Exercise to the Lecture: Intro. to Cosmology
KIT, Wintersemester 2022/2023

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Lectures	Tues. 11:30-13:00, kl. Hörsaal A
Excerices	Thurs.14:00-15:30, kl. Hörsaal A
ILIAS	`https://ilias.studium.kit.edu/goto.php?target=crs_1945182&client_id=produktiv`

Sheet 5 – Due 02.02.2023

1) Doppler shift of the CMB

(a) In the lecture it was shown that the CMB spectrum has the properties of an ideal blackbody radiator. Show that an observer moving with velocity β in direction θ relative to the CMB sees the blackbody spectrum with temperature T in the rest frame as a blackbody spectrum with temperature

$$T' = \frac{T}{\gamma\,(1-\beta\cos\theta)} \tag{1}$$

due to the Doppler effect.

Hint: Use the Lorentz invariance of $\frac{1}{\nu^3}\frac{\mathrm{d}n}{\mathrm{d}\nu} \sim \left(e^{-E/k_\mathrm{B}T}-1\right)^{-1}$.

(b) From the measured cosmic background radiation dipole anisotropy of 3.365 ± 0.027 mK (COBE[1]), determine the velocity of the Earth relative to the CMB rest frame. What is the origin of this relative velocity?

2) Cosmic Background Radiation

In 1990, the cosmic background radiation spectrum was measured with the satellite-based spectrometer COBE (**CO**smic **B**ackground **E**xplorer). It is a nearly perfect blackbody spectrum as shown in Figure 1. The spectral energy density $u(\nu, T)$ is given by Planck's radiation law:

$$u(\nu, T)\mathrm{d}\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{\mathrm{e}^{\frac{h\nu}{k_\mathrm{B}T}}-1}\mathrm{d}\nu\,. \tag{2}$$

(a) Determine the temperature of the cosmic background radiation from figure 1.

(b) Estimate the total power of cosmic background radiation on Earth by graphically integrating the spectrum from figure 1. What temperature would be required for the power on Earth to equal the power of solar radiation?

(c) Determine the photon density of the background radiation by putting Planck's radiation law into the form $u(E,T)\mathrm{d}E$ and integrating over $n(E,T)\mathrm{d}E = \frac{u(E,T)}{E}\mathrm{d}E$. The integral can be solved with $\int_0^\infty \frac{x^2}{\mathrm{e}^x-1}\mathrm{d}x \approx 2.4$.

3) Acoustic Peak of the Multipole Spectrum

In the lecture the power spectrum was introduced via the multipole order ℓ. From the WMAP results it can be seen that the multipole distribution of the CMB has various characteristics. The first peak of the spectrum corresponds to the acoustic peak. Calculate the position of the acoustic peak in the multipole spectrum of the CMB. To do this, proceed as follows:

[1] Kogut, A., et al. "Dipole anisotropy in the COBE DMR first-year sky maps." arXiv preprint astro-ph/9312056 (1993).

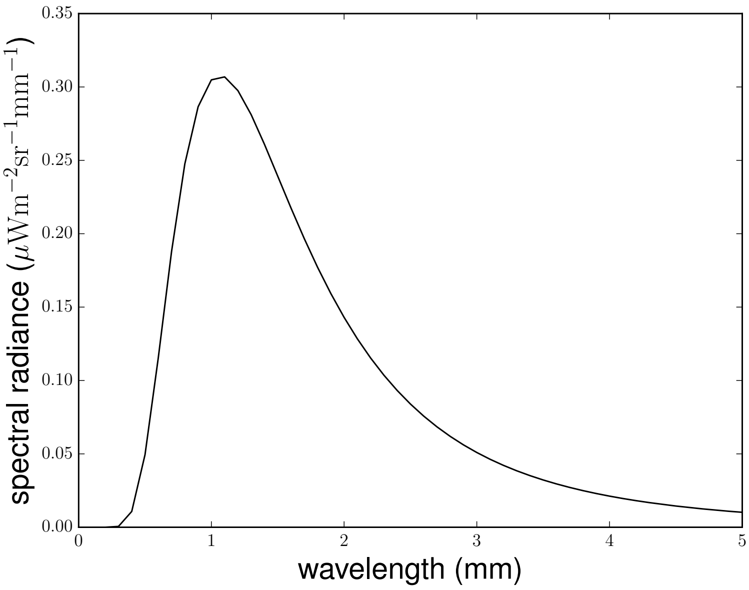

Abbildung 1: Blackbody spectrum of the cosmic background radiation measured with COBE.

(a) The propagation speed of acoustic waves is

$$v_s^2 = \frac{\partial p}{\partial \rho} \quad \text{mit} \quad p = \frac{1}{3}\rho c^2 \; . \tag{3}$$

Determine the speed of propagation of a wave and its horizon until the time of recombination.

(b) Calculate the corresponding present-day magnitude and angular diameter Θ at which an object of this magnitude is seen today. Calculate the multipole order ℓ with $\Theta[°] = 180/\ell$ of the acoustic peak and compare the result with a multipole spectrum from the lecture.