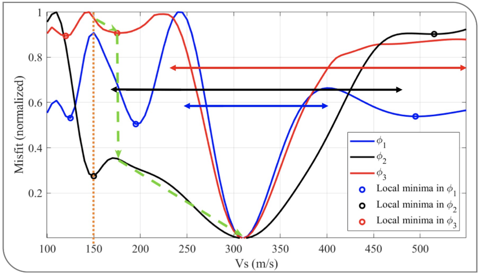
Karlsruhe Institute of Technology

Full Waveform Inversion

Random Objective Waveform Inversion (ROWI)

Thomas Bohlen



Agenda

1. Introduction
2. Multi-Objective Waveform Inversion (MOWI)
3. Random Objective Waveform Inversion (ROWI)
4. Synthetic Application of ROWI
5. 3D 9-C Field Data Application of ROWI
6. Conclusions

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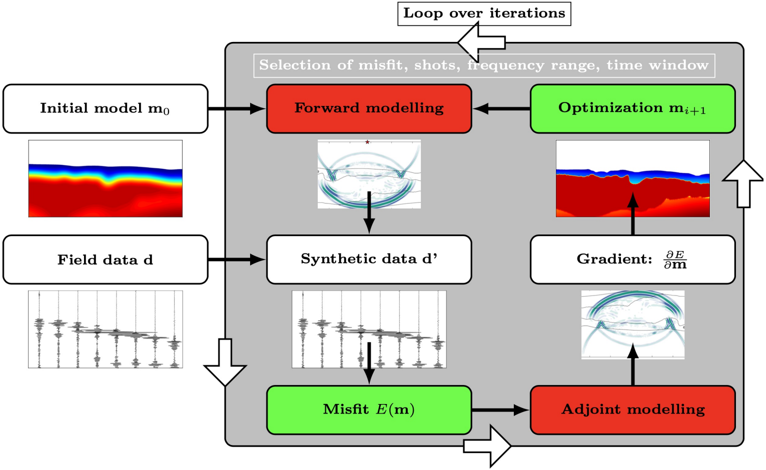
FWI workflow



Random Objective Waveform Inversion (ROWI)

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Problem statement

- High computational cost
- Quantification of uncertainties

Concept of ROWI

1. Choose one shot gather randomly (Shigapov 2019)
2. Choose one misfit function randomly (Pan et al. 2019)
3. Perform one iteration only using preconditioned conjugate gradients

Random Objective Waveform Inversion (ROWI)

General observations

1. Less requirements to the initial model
2. (Much) less computational requirements
3. Obtain mean and standard deviations
4. High data redundancy and sufficient data quality required

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Multi-Objective Waveform Inversion

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- Finding a set of optimal models **m** which minimizes a vector-valued objective function:

$$\mathbf{\Phi}(\mathbf{m}) = \left[\phi_1(\mathbf{m}), \phi_2(\mathbf{m}), \phi_3(\mathbf{m})\right],$$

where

$$\phi_1(\mathbf{m}) = \frac{1}{2}\sum_{src} \|\mathbf{d}(\mathbf{m}) - \mathbf{d}^{obs}\|_2^2, \qquad \text{FWI}$$

$$\phi_2(\mathbf{m}) = \frac{1}{2}\sum_{src} \|\mathbf{e}(\mathbf{m}) - \mathbf{e}^{obs}\|_2^2, \qquad \text{EWI}$$

$$\phi_3(\mathbf{m}) = \frac{1}{2}\sum_{src} \|\mathbf{D}(\mathbf{m}) - \mathbf{D}^{obs}\|_2^2, \qquad \text{SWI}$$

$$\mathbf{e} = \sqrt{\mathbf{d}^2 + \mathcal{H}^2(\mathbf{d})}. \qquad \mathbf{D} = |\mathcal{F}_{2D}(\mathbf{d})|$$

Multi-Objective Waveform Inversion

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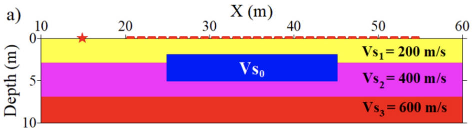
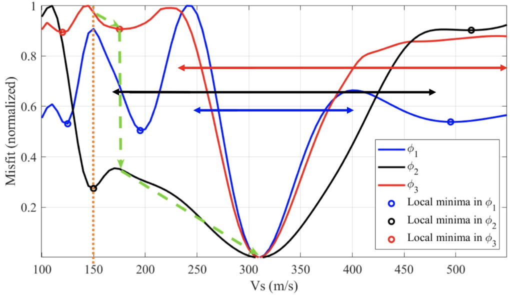
Model

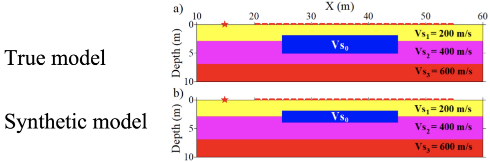


Objective function

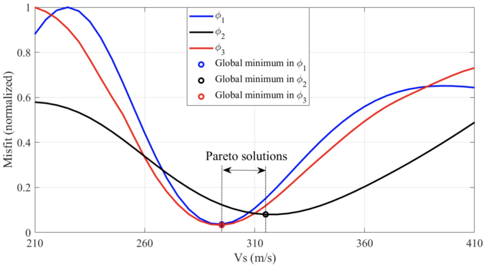


- Change of misfit definition during iterations may help to avoid local minima
- Increase of robustness

Pareto solution: uncertainty estimation



Systematic error between the true model and the synthetic models for simulation



305 ± 5.8 m/s

Pareto solution: It is not possible to reduce one of the multi-objective functions without worsening at least one of the other objective functions

Pareto solution: uncertainty estimation

- Uncertainty estimation

Variance of the Pareto solutions

$$U(\mathbf{m}^{opt}) = \frac{1}{N-1}\sum_{i=1}^{N} |\mathbf{m}_i^{opt} - \boldsymbol{\mu}|^2,$$

Variance of one specific Pareto solution

$$U(\mathbf{m}_j^{opt}) = \frac{1}{N-1}\sum_{i=1}^{N} |\mathbf{m}_i^{opt} - \mathbf{m}_j^{opt}|^2,$$

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Random Objective Waveform Inversion

- Objective function in MOWI

$$\mathbf{\Phi}(\mathbf{m}) = \left[\phi_1(\mathbf{m}), \phi_2(\mathbf{m}), \phi_3(\mathbf{m})\right],$$

$$\phi_1(\mathbf{m}) = \frac{1}{2}\boxed{\sum_{src}}\|\mathbf{d}(\mathbf{m}) - \mathbf{d}^{obs}\|_2^2,\ \phi_2(\mathbf{m}) = \frac{1}{2}\boxed{\sum_{src}}\|\mathbf{e}(\mathbf{m}) - \mathbf{e}^{obs}\|_2^2,\ \phi_3(\mathbf{m}) = \frac{1}{2}\boxed{\sum_{src}}\|\mathbf{D}(\mathbf{m}) - \mathbf{D}^{obs}\|_2^2,$$

Redundant or conflicting

- Objective function in ROWI

$$\underset{\mathbf{m}}{\operatorname{argmin}} \frac{1}{2} \begin{Bmatrix} \|\mathbf{d}_1^{syn}(\mathbf{m}) - \mathbf{d}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{d}_{N_s}^{syn}(\mathbf{m}) - \mathbf{d}_{N_s}^{obs}\|_2^2 \\ \|\mathbf{e}_1^{syn}(\mathbf{m}) - \mathbf{e}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{e}_{N_s}^{syn}(\mathbf{m}) - \mathbf{e}_{N_s}^{obs}\|_2^2 \\ \|\mathbf{D}_1^{syn}(\mathbf{m}) - \mathbf{D}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{D}_{N_s}^{syn}(\mathbf{m}) - \mathbf{D}_{N_s}^{obs}\|_2^2 \end{Bmatrix}$$

Random Objective Waveform Inversion

- Optimization in ROWI

$$\underset{\mathbf{m}}{\operatorname{argmin}} \frac{1}{2} \begin{Bmatrix} \|\mathbf{d}_1^{syn}(\mathbf{m}) - \mathbf{d}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{d}_{N_s}^{syn}(\mathbf{m}) - \mathbf{d}_{N_s}^{obs}\|_2^2 \\ \|\mathbf{e}_1^{syn}(\mathbf{m}) - \mathbf{e}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{e}_{N_s}^{syn}(\mathbf{m}) - \mathbf{e}_{N_s}^{obs}\|_2^2 \\ \|\mathbf{D}_1^{syn}(\mathbf{m}) - \mathbf{D}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{D}_{N_s}^{syn}(\mathbf{m}) - \mathbf{D}_{N_s}^{obs}\|_2^2 \end{Bmatrix}$$

We ***randomly*** choose one of the 3**Ns* objective functions at each iteration and solve it with a preconditioned steepest descent method

$$\mathbf{m}_{k+1} = \mathbf{m}_k - \alpha H_a^{-1} \delta \mathbf{m}_k$$

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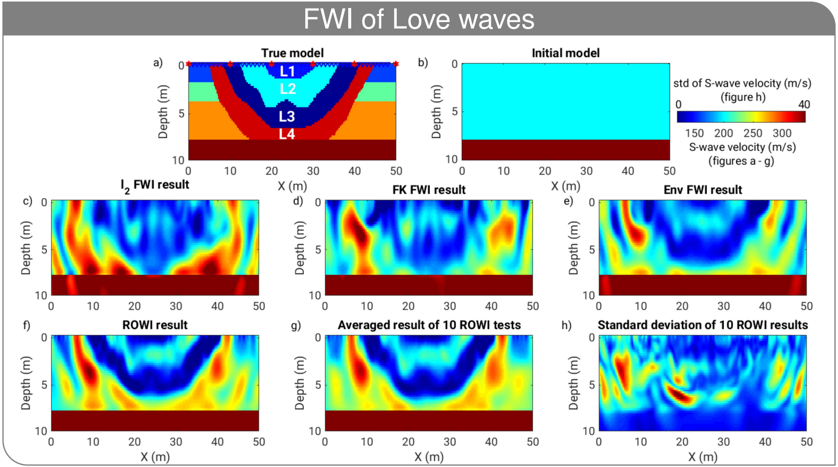
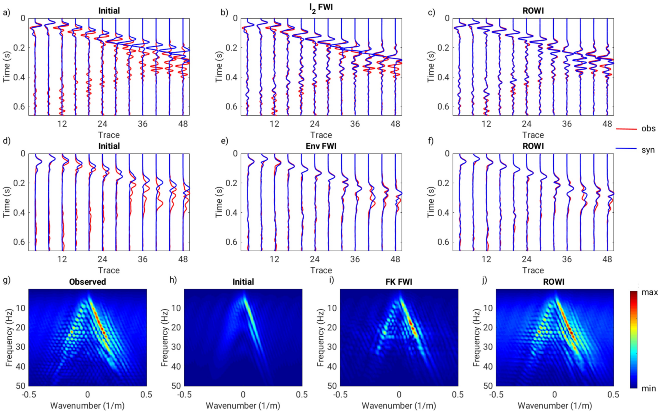
Synthetic application of ROWI



(Pan et al. 2020)

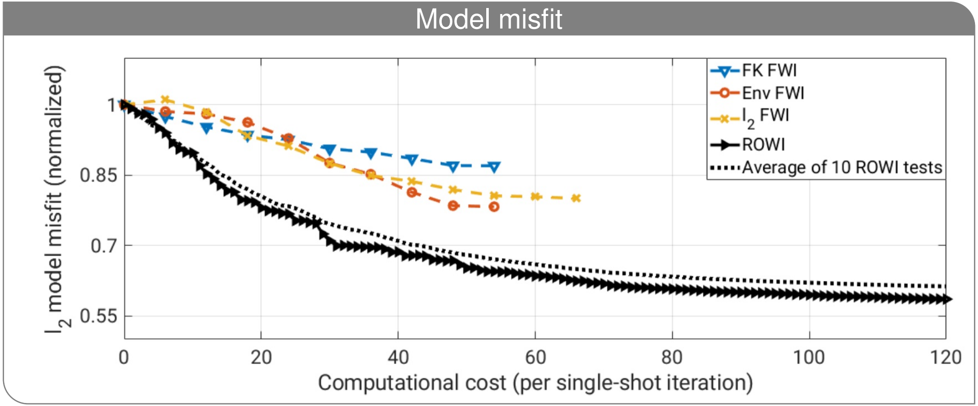
Synthetic application of ROWI

FWI of Love waves



(Pan et al. 2020)

Synthetic application of ROWI



(Pan et al. 2020)

Randon Objective Waveform Inversion (ROWI)

Conclusions from synthetic application

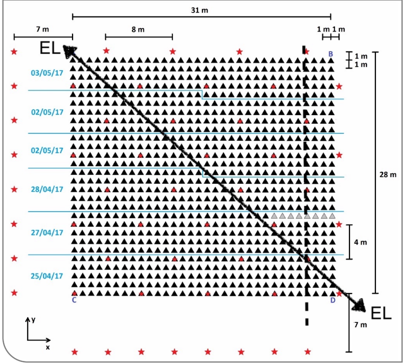
- More robust against initial model
- Faster convergence
- Mean and standard deviations of reconstructed parameters

Agenda

3D 9C Acquisition 2017

Field measurement

Geometry



Area 31m x 27 m, 52 3-C Galpherin source locations (”UVW”), 896 3-C geophones (4.5 Hz, ”XYZ”), 6 days, repetition of all source positions for each geophone patch

(Schaneng 2017)

L2 FWI (full 9C data)

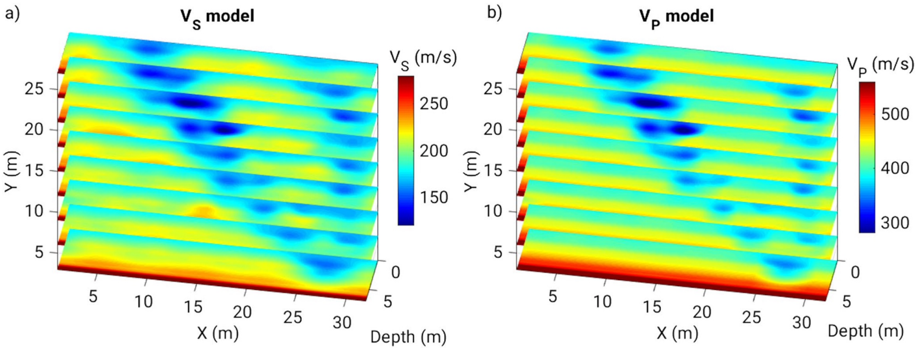
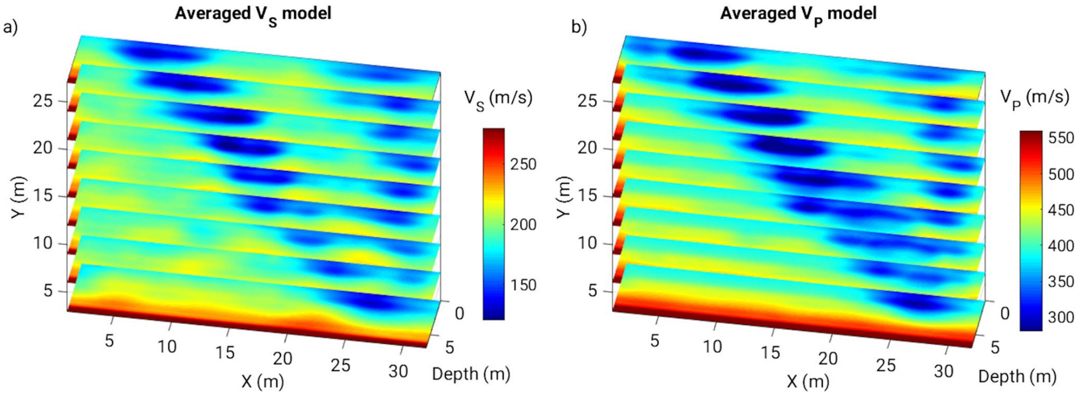


Figure 6. Conventional least squares full-waveform inversion result in the field example. (a and b) represent the final V_S and V_P models, respectively.

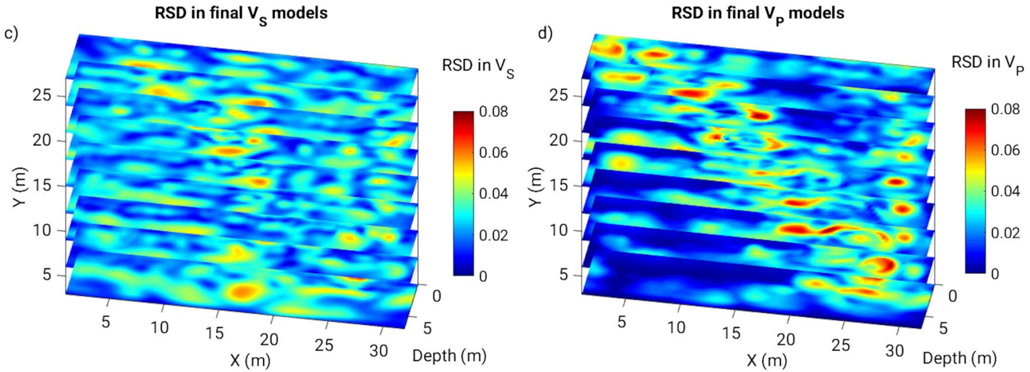
(Pan et al. 2021)

ROWI (full 9C data)



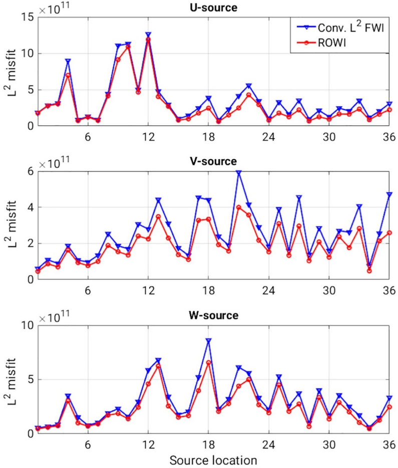
(Pan et al. 2021)

ROWI: Relative Standard Deviation (RSD)



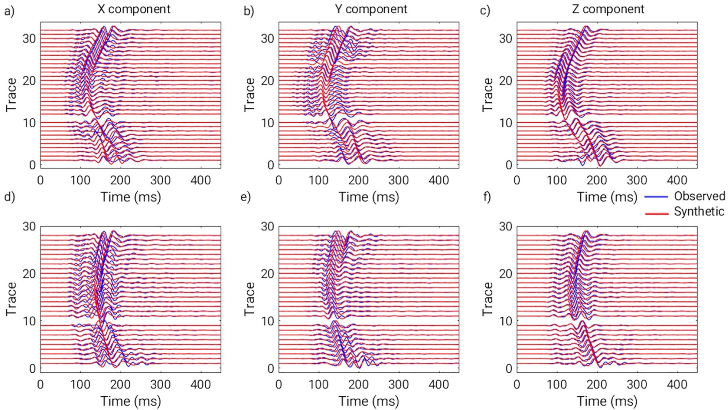
(Pan et al. 2021)

ROWI misfit



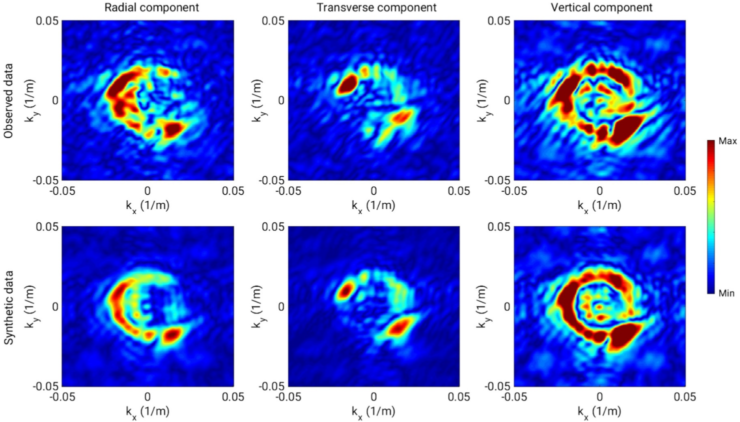
(Pan et al. 2021)

ROWI: Seismogram comparisons



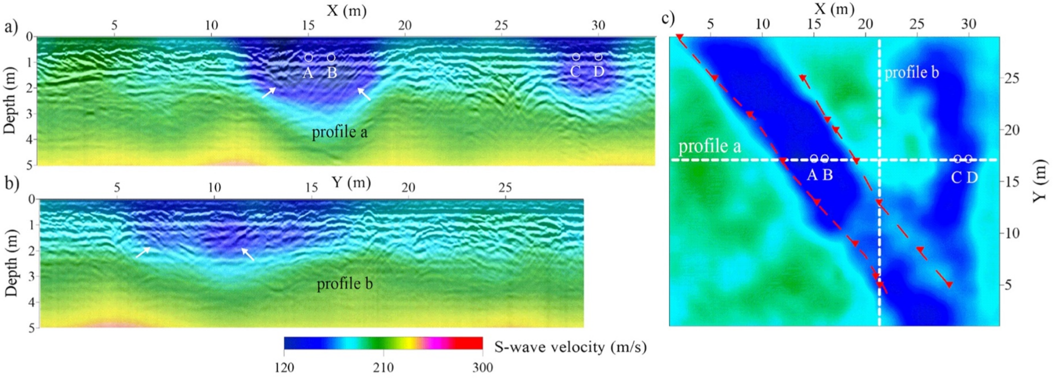
(Pan et al. 2021)

ROWI: Comparison of FK spectra



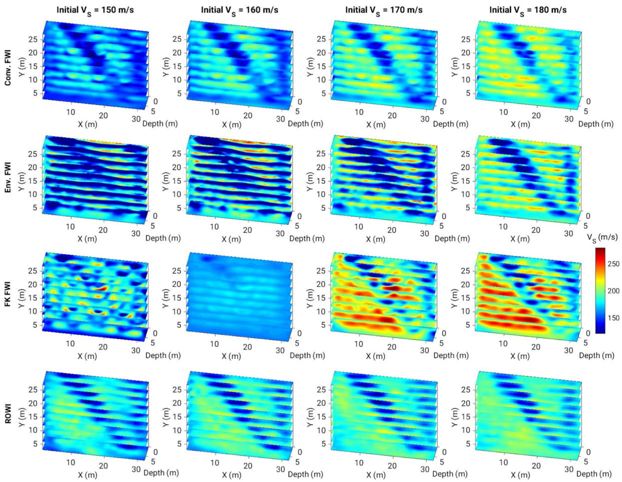
(Pan et al. 2021)

ROWI: Comparison with GPR profiles



(Pan et al. 2021)

Robustness of ROWI



(Pan et al. 2021)

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Conclusions from synthetic and field data applications

- More robust against poor initial models
- Lower computational requirements
- Mean and standard deviations of reconstructed parameters
- Not applicable in case of poor data quality (not shown)

Thank you for your attention

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References

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