Exercise to the Lecture: Intro. to Cosmology
KIT, Wintersemester 2023/2024

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Lectures	Tues. 11:30-13:00, kl. Hörsaal A
Excerices	Thurs.14:00-15:30, kl. Hörsaal A
ILIAS	`https://ilias.studium.kit.edu/goto.php?target=crs_2238560&client_id=produktiv`
Sheet 5, due date	25.01.2024

1) **Doppler shift of the CMB**

(a) In the lecture it was shown that the CMB spectrum has the properties of an ideal blackbody radiator. Show that an observer moving with velocity β in direction θ relative to the CMB sees the blackbody spectrum with temperature T in the rest frame as a blackbody spectrum with temperature

$$T' = \frac{T}{\gamma\,(1-\beta\cos\theta)} \tag{1}$$

due to the Doppler effect. (Useful, albeit detailed, resources: Planck [1] and COBE [2])

Hint: Use the Lorentz invariance of the photon occupation number which is given by the Bose-Einstein distribution $n = \left(e^{-E/k_\mathrm{B}T} - 1\right)^{-1}$.

(b) From the measured cosmic background radiation dipole anisotropy of $3.365 \pm 0.027\,\mathrm{mK}$ (COBE[2]), determine the velocity of the Earth relative to the CMB rest frame (Hint: Expand equation (1) to first order). What is the origin of this relative velocity?

2) **Cosmic Background Radiation**

In 1990, the cosmic background radiation spectrum was measured with the satellite-based spectrometer COBE (**CO**smic **B**ackground **E**xplorer). It is a nearly perfect blackbody spectrum as shown in Figure 1. The spectral energy density $u(\nu, T)$ is given by Planck's radiation law:

$$u(\nu, T)\mathrm{d}\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{\mathrm{e}^{\frac{h\nu}{k_\mathrm{B}T}} - 1}\mathrm{d}\nu\ . \tag{2}$$

(a) Estimate the temperature of the cosmic background radiation from figure 1 (Hint: graphically extract the peak position).

(b) Estimate the total power of cosmic background radiation on the Earth's surface by graphically integrating the spectrum from figure 1 (rough approximation is okay). What CMB temperature would be required for the power on Earth to equal the power of solar radiation given that the irradiance from the sun on the Earth's surface is about $1\,\mathrm{kW\,m^{-2}}$?

(c) Determine the photon density of the background radiation by expresing Planck's radiation law in terms of energy $u(E, T)\mathrm{d}E$ and integrating over the number density $n(E, T)\mathrm{d}E = \frac{u(E,T)}{E}\mathrm{d}E$. The integral can be solved with $\int_0^\infty \frac{x^2}{\mathrm{e}^x - 1}\mathrm{d}x \approx 2.4$.

[1] Planck Collaboration."Planck 2013 results. xxvii. doppler boosting of the cmb: Eppur si muove." (2014).
[2] Kogut, A., et al. "Dipole anisotropy in the COBE DMR first-year sky maps." arXiv preprint astro-ph/9312056 (1993).

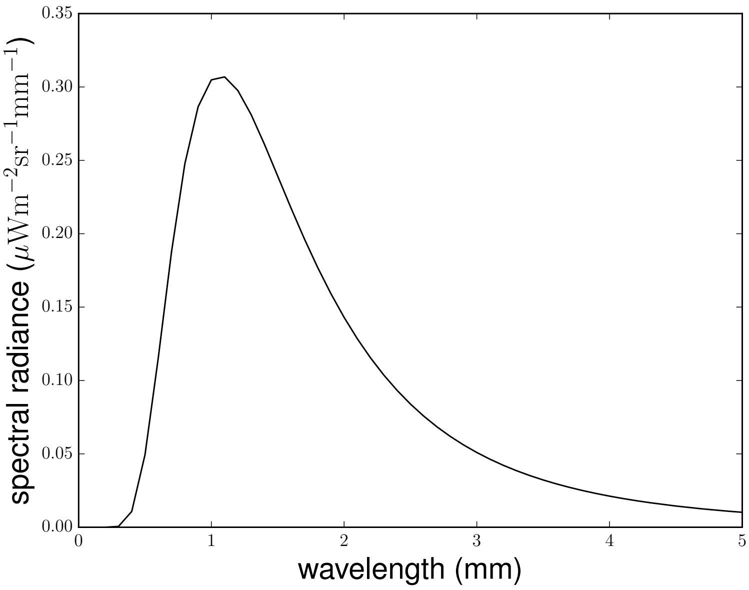

Abbildung 1: Blackbody spectrum of the cosmic background radiation measured with COBE.

3) <u>Acoustic Peak of the Multipole Spectrum</u>

In the lecture the power spectrum was introduced via the multipole order ℓ. From the WMAP results it can be seen that the multipole distribution of the CMB has various characteristics. The first peak of the spectrum corresponds to the acoustic peak. Calculate the position of the acoustic peak in the multipole spectrum of the CMB. To do this, proceed as follows:

(a) The propagation speed of acoustic waves is

$$v_s^2 = \frac{\partial p}{\partial \rho} \quad \text{mit} \quad p = \frac{1}{3}\rho c^2 \, . \tag{3}$$

Determine the speed of propagation of a wave and its horizon until the time of recombination t_r, i.e. the size of the wave in meters at t_r.

(b) Calculate the corresponding present-day magnitude and angular diameter Θ at which an object of this magnitude is seen today (Hint: Find the redshift of recombination from the lecture or elsewhere). Calculate the multipole order ℓ with $\Theta[°] = 180/\ell$ of the acoustic peak and compare the result with a multipole spectrum from the lecture.