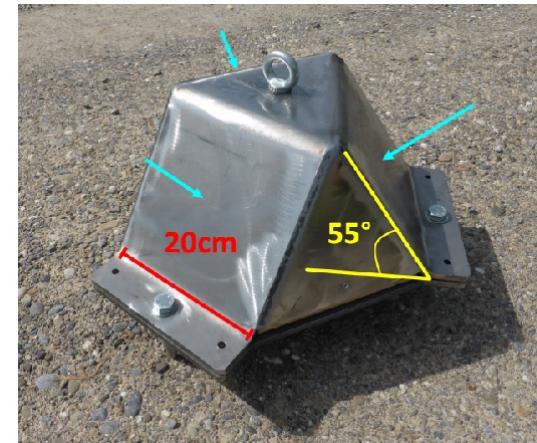
5 Summary

# 9-C Acquisition

Airfield Rheinstetten



3-C Galperin-Source



3-C Geophone



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5 **Summary**

# Summary

- 2-D elastic FWI is applicable to field data and can infer lateral variations of Vs
- Pre-processing:
  - geometrical spreading correction
  - a priori Q-estimation
- Elastic FWI
  - misfit of normalized seismograms
  - source signal inversion
  - viscoelastic forward modelling
- Outlook
  - Inversion of attenuation
  - 3-D elastic FWI

# Thank you for your attention!



# References

- Field data applications
  - Schäfer, M.: *Application of full-waveform inversion to shallow-seismic Rayleigh waves on 2D structures*. Dissertation, Karlsruher Institut für Technologie, 2014.
  - Groos, L.: *2D full waveform inversion of shallow seismic Rayleigh waves*. Dissertation, Karlsruher Institut für Technologie, 2013.
  - L. Groos, M. Schäfer, T. Forbriger, T. Bohlen. *Application of a complete workflow for 2D elastic full-waveform inversion to recorded shallow-seismic Rayleigh waves*. Geophysics 82(2), R109-R117, 2017.
  - F. Wittkamp, T. Bohlen. *Field Data Application of Individual and Joint 2-D Elastic Full Waveform Inversion of Rayleigh and Love Waves*. Conference: Near Surface Geoscience 2016.
- Attenuation
  - L. Groos, M. Schäfer, T. Forbriger, T. Bohlen. *The role of attenuation in 2D full-waveform inversion of shallow-seismic body and Rayleigh waves*, Geophysics, Geophysics 79(6):R247-R261, 2014.
- Spreading correction
  - T. Forbriger, L. Groos, M. Schäfer, *Line source simulation for shallow seismic field data. Part 1: Theoretical background*. Geophysical Journal International, 2014.
  - M. Schäfer, L. Groos, T. Forbriger, T. Bohlen, *Line-source simulation for shallow-seismic data. Part 2: Full-waveform inversion – a 2D case study*. Geophysical Journal International, 2014.