Exercise to the Lecture: Intro. to Cosmology
KIT, Wintersemester 2022/2023

Prof. G. Drexlin, A. Huber, A. Elykov, H.-S. Wu

Karlsruher Institut für Technologie

Lectures	Tues. 11:30-13:00, kl. Hörsaal A
Excerices	Thurs.14:00-15:30, kl. Hörsaal A
ILIAS	`https://ilias.studium.kit.edu/goto.php?target=crs_1945182&client_id=produktiv`

Sheet 4 – Due 12.01.2023

1) **Big Bang Nukleosynthesis**

In the early universe, free protons and neutrons are in thermal equilibrium via the reactions

$$\begin{aligned} \mathrm{n} + \nu_\mathrm{e} &\leftrightarrow \mathrm{p} + \mathrm{e}^- \\ \mathrm{p} + \bar{\nu}_\mathrm{e} &\leftrightarrow \mathrm{n} + \mathrm{e}^+ \\ \mathrm{e}^- + \mathrm{e}^+ &\leftrightarrow \nu_\mathrm{e} + \bar{\nu}_\mathrm{e}\ . \end{aligned} \tag{1}$$

The neutrinos freeze out after a short time, approximately when the interaction rate $\Gamma = n_\nu \langle \sigma v \rangle$ equals the expansion rate H. The expansion rate in the radiation dominated universe is determined by the energy density of relativistic particles $\rho_r = \frac{g\pi^2 T^4}{30}$, with $g = 2 + \frac{7}{8}(4+6)$. The interaction cross section is $\sigma \approx 1{,}3 \cdot 10^{-21} \left(\frac{T}{\mathrm{MeV}}\right)^2 \mathrm{MeV}^{-2}$ and the particle number density of relativistic fermions in thermal equilibrium is $n_\nu \approx \frac{9}{10\pi^2} g' T^3$ mit $g' = 6$.
Note: consider the Friedmann equation in its most general form

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{2}$$

and assume that in the radiation-dominated universe the density ρ_r dominates over all other factors.

(a) Which particles contribute to the relativistic degrees of freedom for the factor g? At what temperature do the neutrinos freeze out?

(b) 0.01 seconds after the Big Bang, the temperature is 10^{11} K; the universe is still radiation-dominated at this time. How long will it take for the universe to cool by expansion to the point where the neutrinos freeze out? Note here that the temperature at this stage of the universe decreases with time over $T \sim t^{-1/2}$.

(c) What was the ratio between protons and neutrons 0.01 seconds after the Big Bang and what was it at the time of the neutrino freeze-out? How does it evolve after that? After that, the ratio does not track temperature, but the number of neutrons decreases due to β decay of free neutrons with a lifetime of about 880s (15 min.).

(d) Deuterium has a binding energy of about $B_\mathrm{D} = 2.2\,\mathrm{MeV}$. Its synthesis starts approximately at $\mathrm{e}^{-B_\mathrm{D}/k_\mathrm{B}T} = \eta$ with $\eta \approx 5 \cdot 10^{-10}$. At what temperature and after what time is this condition satisfied? What is the ratio between protons and neutrons at this time? Why must the temperature be considerably lower than 2.2 MeV for the synthesis of deuterium to begin?

(e) What is the element composition at the beginning of the nucleosynthesis, what at the end? At which element do all relevant reaction paths of nucleosynthesis end? Are there elements that are passed through all reaction pathways?

(f) Almost all neutrons are bound in helium nuclei during nucleosynthesis. Calculate the resulting particle number and mass ratio of helium to hydrogen from the ratio of protons to neutrons at the beginning of nucleosynthesis.